

# A REVIEW OF DIFFICULTIES IN ACHIEVING SHORT BUNCHES IN STORAGE RINGS.<sup>1</sup>

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## Abstract

During the last few years, the potential of electron storage rings for the production of short bunches has been thoroughly investigated. The beam-wall interaction in general was reconsidered for 3<sup>rd</sup> generation synchrotron sources. The mechanism of the so-called "microwave" instability, responsible for bunch lengthening and energy widening, was clarified in all details from a theoretical point of view. Predictions were confirmed with results from tracking code simulations and experimental data. A result of these studies is that harmonic cavities are ineffective for either shortening the bunch or substantially raising the instability threshold. Extensive investigations were performed on quasi-isochronous lattices. Many rings were operated with very low momentum compaction optics, above and below transition energy. Experimental results in full agreement with theoretical prediction confirm that unfortunately short bunches are very hard to produce.

## 1 INTRODUCTION

By the time the first 3<sup>rd</sup> generation storage rings were commissioned, the quasi-isochronous tuning of rings appeared promising to produce short electron bunches and accordingly short and intense X-ray pulses. The quasi-isochronous regime has now been tested. Unfortunately, the bunch lengthens strongly with current and its length becomes independent of both the momentum compaction and the energy. The strong defocusing effect of the self-induced voltage can be reversed when the momentum compaction is negative. But, a negative mass instability then starts developing, blowing up the energy spread. For a positive momentum compaction, the energy spread will start widening above the "microwave" instability threshold. The "microwave" instability mechanism was thoroughly investigated again to determine if, implementing harmonic cavities or a passive cavity-like structure, would help to reach higher peak brilliance on storage rings. Unfortunately, none of these structures will allow to boost the peak brilliance to the level of performances announced for linac driven sources.

## 2 SHORT BUNCHES HISTORY

### *Small alpha implicitly from high brilliance*

3<sup>rd</sup> generation synchrotron radiation storage rings have been designed to provide high quality synchrotron light with a brilliance exceeding by 4 orders of magnitude that of 2<sup>nd</sup> generation machines. Such an improvement was made possible by the extensive use of undulators and wigglers. Undulator spectra are highly peaked when the transverse emittances are small. Accordingly emphasis was put to reach the smallest possible emittances on 3<sup>rd</sup> generation rings, typically below 10 nm for the horizontal emittance and less than 1% of that value for the vertical one.

A reduction of the horizontal emittance is achieved by minimizing the amplitude of the dispersion function in the dipole magnets. As the amplitude of the dispersion behaves as a parabolic function along the dipole length, short dipole magnets were implemented.

As a consequence of the reduction of the dispersion amplitude, the momentum compaction  $\alpha$ ,

$$\alpha = \frac{dL/L}{dp/p} = \frac{1}{L} \int_{\text{dipoles}} \frac{D}{\rho} ds \quad (\text{Equ.1})$$

was decreased by at least one order of magnitude. The natural bunch length

$$\text{scales like } \sqrt{\frac{E^3 \alpha}{f_{\text{rf}} V_{\text{rf}}}} \quad (\text{Equ.2}),$$

with  $f_{\text{rf}}$  the RF frequency,  $V_{\text{rf}}$  the RF voltage, and  $E$  the energy of the electron beam. The bunch lengths, typically of 200 ps in Full Width Half Maximum (FWHM) on 2<sup>nd</sup> generation storage rings, were shortened to a few tens of ps on 3<sup>rd</sup> generation rings.

Incidentally, the need for more RF voltage to compensate for increased synchrotron losses also contributed to further reduce the natural bunch length.

As claimed during the Stanford Workshop on 4<sup>th</sup> generation light sources [1], there is a strong case for short electron bunches, and shortening the electron bunch length by gaining orders of magnitude on  $\alpha$  seemed attractive by the advent of the 3<sup>rd</sup> generation of machines.

### *Strong case for short bunches*

Various synchrotron radiation user communities would benefit from the reduction of the electron bunch length.

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There is a strong scientific case for intense X-ray pulses in the subpicosecond range to perform dynamic studies. FEL users would appreciate the reduction of bunch length by one or two orders of magnitude since the FEL gain is proportional to the peak current. With a bunch length in the range or smaller than the wavelength of the radiation emitted, the emission becomes coherent, and the intensity is then proportional to  $N^2$ , the square of the number of particles per bunch, whereas for the spontaneous emission the flux is only proportional to  $N$ . The tremendous enhancement of photons flux has already been demonstrated experimentally for far infra-red radiation. The possibility to store subpicosecond bunches would boost the performances of infra-red sources.

Optimized luminosity is obtained for an rms bunch length  $\sigma_\tau$  approximately of the order of  $\beta^*$  the betatron value at the interaction point. B-factories and damping rings used to inject in linear colliders are good candidates for smaller bunches. Finally, the muon collider will be operated with bunches in the mm range and in a quasi-isochronous regime.

### 3 QUASI-ISOCHRONOUS REGIME

When the revolution period of particles become nearly independent of energy, the ring is said to be quasi-isochronous. The phase-slippage factor  $\eta = \frac{1}{\gamma^2} - \frac{dL/L}{dp/p}$ ,

is then very small. In electron machines, the phase-slippage factor  $\eta$  is approximately the momentum compaction  $\alpha = \frac{dL/L}{dp/p} = \alpha_1 + \alpha_2 dp/p + \alpha_3 (dp/p)^2 \dots$ , and

the quasi-isochronous regime is reached when  $\alpha$  is small up to higher order terms in  $dp/p$ .

Historically, difficulties to run an accelerator in the quasi-isochronous regime were already met as it corresponds to the situation of transition energy crossing on proton and heavy ion machines. By performing the Q-jump, difficulties to run in the quasi-isochronous regime are avoided.

For running a ring steadily in the quasi-isochronous regime, the higher order terms of  $\alpha$ , need to be minimized permanently.

### 4 SMALL $\alpha$ AND NEGATIVE $\alpha$ TUNING

#### *Lattice tuning*

To reduce the momentum compaction  $\alpha$ , the dispersion function needs to be reduced along the dipole. By forcing the dispersion to be negative within the two dipoles, the momentum compaction, integral (1) can vanish and even be made negative.

At the ESRF, the lattice is a DBA of the Chasman-Green type. The focusing quadrupoles of the achromat, allow to adjust the amplitude of the dispersion function in

the dipoles. By forcing the dispersion function to have its minimum taking a negative value in the middle of the dipole, one can tune  $\alpha$ , to any small positive or negative value and get a small horizontal emittance at the same time.

#### *Necessity of decreasing the 2<sup>nd</sup> order term $\alpha_2$*

Non-linear terms arising from bending magnets and their fringing fields as well as from sextupoles contribute to the generation of an  $\alpha_2$  term.  $\alpha_2$  is the longitudinal chromaticity. Three different families of sextupoles are needed to correct at the same time the two transverse and the longitudinal chromaticities. The RF bucket size  $\Delta p/p$  follows  $1/\sqrt{\alpha_1}$  when 2<sup>nd</sup> order terms can be neglected.

When the 2<sup>nd</sup> order terms are not negligible, the RF bucket size remains smaller than  $\alpha_1/(\alpha_2 \sqrt{3})$ .

With the ESRF experiment, for an  $\alpha_1$  in the  $2.10^{-5}$ ,  $\alpha_2$  needed to be decreased below  $1.75.10^{-3}$  to keep an RF acceptance  $\Delta p/p$  above the quantum limit of  $6.6 \sigma_\delta$ . In figure 1, RF buckets are depicted for three different combinations of  $(\alpha_1, \alpha_2)$ . Obviously, with shorter bunches the bunch density is increased creating more frequent Touschek collisions. The energy excursion occurring from a Touschek collision can easily reach 10% of the beam energy. Consequently, the RF bucket size must be huge to get a decent Touschek lifetime.

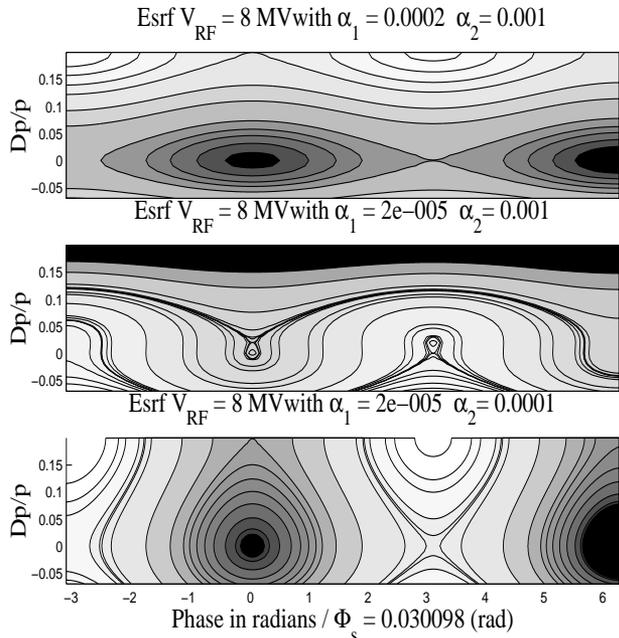


Figure 1- Necessity of decreasing  $\alpha_2$

NB: RF bucket sizes are also further squeezed due to energy independent terms increasing the spread in circumference length. This term is a function of both transverse emittances. Accordingly, a small emittance lattice is also mandatory to reach very small  $\alpha$  values while maintaining a viable RF bucket size.

## ESRF experiment with short bunches

In order to reach bunch lengths of 1ps in rms value on the ESRF ring, attempts were made to lower  $\alpha_1$  down to the  $10^{-6}$  range. Only with  $\alpha_1 = 2 \cdot 10^{-5}$ , the reduction of  $\alpha_2$  was successful leaving a decent lifetime for measurements. The bunch lengths were measured with a 2ps resolution streak camera. They show that as soon as a substantial current is stored, the bunch length is independent of both the momentum compaction and the energy. The impedance of the ring makes the bunch lengthen strongly due its strong self-induced voltage.

Incidentally, but as an experimental difficulty to be mentioned, the RF system must be very clean down to very low frequencies to avoid driving coherent longitudinal oscillations. Indeed, for very small momentum compaction, the synchrotron frequencies reached are small since they are in the few hundred Hz..

## 5 COLLECTIVE EFFECTS

Careful attention was paid to smooth the vacuum chamber and reduce the impedance of 3<sup>rd</sup> generation rings. Steep step transitions and cavity-like structures have been avoided as much as possible. Vacuum ports are attached to ante-chambers and bellows are RF shielded.

The strong lengthening with current, the independence of the bunch length with both the energy and the momentum compaction at high currents, can be explained by an inductive impedance model. In 3<sup>rd</sup> generation machines, experiments have shown that the theoretical model of an inductive impedance matches nicely bunch lengths measurements [ESRF, Super-Aco[5], ALS[4]...].

However, a purely inductive impedance model can not justify the energy widening observed above peak current of 450 A in the ESRF case. Only, a high frequency resistive interaction can be at the origin of an energy spread blow-up. The mechanism involved is then the well-known “microwave instability”.

The nature of the impedance of 3<sup>rd</sup> generation rings makes them inappropriate to the production of short and intense bunches, and in particular not in the quasi-isochronous regime. The path followed to draw this conclusion of major importance is presented in [2]. A short overview is given here below.

The resolution of the Haissinski equation [3], a multiparticle tracking code of the longitudinal motion and analytical derivations were used when possible. The simplified and academic case of a purely inductive impedance is studied first.

### *a- Inductive impedance*

For a purely inductive impedance, the self-induced voltage is the derivative of the bunch line density which is gaussian at zero current.

## Positive $\alpha$

When the momentum compaction is positive, with increasing current, the slope of the effective voltage subsides around the synchronous phase. The bunch is defocused and lengthens. For increasing current, the bunch profile becomes more and more like a parabola.

The Haissinski equation is a transcendental equation, for which an explicit solution can not be written in the case of a purely inductive impedance. However, one can demonstrate that there always exists a solution whatever the current for the case of a positive momentum compaction and an inductive impedance[2]. As for the coasting beam, for a purely inductive impedance and for a positive momentum compaction, the beam is always stable. The bunch lengthens in a pure potential well regime and its energy distribution remains unchanged and equal to the natural one. Writing the Haissinski equation as  $\lambda'(x) = \frac{-x\lambda(x)}{1+\Delta\lambda(x)}$  with  $\Delta = \frac{2\pi I(L\omega_0)}{V_{rf} h \cos\phi_s (\omega_0 \sigma_{\tau_0})^3}$ ,  $x = \tau/\sigma_{\tau_0}$

a Runge-Kutta algorithm allows to compute rapidly the temporal distribution  $\lambda(\tau)$ .

At high currents, the bunch length becomes independent of both the momentum compaction  $\alpha$  and of the energy, but depends on the power 1/3 of ratio between the induced voltage and the external voltage. The quasi-isochronous regime is then useless in that to produce short and intense bunches.

### *Negative momentum compaction*

The strong defocusing effect can be reversed when the momentum compaction takes a negative value. The synchronous phase is then on the rising slope of the RF wave. The self-induced perturbing voltage now tends to increase the RF gradient around the synchronous phase, and the bunch self-focuses. Unfortunately, this self-focusing effect is limited.  $\Delta$  is now negative as  $\cos\phi_s$  is negative. There is a threshold  $\Delta_{limit} = -1.55$ , at which, the Haissinski equation presents a singularit. It corresponds to a current threshold at which the effective gradient field would take infinite values [2]. Beyond this value, a negative mass instability for bunched beam starts developing. The rms energy spread  $\sigma_s$  and the rms bunch length  $\sigma_{\tau}$  both widen in parallel, such that the coefficient  $\Delta$  remains constant.

This result was also confirmed by running the multiparticle tracking code, a similar threshold was found [2]. Above that threshold, the bunch lengthens and bunch distributions keep the shape presented at threshold. The line density shows a high density core and long low density tails. The energy spread profile remains gaussian.

This mechanism of the negative mass instability is a well known mechanism. Its appearance in 3<sup>rd</sup> generation rings was proved experimentally and unfortunately ruins the hopes of using negative momentum compaction to produce short and intense bunches.

NB: The purely inductive model is not fully satisfying for fitting experimental results when the momentum compaction is negative. Indeed, the narrow high density peak interacts with the high frequency terms. The impedance presents some resistive components in the high frequencies. The high frequency resistive term is the source of additional energy widening and it also generates a strong skewness on the bunch profile.

### b- Broadband resonator impedance

A purely inductive impedance can not correctly represent a the true impedance of a ring. Unavoidable discontinuities capture some fields, the power of which is dissipated in the walls.

Of the few different representations commonly used[8],[9] , the broadband resonator model proved to be mathematically handy while possessing the correct physical features. It represents an impedance which is inductive up to a resonant frequency,  $f_r$ , at which the inductive part falls and where the strong resistive components are present.

A “microwave” instability develops in the presence of a high frequency resistive interaction. As depicted in figure 2, the self-induced perturbation voltage oscillates with a wavelength  $\lambda_r \approx c/f_r$  and accordingly the amplitude of the longitudinal focusing force oscillates with that wavelength along the bunch.

We first consider the situation in which the bunch length contains more than one wavelength. This creates some local stability islands or fixed points on the isohamiltonian curves. The isohamiltonian curves represent the motion in the longitudinal phase space in the absence of radiation damping and quantum fluctuation. Particles gather in micro-bunches and glide along those stability islands. The micro-bunch structure is then diluted due to the global synchrotron motion generated by the external RF voltage (See figure 3). Very small drops keep reappearing which are immediately diluted. The bunch density stabilizes with a temporal distribution of a parabolic shape and an energy distribution of gaussian shape both with FWHM values which increase with the power 1/3 of the current.

An analytical formula for the threshold of the “microwave” instability has been derived from a dispersion relation deduced from the linearization of the Vlasov equation. The stability criterion, here below, also gives the bunch dimensions above threshold:

$$\frac{\hat{\Lambda} R_{\text{eff}}}{\sigma_{\tau}^2} < 1.5 \text{ with } Z_{\text{eff}} = R_{\text{eff}} = \frac{\sum_p Z(p\omega_0) \tilde{\lambda}_{p_r}^2(p)}{\sum_p \tilde{\lambda}_{p_r}^2(p)}$$

$$\text{with } \hat{\Lambda} = \frac{\alpha I}{\sqrt{2\pi} E_e \omega_0 \sigma_{\tau}} \text{ and } \sigma_{\tau} = \omega_s \sigma_{\tau}.$$

The effective impedance  $Z_{\text{eff}} = R_{\text{eff}}$  is only resistive since the perturbation  $\tilde{\lambda}_{p_r}$  of the stationary distribution is centered around  $p_r$  ( $p_r = f_r/f_0$ ) and the overlap with the reactive part of the impedance is globally zero.

This criterion was first derived by Keil & Schnell for a coasting beam and applied to proton beams by Bousard. The extensive derivation for electron bunched beams can be found in [2], [6],[7]. The validity of the formula was confirmed by the results from the tracking code.

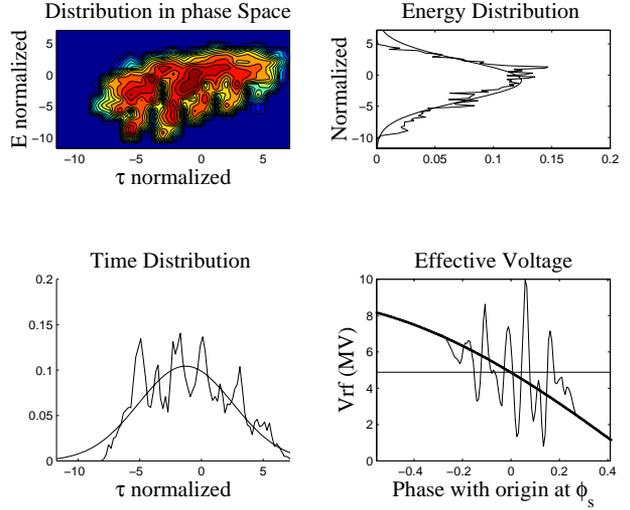


Figure 2 Microwave mechanism

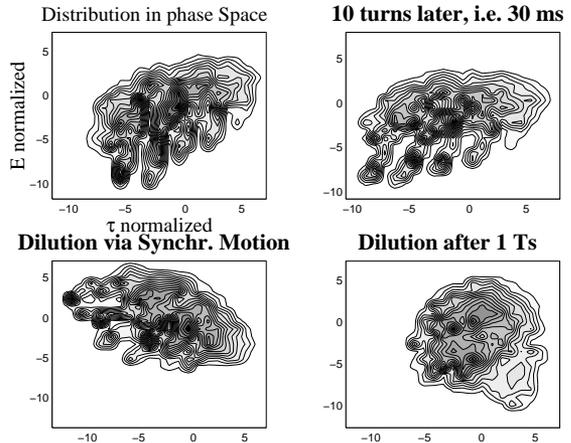


Figure 3 Dilution of micro-structure

## 6 HIGH PEAK BRILLIANCE IN STORAGE RINGS

### No Possibility of bunch shortening

The condition of appearance of the “microwave” instability is that the phase advance of the wake field is larger than  $2\pi$  over the bunch length of  $2\sigma_{\tau}$  leading to the condition:  $\omega_r \sigma_{\tau} > \pi$  with  $f_r$  the resonant frequency and  $\sigma_{\tau}$  the rms bunch length (the bunch eventually lengthens due to its interaction with the inductive part of the resonator).

When the condition  $\omega_r \sigma_r > \pi$  is not fulfilled, the resistive interaction leads to a bunch self-focusing. However, the bunch breaks into two subbunches or a short bunch with a long tail. The energy spread can start widening; indeed the particles of the tail of the bunch have a very slow synchrotron speed. The energy spread profile is eventually not any more gaussian but the superposition of two gaussian distributions with different rms values. The strong diffusion process either to the tail or to a second sub-bunch, makes the passive-cavity like structures unusable for the production of short and intense bunches.

NB: At that point, it is worth mentioning that while running the ESRF at 2 GeV, the bunch length, for the lowest currents allowing streak camera measurements was already close to 10ps, to be compared with the 2ps theoretical zero current value at that energy and for an RF voltage of 8 MV.

### *Harmonic cavity*

Reaching small and intense bunches is not possible in storage rings, since the bunch either lengthens and possibly widens in energy.

One could nevertheless expect reaching high peak current since the peak current increase with the power  $2/3$  of the current. In 3<sup>rd</sup> generation rings, the transverse single bunch current limit can be raised either by increasing the chromaticity or by using a feedback system. The increase of the chromaticity is not a good solution as it deteriorates the off-momentum dynamic aperture and accordingly reduces the Touschek lifetime.

By means of an harmonic cavity, the bunch can be lengthened to at least a factor 3 and the Touschek lifetime raised by the same amount. The RF voltage slope is made about flat around the synchronous phase along the desired width of the bunch. For a long bunch, the perturbing voltage is spread over a wider range, and has consequently a smaller amplitude. The current threshold is for that reason higher. However, with a lengthened bunch, the spectrum is shrunked, accordingly the effective resistance  $R_{\text{eff}}$  is larger. More detailed analysis are in preparation [10].

The harmonic cavity fullfills its goal as to increase the Touschek lifetime but the “microwave” instability threshold is not raised by more than a factor of 2 for a bunch lengthened by a factor of 3[7]. Harmonic cavities are not the solution either to increase the peak brilliance by orders of magnitude.

## 7 CONCLUSIONS

It has been demonstrated that storing subpicosecond bunches with high current cannot be achieved on storage rings. In the range of bunch length explored on 3<sup>rd</sup> generation rings, the bunch lengthens in its inductive wakefield. A “microwave” instability starts appearing at

high currents due to the presence of high frequency resistive components, which mechanism has been fully analyzed. Running the ring with a negative momentum compaction is also not a solution since a negative mass instability develops which blows the energy spread of the electron beam.

### *Future possibilities*

Peak brilliance of up to  $10^{31}$  @ 10 keV in standard units are announced for the linac driven sources (LCLS[11], Tesla). The 8 orders of magnitude difference, above performances achieved presently on storage rings, have two origins. Two orders come from the reduction of the bunch length down to 67 fs in rms value. The other six come from a new principal, SASE (Self Amplified Spontaneous Emission). The radiation will be coherent as the bunch microbunches itself at the optical wavelength of its own radiated field. If SASE works as predicted, the linac driven sources will constitute genuine 4<sup>th</sup> generation sources for which the figure of merit will be the peak brilliance.

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