

# THE TRANSVERSE WAKE FIELDS IN THE TESLA-FEL TRANSFER LINE \*

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## Abstract

For TESLA-FEL operation the electron beam is extracted from the regular accelerating structure at the desired energy level and then transferred by a long transfer line to the experimental hall [1]. It is required that longitudinal and transverse emittance be kept small. Wake fields, excited from various devices e.g. pumping holes, quadrupole tapers, beam position monitors, can cause emittance growth. In addition the wake fields of the short bunches due to the surface roughness in this very long line can give a large contribution.

In this paper the transverse wake fields in a vacuum chamber with small protrusions on the surface are calculated for short bunches and an analytical expression for very short bunches is presented. The effect of these wake fields on the beam emittance growth in the TESLA-FEL transfer line is discussed.

## 1 INTRODUCTION

The use of very short, intense bunches is essential for the Linear Collider projects. For the FEL-operation provided at TESLA bunchlengths  $\sigma_z$  of less than  $25\mu\text{m}$  are aimed for. Since only one large experimental area is planned at the interaction point, the beam has to be extracted from the main accelerator, when it has reached the desired energy level. Then it is guided through an up to 12 km long transfer line to the undulators. Naturally it is required that longitudinal and transverse emittance be kept small. Wake fields, excited from various devices like pumping holes, quadrupole tapers, beam position monitors, can cause emittance growth.

In addition the fields of short bunches due to the surface roughness wake field effect in vacuum chamber can give some contribution. Extruded tubes, commonly used for such purpose, have an rms roughness depth of around 350 nm. Therefore it is worth to know, whether the wake fields excited by a  $25\mu\text{m}$  bunch can lead to a large energy spread and transverse instability.

## 2 ANALYTICAL APPROACH

Since it is expected, that the monopole wake fields of a waveguide with a rough surface behave in the same way as those inside a waveguide with a dielectric layer ([2, 6]), it

is feasible to start with the wake fields in a dielectric lined waveguide. The case of the wake field inside a wave guide covered with a thin dielectric layer was studied in e.g. ([2, 4]).

### 2.1 Monopole Case

In the monopole case ( $m = 0$ ) the wave number of a wave guide covered with a thin dielectric layer is given by

$$k_0^2 = \frac{2\varepsilon}{(\varepsilon - 1)a\delta} \quad (1)$$

where  $\varepsilon$  is the relative permittivity,  $a$  the tube radius and  $\delta$  the thickness of the dielectric layer. The longitudinal wake function is

$$W_0^{\parallel}(s) = \frac{Z_0 c}{\pi a^2} \cos(k_0 s) \quad (2)$$

using the impedance of free space  $Z_0 = \sqrt{\mu_0/\varepsilon_0}$ .  $c$  denotes the speed of light.

### 2.2 Dipole Case

In the dipole case the wave number is the same,

$$k_1^2 = \frac{2\varepsilon}{(\varepsilon - 1)a\delta} \quad (3)$$

as in the monopole case. The longitudinal wake function is given by:

$$W_1^{\parallel}(s) = 2 \left(\frac{r_0}{a}\right) \left(\frac{r_1}{a}\right) \frac{Z_0 c}{\pi a^2} \cos(k_1 s) \quad (4)$$

where  $r_0$  and  $r_1$  are the offset of the driving charge and the witness respectively. Note that for  $r_0 = r_1 = a$  the amplitude of the longitudinal dipole wake is twice as large as in the monopole case. Finally the transverse dipole wake function reads:

$$W_1^{\perp}(s) = 2 \left(\frac{r_0}{a}\right) \frac{Z_0 c}{\pi a^3 k_1} \sin(k_1 s). \quad (5)$$

For the verification of the applicability of this model to dipole wakes in rough surface tubes, different numerical simulations using the program package MAFIA [5] are performed. The results are compared to the predictions derived above.

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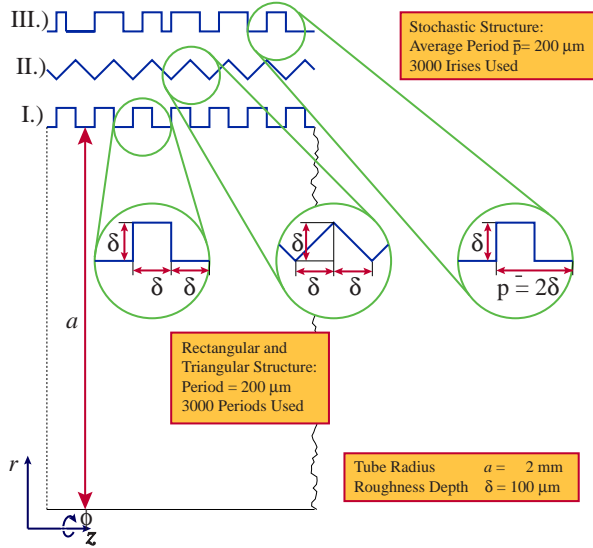


Figure 1: The surface roughness of the tube is modelled in 3 different manners. I.) periodically rectangular, II.) periodically triangular, III.) stochastically with rectangular shape. The Depth of the roughness is  $100\mu\text{m}$ , the period (I., II.) or average period (III.) respectively is  $200\mu\text{m}$

### 3 NUMERICAL RESULTS

The shape of the single protrusions has, of course, a major effect on the strength and the shape of the wake field ([3]). They have to be considered by an effective permittivity  $\varepsilon$ , which is expected to be around 2.

A cylindrical symmetric structure is assumed, with the beam pipe radius  $a = 2\text{mm}$  and a roughness depth  $\delta = 100\mu\text{m}$ . Although the simulated roughness is much larger than in reality, the derived results allow an extrapolation to a real transfer line. Three kinds of corrugations are studied: a periodic rectangular one (Fig. 1.I), a periodic triangular one (Fig. 1.II), each of them with a periodicity of the roughness of  $p = 200\mu\text{m}$ , and finally a stochastically distributed rectangular shaped roughness with an average periodicity of  $\bar{p} = 200\mu\text{m}$  (Fig. 1.III), to show, that the given formulae are not restricted to the case of a perfect periodic surface structure only. In every simulation 3000 periods are used. The bunch length used for the calculations is  $\sigma_z = 250\mu\text{m}$ . For all calculations the offset of the driving and the witness charge is  $r_0 = r_1 = a$ . The longitudinal monopole and dipole wake fields are calculated as well as the transverse dipole wake field.

#### 3.1 Periodic Rectangular

First of all the periodic rectangular structure is studied (Fig. 2). The wake field of both, the monopole and the dipole field approaches a periodic function with constant amplitude and the same frequency. The longer the simulated tube is, the more exact a periodic function is matched. The amplitude of the dipole wake ( $r_0 = a$ ) is twice as large as that of the monopole wake. The wavelength is  $1.37\text{mm}$ .

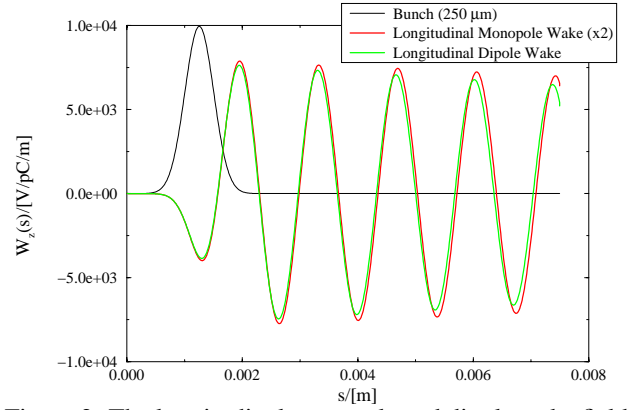


Figure 2: The longitudinal monopole and dipole wake field of the periodic rectangular structure (Fig. 1.I). The bunch length is  $\sigma_z = 250\mu\text{m}$ . The monopole wake is scaled by a factor of 2.

Using eq. 1 this leads to an effective equivalent relative permittivity  $\varepsilon = 1.9$  of a dielectric layer with the thickness  $\delta = 100\mu\text{m}$

#### 3.2 Periodic Triangular

The same observations as in 3.1 concerning the amplitude of the wake field can be made for the periodic triangular structure (Fig. 3) too, but it takes a longer time to reach a constant amplitude. Since the wavelength is shorter ( $\lambda = 1.022\text{mm}$ ) the equivalent dielectric layer has the relative permittivity  $\varepsilon = 1.36$ .

#### 3.3 Stochastic Rectangular

The longitudinal wake field generated in the stochastic structure rings at the same frequency as in the periodic structure (Fig. 4). Thus the equivalent permittivity is the same as in 3.1. The factor 2 between the amplitude of the monopole and the dipole wake holds here too.

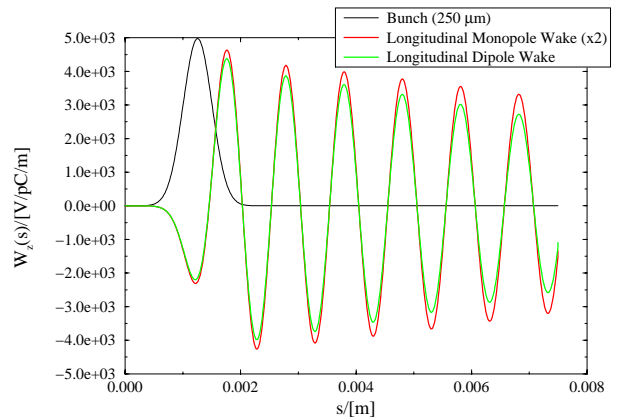


Figure 3: The longitudinal monopole and dipole wake field of the periodic triangular structure (Fig. 1.II). The bunch length is  $\sigma_z = 250\mu\text{m}$ . The monopole wake is scaled by a factor of 2.

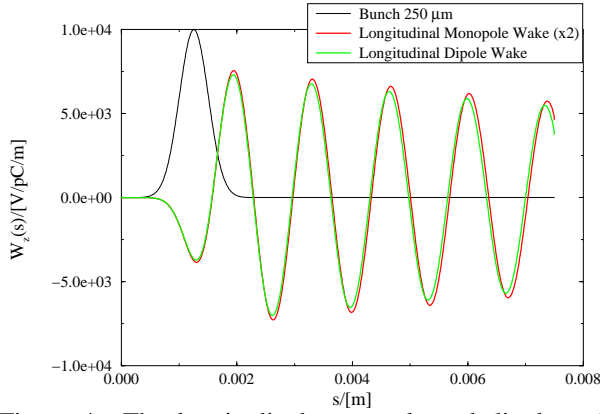


Figure 4: The longitudinal monopole and dipole wake field of the stochastic rectangular structure (Fig. 1.II). The bunch length is  $\sigma_z = 250 \mu\text{m}$ . The monopole wake is scaled by a factor of 2.

### 3.4 Transverse Dipole Wake Fields

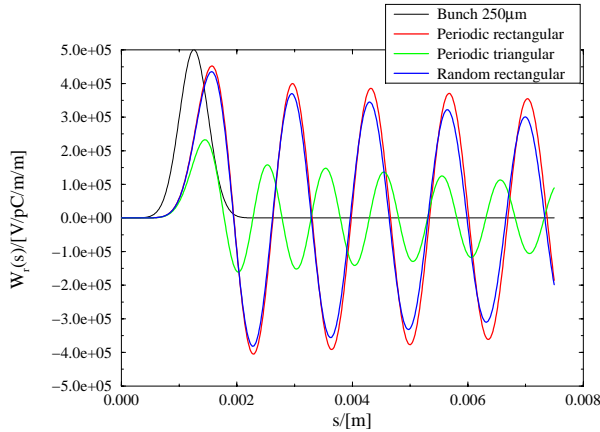


Figure 5: The transverse dipole wake field of the three examined structures. The bunch length is  $\sigma_z = 250 \mu\text{m}$ .

The transverse dipole wake fields of the three structures are shown together in Fig. 5. The function is sinus-like in comparison to the monopole wakes. The frequency of the periodic rectangular and the stochastic rectangular are obviously the same, while the “damping” of the amplitude is larger in the latter case. The frequency of the wake field of the triangular structure is higher, but the amplitude is much smaller than in the rectangular case.

## 4 DISCUSSION

The effect of these wake fields and the applicability of the results to a 3 dimensional structure have to be discussed.

### 4.1 Consequences of Transverse Wakes

To describe the consequences of the transverse wake fields on the bunch in the transfer line the gradient  $G(s)$  of the wake function as the quotient of the transverse dipole wake function and the offset to the axis is introduced:

$$G(s) = \frac{W_1^\perp(s)}{r_0} = \frac{2QZ_0c}{\pi a^4 k_1} \int_{-\infty}^s \sin(k_1(s-s')) \frac{1}{\sqrt{2\pi}\sigma_z} e^{-\frac{s'^2}{2\sigma_z^2}} ds'. \quad (6)$$

Assuming that a bunch of an energy  $E$  enters the tube at an offset  $r_0$  it will double its offset after a certain distance, to which in this paper is referred to as the instability length:

$$z_{inst} = \sqrt{\frac{E}{G(2\sigma_z)}} \quad (7)$$

To apply this results to the TESLA structure, an initial energy of the bunch at the entrance of the transfer line of 10 GeV is assumed. The bunch length is  $\sigma_z = 25\mu\text{m}$  and it is carrying a charge of 1nC. The tube radius is 1 cm and the roughness depth is 350 nm. According to the model, the instability length is  $z_{inst} = 258\text{m}$

### 4.2 Validity in 3D Structures

The model used above is that of a disk-loaded structure. Due to the cylindrical symmetry of the surface roughness the fields can propagate freely into the grooves. Since in reality the cavities are bounded at all sides, the fields will not propagate inside, but decay exponentially. In order to apply the presented formulae to the 3D case, one can introduce the effective roughness depth  $\delta_\perp$ , which is related to the aperture of the cavities of the surface roughness  $\delta_\parallel$  by:

$$\delta_\perp = \frac{\delta_\parallel}{\pi} \quad (8)$$

Assuming that roughness depth and aperture are of the same order in average, the effective depth in 3D-structures is three times less.

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