A 3-D TOUSCHEK SCATTERING THEORY*

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Abstract

A generalized Touschek scattering theory based on a 3-dimensional Gaussian velocity distribution is developed to improve the accuracy of lifetime calculations. The new theory agrees with the measurements better than the classical Touschek scattering theory if the dispersion is large, rf bucket size is large, or vertical velocity spread is large. Agreement with measurements is not perfect and some possible additional beam loss mechanisms are discussed.

1. THEORY

Electrons in a bunch are under the influence of the transverse and longitudinal focussing forces and, therefore, undergo betatron and synchrotron oscillations. A binary Coulomb collision between a pair of electrons may transfer their transverse momenta into longitudinal momenta. The colliding particles are lost if the longitudinal momenta after the collision is outside the momentum acceptance of the accelerator [1]. Many authors reviewed the original theory, which has subsequently been known as the "flat beam model" [2], because the beam volume is assumed to be 3-D in configuration space but 1-D in velocity distribution.

Electrons reach a thermal equilibrium when the radiation damping is balanced by quantum fluctuations and intrabeam scattering. We assume that the system is linear. Liouville 's theorem states that the phase space density is constant along the particle trajectory which is an ellipse with area given by the Courant Snyder constant. Furthermore, we assume that the core distribution is formed by particles which undergo many collisions in one damping time, which , according to the central limit theorem, gives a Gaussian distribution:

$$\Psi(\vec{x},\vec{x}') = \Psi_0 \exp\left(-\frac{H_x[x_\beta, x_\beta']}{2\varepsilon_x} - \frac{H_y[y,y']}{2\varepsilon_y} - \frac{z^2}{2\sigma^2} - \frac{\delta_p^2}{2\sigma_\delta^2} \right)$$

where

$H_x(x_\beta, x_\beta') = (\gamma_x x_\beta^2 + 2\alpha_x x_\beta x'_\beta + \beta_x x'^2_\beta)$ is the Courant-Snyder invariant for the off momentum particles, α_x, β_x, γ_x, and η_x are the lattice functions, $x_\beta = x - \delta_p \eta_x$, $x'_\beta = x' - \delta_p \eta'_x$, $\delta_p = dp/p$ is the momentum deviation, $\Psi_0 = N_0/(8\pi^3 \varepsilon_x \varepsilon_y \varepsilon_z)$ is the normalization constant, ε_x and ε_y are the emittances, N_0 is the total number of particles in the beam, σ_z is the bunch length, and σ_δ is the momentum spread. We assumed that the vertical dispersion is zero.

Beam loss rate can be calculated by considering a small volume $d^3\vec{x} = dxdydz$ in which source particles with velocities between $\vec{v}_1$ and $\vec{v}_1 + d\vec{v}_1$ are incident upon the target particles in the same bunch at the same location with velocities between $\vec{v}_2$ and $\vec{v}_2 + d\vec{v}_2$. The number of particles scattered into a solid angle $d\Omega'$ in unit time from this collision is the product of the flux of the incident particle, the differential cross section, and the number of target particles. The collision rate in the beam coordinate system is [3]:

$$\frac{dN}{dt} = -\iint_{\vec{x},\vec{v}_1} \iint_{\vec{v}_2,\Omega'} \left[\Psi_b(\vec{x},\vec{v}_1) \left\{ \left| \vec{v}_1 - \vec{v}_2 \right| d\sigma(\Omega;\Omega')/d\Omega \right\} \Psi_b(\vec{x},\vec{v}_2) \right] d^3\vec{x}\, d^3\vec{v}_1\, d^3\vec{v}_2\, d\Omega'$$

where $\Psi_b(\vec{x},\vec{v})$ is the distribution function in the beam frame and $d\sigma(\Omega;\Omega')/d\Omega$ is the differential cross section.

In the center-of-mass coordinate system of the colliding particles, let $\vec{v}_1'$ and $\vec{v}_2'$ be the velocity of each particle after the collision, $\vec{V} = (\vec{v}_1 - \vec{v}_2)$ and $\vec{V}' = (\vec{v}'_1 - \vec{v}'_2)$ be the relative velocities before and after the collision. For elastic collisions $|\vec{V}| = |\vec{V}'|$. Let the z-axis be in the direction of beam propagation, y vertically upward, and x radially outward. Let the scattering angle (angle between $\vec{V}$ and $\vec{V}'$) be θ, the polar angle (angle to the z-axis) for $\vec{V}$ and $\vec{V}'$ be χ and χ', and the azimuth (angle around the z-axis) for $\vec{V}$ and $\vec{V}'$ be ϕ, and ϕ'.

The particle is lost if the longitudinal velocity after collision is larger than the maximum positive velocity allowed in the accelerator, $\gamma v \cos\chi' > dv_{max}$, or, $0 < \chi' < \cos^{-1}\mu$, where $\mu = dv_{max}/(\gamma v)$, and $|v| > dv_{max}/\gamma$. We now change variables from (v_{2x}, v_{2y}, v_{2z}) to the variables (v_{1x}, v_{1y}, v_{1z}) minus

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the center of mass variables, (v, χ, ϕ), according to the following relationships: $\mathrm{v}_{2x} = \mathrm{v}_{1x} - 2\mathrm{v}_x$, $\mathrm{v}_{2y} = \mathrm{v}_{1y} - 2\mathrm{v}_y$, $\mathrm{v}_{2z} = \mathrm{v}_{1z} - 2\mathrm{v}_z$ $d^3\vec{\mathrm{v}}_2 = -d^3\vec{\mathrm{V}} = -8d^3\vec{\mathrm{v}}$, $\mathrm{v}_x = |\mathrm{v}|\sin\chi\cos\phi$, $\mathrm{v}_z = |\mathrm{v}|\cos\chi$ $\mathrm{v}_z = |\mathrm{v}|\cos\chi$, $\mathrm{v'}_x = |\mathrm{v}|\sin\chi'\cos\phi'$, $\mathrm{v'}_y = |\mathrm{v}|\sin\chi'\sin\phi'$ $\mathrm{v'}_z = |\mathrm{v}|\cos\chi'$, and $d\Omega' = \sin\chi' d\chi' d\phi'$.

The differential cross-section for elastic Coulomb scattering is according to Moller: $d\sigma / d\Omega' = 4r_e^2 c^2 / V^4 (4/\sin^4\theta - 3/\sin^2\theta)$, where r_e is the classical electron radius, and c is the velocity of light. The scattering angle θ is related to the new variables by the relationship: $\cos\theta = \cos\chi\cos\chi' + \sin\chi\sin\chi'\cos(\phi - \phi')$, which can be obtained by considering the triangle formed by the vectors $\vec{\mathrm{v}}$ and $\vec{\mathrm{v}}'$, and the fact that $|\mathrm{v}| = |\mathrm{v}'|$.

Then the beam lifetime at a point, s, is:

$$\frac{1}{\tau_s} = -\frac{1}{N}\frac{dN}{dt} = \frac{N_0 r_e^2 c K(\zeta)}{32\pi^3 \tilde{\varepsilon}_x \varepsilon_y \sigma_l (\beta\gamma)^3 \delta_A^2}, \text{ where}$$

$$\tilde{\varepsilon}_x = \sqrt{\varepsilon_x^2 + H_x(\eta_x, \eta_x')\varepsilon_x(\sigma_p / p)^2}, \zeta = \left(\frac{\delta_A}{\gamma}\right)^2,$$

$$K(\zeta) = \zeta \int_{u=\zeta}^{\infty} \int_{\xi=-\xi_0}^{\xi_0} \exp\left\{-u\xi^2\right\}$$

$$\int_{\phi=0}^{2\pi} \exp\left\{-u\left[\left(\frac{\cos^2\phi}{\sigma_{x'}^2} + \frac{\sin^2\phi}{\sigma_{y'}^2}\right)\sin^2\chi + \frac{2(\alpha_x\eta_x + \beta_x\eta_x')}{\varepsilon_x}\cos\chi\sin\chi\cos\phi\right]\right\} K_4 \, d\phi \, d\xi \, \frac{du}{u}$$

$$K_4(u,\chi) = \int_{\chi'=0}^{\cos^{-1}\sqrt{\zeta/u}} \int_{\phi'=0}^{2\pi} \left[\frac{4}{\left\{1 - [\cos\chi\cos\chi' + \sin\chi\sin\chi'\cos\phi']^2\right\}^2} - \frac{3}{1 - [\cos\chi\cos\chi' + \sin\chi\sin\chi'\cos\phi']^2}\right] d\phi' \sin\chi' \, d\chi'$$

$$\xi_0 = \left(\frac{\gamma}{\delta_p}\right)\left(\frac{\tilde{\varepsilon}_x}{\varepsilon_x}\right), \text{ and } \cos\chi = \frac{\xi}{\xi_0}.$$

The total beam lifetime can be obtained by averaging $1/\tau(s)$ over s around the entire storage ring. A computer code is written to calculate lifetime given by above equations.

2. ALS DATA

Beam lifetime of real machines such as the ALS is clearly affected by many other processes which are not included in the theory described above. For example, if the pulse shape N(z) is not Gaussian, it can be shown that lifetime is modified by the form factor, $F_\tau = (2\sqrt{\pi}\sigma_z / N_0^2) \int_{-\infty}^{\infty} N^2(z) dz$. Touschek scattered particles, which are "almost lost", forms the high energy tails in the particle velocity distribution and may significantly reduce beam lifetime. Resonances and nonlinear effects will also reduce beam lifetime. We attempt to include all these effects in a heuristic form factor, $F_\tau = \tau_{measured} / \tau_{theory}$, for fitting theory with experiments.

We adopt the following fitting procedure to make the many-parameter fitting as unambiguous as possible: (1) identify parameter regime where lifetime depends on minimum number of parameters, (2) fit to the functional dependencies of lifetime on the parameters.

Typical ALS beam parameters used for the present study are: f_s= 0.0075f_o, rf bucket height (δ_{RF}/E)=0.027, beam energy=1.522 GeV, single bunch current = 1.4 mA, natural emittance = 3.4 x 10^{-9} m rad, natural energy spread (σ_E/E) = 6.45 x 10^{-4}, natural bunch length = 5.92 x 10^{-3} m, momentum compaction factor = 1.6 x 10^{-3}, averaged = β_x 7.85 m, averaged β_y = 8.34 m, circumference = 196.8 m, emittance ratio = 1%, , unless otherwise specified. During lifetime and beam size measurements the current per bunch was varied by varying the number of bunches while keeping the total current constant at 8 mA.

As a first step, we consider the functional dependency of the current-lifetime product on beam current as shown in figure 1. We have deliberately chosen the synchrotron tune to be 0.0055 (rf bucket size = 0.018), which is lower than the nominal value, so that the lifetime is independent of the dynamic momentum aperture. By choosing a lower rf voltage we have one less parameter to worry about at this stage of fitting.

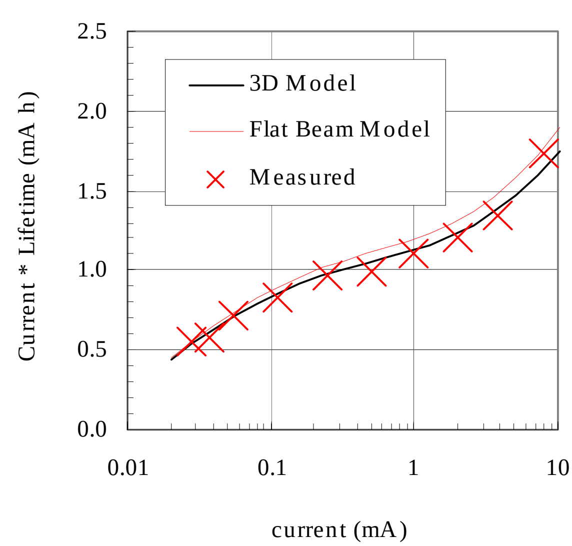

Figure 1. The current-lifetime product versus beam current. In this parameter regime the differences between the flat beam model (upper curve) and the 3-D model is expected to be small as shown.

Intrabeam scattering is negligible in the ALS for currents below 0.5 mA. We fit 2 parameters, the gas scattering time (τ_{gas}) and the form factor F_τ for the low current part of the data shown in figure 1 by using the relationship: $1/\tau_{maesured} = 1/\tau_{gas} + 1/(F_\tau \tau_{theory})$. The gas density is independent of current per bunch because we keep the total current constant. We find a good fitting for F_τ = 0.7 and τ_{gas} = 50 hours. The fitted gas scattering time is consistent with measurements using other techniques. [5]

Intrabeam scattering and instabilities increase beam lifetime for higher currents. We use the fitting parameter $F_\varepsilon = \gamma^{measured} / \gamma^{BM}$ where $\gamma^{measured}$ is the measured emittance growth rate and γ^{BM} is the calculated IBS growth rate according to Bjorken and Mtingwa [3]. The value of $\gamma^{measured}$ is expected to be different from the value of γ^{BM} because of at least two reasons: (1) presence of instabilities, (2) presence of high energy tails. Raubenheimer [4] has shown that intrabeam Coulomb scattering creates a small number of high energy electrons which do not belong to the Gaussian core of the distribution. The core emittance of the Gaussian distribution excluding the high energy tail is significantly smaller (by about a factor of 0.5) than the "total emittance" that includes the core and the high energy tail. We expect that growth of the core emittance will make the lifetime longer but the tail will make lifetime shorter through creation of a beam halo. We find F_ε = 0.5 gives a good fitting which is shown as the solid line in figure 1. The result is consistent with the beam size and the bunch length measurements [5], [6].

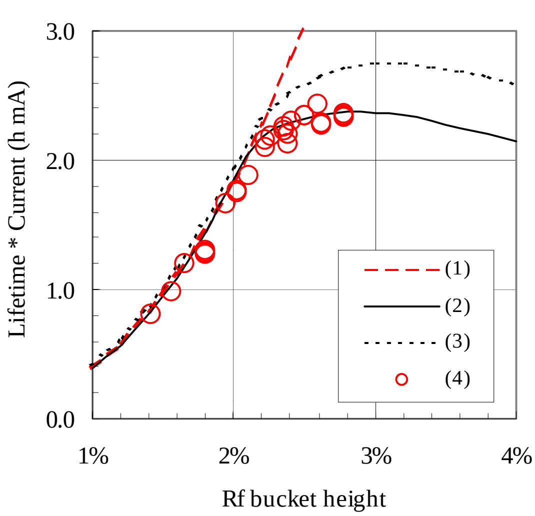

Figure 2. The dependencies of the measured and calculated current-lifetime products on rf bucket size. The broken line (1) represents the 3-D model where the momentum acceptance is assumed to be equal to the rf bucket height. , the solid line (2) represents the 3-D model for δ_1 =2.2 % and δ_0 = 4%, and the dotted line (3) represents the flat beam model for δ_1 =2.2 % and δ_0 = 4%. ALS data is shown in circles.

Figure 2 shows the dependencies of the measured and calculated current-lifetime products on rf bucket height. The broken line represents the 3-D model where the momentum acceptance is assumed to be equal to the rf bucket height. ALS data (circles) show that lifetime saturates for rf bucket sizes larger than about 2.2 %, presumably because the momentum acceptance is smaller than the rf bucket height. The momentum acceptance for such large momentum deviations is determined by nonlinear beam dynamics. The "dynamic momentum acceptance" can be estimated from the data in the following way. The dynamic momentum acceptance of the accelerator, $\delta_A(s)$, can be approximated as a square function with a constant value (δ_0) at non-dispersive regions and another constant (δ_1) at dispersive regions. We have the best fitting for δ_1 =2.2 % and δ_0 = 4% for the ALS data which is shown as the solid line in figure 2. The results agree with models calculations. [7] The solid line also shows that beam lifetime becomes shorter for larger rf bucket heights, which, we believe, is due to the shortening of the bunch length. The dotted line represents the flat beam theory which is about 20 % longer than the new theory for the nominal condition Smaller dynamic momentum aperture in the dispersive region makes the difference between the flat beam model and the 3-D model more pronounced as shown in figure 2.

3. CONCLUSIONS

New theory fits the ALS lifetime data better by about 20 % than the flat beam theory . For nominal conditions, ALS lifetime is smaller than predicted by the new theory by a factor of 0.7. High energy tails produced by "almost lost" particles and resonances can be the cause of the shorter beam lifetime.

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