# EXPLANATION OF SEXTUPOLE INSTABILITY IN CERN PS BOOSTER

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*Abstract*

Dual harmonic RF systems have been discussed for many years: to promote Landau damping, to reduce transverse space-charge, and to improve Touschek lifetime. Since its introduction into the CPS booster in 1982, the dual harmonic acceleration process suffered from an unexplained longitudinal instability occurring when the 2nd harmonic cavity is anti-phased and controlled by the 1st harmonic gap signal. The instability does not occur when the beam fundamental is used as reference, nor when the RF harmonics are in-phase. The impetus for the present study arises from the conversion form harmonic numbers h=5 & 10 to h=1 & 2 for LHC operation. The instability has recently been diagnosed as a sextupole mode. In this paper, which is a synopsis of two laboratory notes [3,4], we present experimental results from machine development (M.D.) periods, and a detailed theoretical explanation for the instability (and its correction) that considers feedback from the beam versus the cavity fundamental.

## 1. INTRODUCTION

The present understanding of the PS Booster instability builds upon two pieces of work. In 1994, Chapochnikova [1] pointed out that (with dual harmonic) beyond a critical bunch length Landau damping of all azimuthal modes is lost. In 1996, Blas [2] proposed that the instability is associated with the low level RF system loop delays and the magnitude of the beam transfer function from modulations of the 2nd harmonic RF to modulations of the 1st harmonic beam current.

### 1.1. Machine Development Results

Also contributing to the explanation were the M.D. experiments performed in 1997 and described by Koscielniak [3]. The instability was diagnosed as a beam sextupole mode ($m$=3) with oscillation frequency 3 $f_s$ and 3 nodes in the bunch shape; see Fig.1. There was no evidence of modes $m$=1 or $m$=5. The instability is only weakly dependent on beam current, and this excludes impedance or beam loading or space-charge as the source of the instability. Further corroboration of "not a beam-loading instability" is the fact that RF feedback has been implemented, with a substantial reduction of the apparent cavity impedance, but the instability survives. There is no instability when the 2nd harmonic RF is driven in-phase with the 1st harmonic RF. Further, the instability is only weakly dependent on the control technology (analogue or digital).

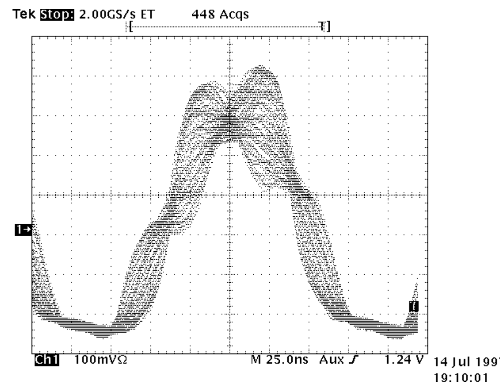


Figure 1: bunch shape with 3 nodes.

These observations suggest that the instability is intrinsic to the beam, albeit modified by the action of control loops. Another feature of the instability is that the growth rate increases when the gain of the second harmonic corrector (SHC) loop is reduced, which suggests the instability would be even stronger without the SHC loop. *A key observation* is that short bunches (i.e. small longitudinal emittance) are stable, whereas long bunches of equal intensity (but large emittance) are unstable; and this indicates some kind of 'critical' bunch length.

### 1.2. Beam Transfer Functions

Adding a second harmonic voltage component to the RF waveform has three consequences:

- There are twice as many inputs and outputs
- All beam transfer functions are altered
- One must answer: "how do we synchronize the two RF waves?"

The implication of the first item is that the beam response is given by a matrix, with each of the elements given by a frequency dependent transfer function:

Equation 1: $$\begin{bmatrix}\phi_{b1}\\ \phi_{b2}\end{bmatrix}=\begin{bmatrix}B_{11}(s) & B_{12}(s)\\ B_{21}(s) & B_{22}(s)\end{bmatrix}\begin{bmatrix}\phi_{v1}\\ \phi_{v2}\end{bmatrix}.$$

Each of the phase modulation transfer functions $B_{ij}$ depends on the beam distribution (e.g. bunch length) and on the relative amplitude and phasing of the voltage components. For example, voltages in-phase gives *short bunch operation*, whereas voltages anti-phased gives *long bunch operation.* Moreover, each of the TFs is contributed to by many azimuthal modes $m$. However, since they are the only ones observed we shall restrict to $m$=1,3. The phase of the 2nd harmonic beam component is not monitored and the matrix components $B_{21}$ and $B_{22}$ are not essential to our discussion.

For the case of RF voltages in-phase, the synchrotron frequency is a monotonic decreasing function of amplitude; i.e. similar to the case of single RF system, except the frequency spread is larger and the small amplitude oscillation frequency is higher.
For the case of RF voltages anti-phased (voltage ratio $V_2/V_1 = -1/2$) and non-accelerating RF buckets, the computer program BTF [1,5] can calculate the relevant transfer functions. Examples are given in Figs. 2 & 3.

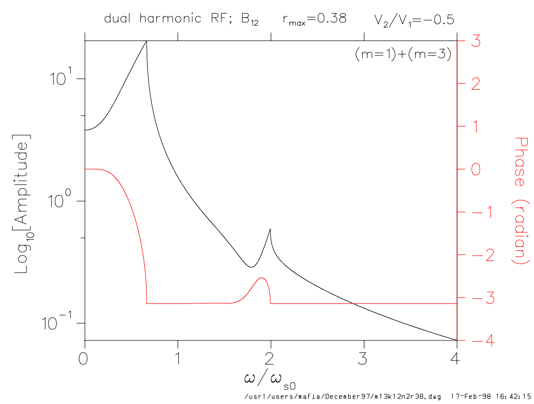


Figure 2: $B_{12}$ for small emittance bunch

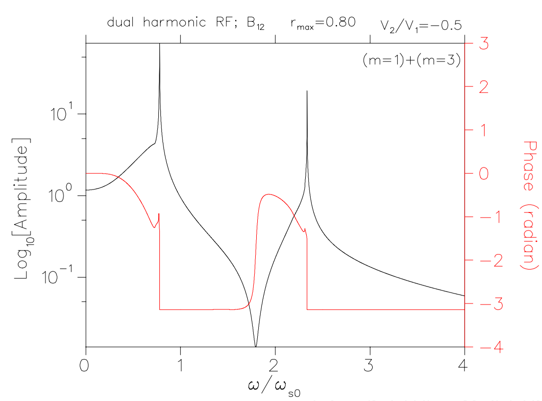


Figure 3: $B_{12}$ for large emittance bunch

The incoherent synchrotron frequency initially increases with amplitude, reaches a maximum $\omega_c = 0.7786\omega_{s0}$ at a critical amplitude $r_c$=0.7235 (i.e. bunch length 233.105°), and then decreases toward zero. This implies the oscillation frequency is not a single valued function of amplitude ($r$); and so two P.V. integrals and residues must be calculated. Moreover, where $\partial\omega_s/\partial r = 0$ the local density of oscillators per unit frequency becomes infinite, leading to a resonant response of the form: Equation 2: $B_{1k}(s) = \sum_m -\beta_{m,1k}\, j\omega_m^2 \Big/ \sqrt{s^2+\omega_m^2}$ , where $k$=1 or $k$=2 and $\omega_m = m\omega_c$.

## 2. CONTROL STRATEGIES

The issue of "how to synchronize" can be answered in three possible ways:

- Perfect feed-forward of 2nd harmonic (academic)
- 2nd harmonic RF locked to voltage fundamental
- 2nd harmonic RF locked to beam fundamental.

The latter two cases of dual harmonic are elaborated in the following sections and analysed according to the Nyquist criterion. Consider a system with forward gain $A$, and gain $B$ in the feedback path. The open loop gain is $A{\times}B$. Assuming *negative feedback*, if there are frequencies at which the phase-shift $\angle AB = -\pi$ and the gain $|AB| \geq 1$ then the system will be unstable when the loop is closed. If *positive feedback* is used, the system is unstable if the clockwise phase-shift $\angle AB = 0$ and the gain $|AB| \geq 1$ for certain frequencies.

### *2.1. 2nd RF locked to voltage fundamental*

The control scheme is shown in Figure 4. $K_1$ is the beam phase loop gain and $K_2$ is the second harmonic corrector (SHC) gain. It should be evident that the beam response $B_{12}$ is outside the domain of control of the SHC loop. Because there are several loops we must be careful to specify which are closed and which is open. Suppose the SHC loop is closed and the phase loop (at fundamental) is opened just before the summing point of $\Delta\phi_{v1}$ with $\phi_{v1}^{ref}$ . If the SHC gain is large, then phase $\phi_{v2}$ is a copy of $\phi_{v1}$. Hence the open loop transfer function is:

Equation 3: $\Delta\phi_{v1} = \dfrac{K_1 e^{-sT}}{s}\{B_{12}(s) + B_{11}(s) - 1\}\phi_{v1}^{ref}$ .

For short bunches, the dipole and sextupole resonances of $B_{11}$ and $B_{12}$ are both damped progressively as $m$=1,3,5; so if the gain-delay product is small enough for stability at $m$=1, then it is also small enough at the higher resonances. As the bunch is lengthened, so the sextupole mode becomes more prevalent, particularly for excitation by 2nd harmonic, and the form of $B_{12}$ departs from that of $B_{11}$. For long bunches both $B_{11}$ and $B_{12}$ are multiply resonant (with no Landau damping) at the same frequency locations. Despite these changes, the gain and phase-shift are probably *not* adequate to induce an instability (even for bunches beyond the critical length) *unless* there are delays in the feedback. However when the phase-loop feed-back is delayed, then it is almost inevitable that the additional negative phase-shifts will induce an instability where the sum of $B_{12}$ and $B_{11}$, may be large; such as in the vicinity of $3\omega_c$. The open-loop Bode plot for beam and phase loop with delay T=15µs confirms that the phase passes through 0 clockwise where the gain is large, and anti-clockwise where the gain is small; and so the system is unstable

### 2.2. Physical Explanation

In simple terms, the mechanism of instability is as follows: the beam response is resonant at $3\omega_c$ and at this frequency the loop delay is large enough that the correction pushes the beam in the wrong direction because the correction uses information which is too old.

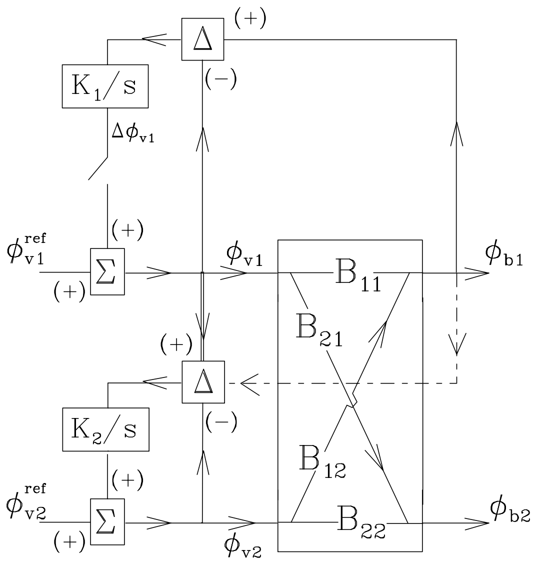


Figure 4: system schematic for 2nd harmonic locked to voltage (bold line) or beam (dashed) fundamental

### 2.3. 2nd RF locked to beam fundamental

Suppose that $\phi_{b1}$ replaces $\phi_{v1}$ as the input to the phase discriminator of the SHC, as indicated by the dashed line in Figure 4. Imagine, now, that the SHC loop is closed on the beam fundamental. This situation constitutes a feedback with a slightly unusual topology. The beam response $B_{12}$ is inside the SHC loop and also partially inside the phase-loop. As a consequence, it is the joint beam response $\phi_{b1} = \left[B_{11}/(1-B_{12})\right]\phi_{v1}$ which under the control of the phase-loop (assuming the SHC gain is large enough). At d.c. the response is exactly unity whether bunches are long or short, so d.c. errors are not amplified. At HF the response tends to zero. Further, the resonances of $B_{12}$ and $B_{11}$ occur at approximately the same frequencies, and exactly so for bunches longer than the critical length. When one imagines the response of long bunches, at the dipole and sextupole resonances, either or both of two things can happen. (i) The transfer function gain in the vicinity of the resonant frequencies $m\omega_c$ will be much reduced; and/or (ii) the denominator of the transfer function behaves as a *phase advance network*. The details depend on bunch length; but typically the phase-advancing effect dominates at *m*=3 and the gain reduction at *m*=1.

In more detail, let us ask: "is the system stable when the beam-phase loop is closed?" We answer in terms of the gain and phase of the open-loop transfer function:

Equation 4: $$\Delta\phi_{v1} = \frac{K_1 e^{-sT}}{s}\left\{\frac{B_{11}(s)}{1-B_{12}(s)} - 1\right\}\phi_{v1}^{ref},$$

with loop delay T=15µs and $K_1 \cong 25$ kHz.

In the previous synchronisation scheme discussed, the system was stable until we introduced the effect of loop delay. However for the scheme discussed here, there may be large phase-shifts due to loop delay but in the vicinity of the *m*=3 resonance, the phase passes through 0 clockwise where the gain is small, and anti-clockwise where the gain is large; and so the system is stable according to the Nyquist condition.

### 2.4. Physical explanation

The mechanism of stabilisation is as follows. Suppose that either due to drive by $\phi_{v1}^{ref}$ or spontaneous self-excitation, the beam phase starts to oscillate; then the SHC loop will generate a signal $\phi_{v2}$ that, in turn, acts through $B_{12}$ (i.e. through the 2nd harmonic cavity) to modify the initial oscillation $\phi_{b1}$. This signal (which includes a phase advance) is then acted upon by the beam fundamental phase-loop. Actually, to be strictly correct, the time-ordering depends on the relative band-widths of the two loops.

## 3. CONCLUSION

The mechanism for the instability when feedback is from the cavity has two ingredients:

- the large gain of the BTF when the bunch length approaches or exceeds the critical length,
- the large phase-shifts that are contributed by the long loop delays.

The critical length is that for which the derivative of synchrotron frequency with respect to action is zero

The explanation for the stabilization when the feedback is from the beam, is due to modification of the beam transfer function via either a phase-advance mechanism or/and a reduction of the gain.

## REFERENCES


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