MOMENTUM APERTURE OF THE ADVANCED LIGHT SOURCE

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Abstract

The lifetime of a low energy, small emittance synchrotron radiation source like the ALS is usually limited by the momentum aperture of the ring. Large momentum apertures are reached by providing enough rf-voltage for a large bucket and by avoiding the degradation of the momentum aperture due to linear or non-linear synchro-betatron coupling where the particle exceeds the transverse aperture of the machine. We show measurements of the changing momentum aperture of the ALS due to transverse aperture restrictions, coupling, implementation of insertion devices, and sextupole settings.

1 THE MOMENTUM APERTURE

When electrons scatter within a bunch, they may transfer enough momentum to be outside the momentum aperture of the storage ring. This effect is proportional to the electron intensity within a bunch. Assuming a flat beam, and thus the main contribution of the velocity spread coming from horizontal motion (σ'_x), the Touschek lifetime becomes [1]:

$$\frac{1}{\tau_{tou}} \sim \frac{1}{E^3} \frac{I_b}{V_b \sigma'_x} \frac{1}{\varepsilon^2} f(\varepsilon, \sigma'_x, E) \tag{1}$$

 I_b is the bunch current, V_b the bunch volume, and ε is the momentum aperture of the storage ring. The momentum aperture is determined by two different processes: the height of the rf-bucket and the maximum stable transverse amplitudes.

The height of the rf-bucket provided by the accelerating voltage in the cavity is:

$$\varepsilon_{RF} \sim \pm \sqrt{\frac{V_{RF}}{\alpha h E}}$$
 (2)

where ε_{RF} is the maximum relative momentum deviation $(\frac{dP}{P}_{max})$, V_{RF} is the rf-voltage, α the momentum compaction factor, and *h* the harmonic number. Equation 2 is only valid for rf-voltages much higher than the energy loss per turn. Presently ε_{RF} is limited to $\approx 3\%$ in the ALS.

The transverse motion of particles is limited by the vacuum chamber aperture, which for the horizontal case is reduced by the off energy orbit. The invariant physical horizontal aperture is:

$$A_{phys,x}(\delta) = \min_{s \in [0,L]} \frac{(x_{vc.}(s) - \eta(s)\delta)^2}{\beta_x(s)}$$
(3)

With η , β being the dispersion and β -function, δ the relative momentum deviation, and x_{vc} , the vacuum chamber aperture. Dispersion is usually only present in the horizontal plane, thus the vertical physical aperture is computed by equation 3 without dispersion, leaving $A_{phys,y}$ momentum independent.

The motion is also confined by dynamic effects, leading to a dynamic aperture $A_{dyn,x}$. One can think of the dynamic limit as follows: Particles get lost when their tune satisfies a resonance condition. From knowing the tune shift terms with amplitude and energy, $\frac{\partial \nu_y}{\partial A_x}$, $\frac{\partial \nu_y}{\partial A_y}$, and $\frac{\partial^2 \nu_y}{\partial \delta^2}$ one can compute the tune shift due to momentum and transverse deviations:

$$\Delta \nu_y = \frac{\partial \nu_y}{\partial A_x} A_{dyn,x} + \frac{\partial \nu_y}{\partial A_y} \kappa A_{dyn,x} + \frac{\partial^2 \nu_y}{\partial \delta^2} \delta^2 + \cdots$$
(4)

where $\Delta \nu_y$ is the distance to the closest 'deadly' resonance, and κ is the emittance coupling factor. This defines a momentum dependent dynamic aperture. The transverse momentum dependent aperture is given by the minimum of the physical and the dynamic aperture.

After a momentum change due to a scattering within the bunch, the particle starts a betatron oscillation around the dispersion orbit. The induced linear invariant amplitude is:

$$a_{ind,x}(s,\delta) = H(s)\delta^2 \tag{5}$$

where: $H(s) = \gamma(s)\eta(s)^2 + 2\alpha(s)\eta(s)\eta(s)' + \beta(s)\eta(s)'^2$. In the presence of coupling, the invariant particle amplitude couples in the vertical plane by $a_{ind,y}(s,\delta) = \kappa a_{ind,x}(s,\delta)$.

The induced amplitude should not exceed the maximum allowable transverse amplitude:

$$H(s)\delta^{2} \leq \min\left[A_{phys,x}(\delta), A_{dyn,x}(\delta), \frac{1}{\kappa}A_{phys,y}(\delta)\right]$$
(6)

Solving this equation for the maximum δ around the ring gives a position dependent momentum aperture, which we call $\varepsilon_{trans}(s)$. The absolute momentum aperture is the smaller of ε_{RF} and $\varepsilon_{trans}(s)$ at any position in the ring. As bunch volume as well as momentum aperture vary around the ring, the Touschek lifetime has to be averaged around the ring.

A. Nadji et al [2] and others pointed out that the straight forward application of the above formulas leads to wrong results for large momentum deviations. Strong sextupoles lead to higher order dispersion which alters the off momentum closed orbit. In addition, the Twiss functions also vary with the beam energy, thus changing the H(s) function into $H(\delta, s)$. However, at the ALS the linear, momentum independent calculations of the optical functions and the closed orbit vary only up to 5 % for momentum deviations of up to 5 %.

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Figure 1 shows a comparison of the maximum allowable apertures and the induced oscillation amplitudes at different locations in the ring. The thin lines represent the in-



Figure 1: Contributions of the different aperture effects to the total momentum aperture. The thin lines are the invariant induced amplitudes at different positions in the ring.



Figure 2: Momentum aperture along one cell of the ALS storage ring.

duced amplitudes in the straight section (lowest), and in the arc section (highest). Thus we can distinguish in principle two different sections (see figure 2). In the straight sections the momentum aperture will be defined by the smallest of the rf-bucket height or the dynamic momentum aperture. In the arcs the momentum aperture will be defined by the dynamic momentum aperture, or for large coupling by the vertical physical aperture.

2 MEASUREMENTS OF THE MOMENTUM APERTURE

Measurements of the Touschek lifetime as a function of the rf-voltage were done under different storage ring conditions. The synchrotron tune was measured simultaneously, from which one can calculate the bunch length and the rfbucket height.

Figure 3 shows measurements with 1% coupling, while in figure 4 the coupling was adjusted to $\approx 10\%$ with the help of skew quadrupoles. In addition a wiggler (2 T peak field, 3 m long) was open (circles) or closed (crosses) in both cases.



Figure 3: Beam lifetime as a function of the rf-bucket height with no additional coupling and wiggler open (circles) and wiggler closed (crosses). 8 mA in 8 bunches were stored at 1.5 GeV.



Figure 4: Beam lifetime as a function of the rf-bucket height with additional coupling of 10 % and wiggler open (circles) and wiggler closed (crosses). 8 mA in 8 bunches were stored at 1.5 GeV.

The data are fitted by applying equation 1 with the following fit parameters: Assuming an initial 1% coupling the bunch volume is corrected by a constant factor that takes into account any volume changes like variation of the coupling, instabilities, etc. The bunch volume is also adjusted according to the changing rf-voltage. The other parameters are the momentum apertures ε_{trans} in the straight section and in the arc.

Table 1 summarizes the results for the measurements shown above as well as for other measurements with smaller vertical apertures and different sextupole settings.

For low coupling (1%) there is almost no difference in the momentum aperture no matter if the wiggler is closed or open. For high coupling (10%) the momentum aperture stays the same for the case without wiggler. If the vertical aperture is reduced in this case, the momentum aperture reduces as is expected for this reduced physical aperture. If coupling is introduced and the wiggler is closed, the aperture is degraded. This limits the performance of the ALS, where additional coupling is usually introduced by using skew quadrupoles to increase the lifetime for user operations [3].

The momentum aperture degradation for high sextupole settings (high chromaticities $\xi_{x,y}$) is the same with and without wiggler, indicating that for these cases the sextupoles seem to be the major cause of the reduced dynamic aperture.

Table 1: Measured momentum aperture in the straight and in the arc section for different machine conditions.

κ	$\xi_{x,y}$	A_y	measured ε [%]	
[%]		[mm mrad]	straight	arc
Wiggler off				
1	1.0	3.0	> 2.8	2.3
10	1.0	3.0	> 2.8	2.3
15	1.0	0.56	> 2.8	1.4
10	0.0	3.0	> 2.8	2.8
10	6.0	3.0	1.5	1.4
Wiggler on				
1	1.0	3.0	> 2.8	2.2
10	1.0	3.0	> 2.8	1.7
10	1.0	0.56	1.6	1.3
10	0.0	3.0	> 2.8	1.7
10	6.0	3.0	1.6	1.4

Assuming tune shifts with amplitude and energy calculated by the optics model, equation 4 is used to calculate a maximum tune shift which corresponds to the observed momentum apertures. This allows also to predict a transverse on-momentum dynamic aperture, which can be compared with independent measurements.

The predicted transverse apertures are in good agreement in some cases and not in others. From the two low coupling cases one calculates a $\Delta \nu_y = 0.08^{-1}$ and a horizontal on-momentum dynamic aperture of $A_x \approx 1.1 \cdot 10^{-5}$ m rad. This aperture agrees well with independent measurements of the dynamic aperture with scrapers [3] and beam kicker magnets. When coupling is introduced, the predicted aperture is 50 % higher than measured with a beam kicker.

For the cases with high chromaticities, the simple model agrees best with $\Delta \nu_y = 0.05$. But the parabolic shape of the momentum aperture in the model is not represented in the measurements. Up to now no kicker measurements exist for these cases.

In figure 5 we show tracking results for a machine with 10% coupling, and chromaticities of 1 and 6 respectively. The known normal (but not skew) quadrupole gradient errors were included to obtain a more accurate model of the present linear optics of the ALS [4]. Real physical apertures were also included in the tracking to confine the transverse motion. In figure 5 we also plot the momentum apertures obtained from equation 4 with the tune shift $\Delta \nu_y$ calculated from the measurements. The off-momentum transverse apertures from the tracking agree well with the

¹The resonance which is approached is the $\nu_y = 8$ integer resonance.

measurements. The on-momentum transverse apertures are much higher than the ones predicted from the simple aperture model. Kicker measurements also give much lower on-momentum apertures. This is why we think that our simple model represents the real apertures better than the model which is used in the tracking. Perhaps orbit errors or other effects which have not been included in the tracking would give more realistic results. Further studies are presently under way.



Figure 5: Horizontal aperture as a function of the momentum deviation for $\xi_{x,y} = 1.0$ (solid line) and $\xi_{x,y} = 6$ (dashed line). Tracking results are plotted with circles, while the lines represent the aperture obtained from the simple model with $\Delta \nu_y(\xi_{x,y} = 1.0) = 0.08$ and $\Delta \nu_y(\xi_{x,y} = 6) = 0.05$.

3 CONCLUSION

Through measurements of the Touschek lifetime we are able to derive the momentum aperture of the Advanced Light Source. We find that the momentum aperture is not entirely determined by the rf-voltage (ε_{RF}). Also the physical apertures are large enough (for coupling up to 10%) that they do not limit the momentum aperture. The aperture is defined by dynamic effects, which are only partially described by our models. It is important to understand these dynamic limits to improve the lifetime in the ALS and accurately predict the lifetime (and dynamic aperture) in future machines.

4 **REFERENCES**

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