

# CAPILLARY WAVEGUIDE FOR LASER ACCELERATION IN VACUUM, GASES, AND PLASMAS

MING XIE, Lawrence Berkeley National Laboratory, Berkeley, USA

## Abstract

I propose a new method for laser acceleration of relativistic electrons using the leaky modes of a hollow dielectric waveguide. The hollow core of the waveguide can be either in vacuum or filled with uniform gases or plasmas. In case of vacuum and gases,  $TM_{01}$  mode is used for direct acceleration. In case of plasmas,  $EH_{11}$  mode is used to drive longitudinal plasma wave for acceleration. Structure damage by high power laser is avoided by choosing a core radius much larger than laser wavelength.

## 1 EIGENMODE PROPERTIES

The capillary waveguide considered here is made of a hollow core with an index of refraction  $\nu_1$  and radius  $R$ , embedded in a dielectric medium with an index of refraction  $\nu_2$ . We are interested in the regime with  $\lambda_1/R \ll 1$ , where  $\lambda_1 = \lambda/\nu_1$  and  $\lambda$  is the wavelength in vacuum. As a result, the EM wave in the core is dominantly transverse. Assuming  $\sqrt{\nu^2 - 1} \gg \lambda_1/R$ , where  $\nu = \nu_2/\nu_1$ , the eigenmodes of the waveguide can be solved following the same procedure by Marcatili and Schmeltzer [1].

Expressing the eigenmodes in the following form

$$\begin{Bmatrix} \mathbf{E} \\ \mathbf{H} \end{Bmatrix} = \begin{Bmatrix} \mathbf{E}_{lm}(r, \phi) \\ \mathbf{H}_{lm}(r, \phi) \end{Bmatrix} e^{i(\beta_{lm}z - \omega t) - \alpha_{lm}z}, \quad (1)$$

the eigenvalues are given by

$$\beta_{lm} = \frac{2\pi}{\lambda_1}(1 - 1/2\gamma_g^2), \quad \alpha_{lm} = \chi/\gamma_g^2 R, \quad (2)$$

where  $\gamma_g = 2\pi R/U_{lm}\lambda_1 \gg 1$ , and  $U_{lm}$  is the  $m$ th root of the equation,  $J_{l-1}(U_{lm}) = 0$ . There are three types of modes, corresponding to

$$\chi = \begin{cases} \frac{1}{\sqrt{\nu^2 - 1}} & : TE_{0m} (l = 0) \\ \frac{\nu^2}{\sqrt{\nu^2 - 1}} & : TM_{0m} (l = 0) \\ \frac{\nu^2 + 1}{2\sqrt{\nu^2 - 1}} & : EH_{lm} (l \neq 0) . \end{cases} \quad (3)$$

For laser acceleration, we are interested primarily in two modes:  $TM_{01}$  mode for acceleration in vacuum and in gases, and  $EH_{11}$  mode for acceleration in plasmas. Correspondingly, we consider three cases:  $\delta\nu_1 = 0$  when the core is in vacuum,  $\delta\nu_1 > 0$  and  $\delta\nu_1 < 0$  when the core is filled with gases and plasmas, respectively, where  $\delta\nu_1 = \nu_1 - 1$  and  $|\delta\nu_1| \ll 1$ . It is noted that  $EH_{11}$  mode is often designated as  $HE_{11}$  mode elsewhere in literature, in this paper we follow the notation in reference [1].

The  $TM_{01}$  mode is given to the leading order by

$$(r \leq R) : \begin{cases} E_z = E_a J_0(k_{r1}r) \\ E_r = -i\gamma_g E_a J_1(k_{r1}r) \\ H_\phi = E_r/Z_0, \end{cases} \quad (4)$$

similarly,  $EH_{11}$  mode is obtained as

$$(r \leq R) : \begin{cases} E_y = E_0 J_0(k_{r1}r) \\ H_x = -E_x/Z_0, \end{cases} \quad (5)$$

where  $E_a$  is the peak acceleration field,  $E_0$  the peak transverse field,  $Z_0$  the vacuum impedance, and  $k_{r1} = (U_{lm} - i\chi/\gamma_g)/R$ . Only the dominant transverse components are specified for  $EH_{11}$  mode. For  $r \geq R$ , all fields have the radial dependence  $\exp(ik_{r2}r)/\sqrt{r}$ , where to leading order  $k_{r2} = k_1\sqrt{\nu^2 - 1}$ . A non-vanishing imaginary part of  $\nu$  due to slightly lossy dielectric medium will give rise to exponential decay of fields in radial direction.

Notice  $EH_{11}$  mode is linearly polarized, while  $TM_{01}$  mode is radially polarized. However, when necessary, linearly polarized mode can be formed by a proper mixing of  $TM_{01}$  with another hybrid mode,  $EH_{21}$  mode, while preserving  $E_z$  of  $TM_{01}$  mode on the axis. For the three modes we have  $U_{11} = 2.405$ , and  $U_{01} = U_{21} = 3.832$ .

An important concern for using a waveguide for laser acceleration is power damage on the structure. To evaluate surface field, we expand the dominant transverse field at  $r = R$  using the expression for  $k_{r1}$  and obtain

$$\frac{E_s}{E_a} = \chi |J_0(U_{01})| : TM_{01}, \quad (6)$$

$$\frac{E_s}{E_0} = \frac{\chi}{\gamma_g} |J_1(U_{11})| : EH_{11}. \quad (7)$$

For  $TM_{01}$  mode, surface field is of the same order as the peak acceleration field. For both modes surface fields are much smaller than the peak transverse field.

Coupling between the waveguide modes to free space Gauss-Laguerre modes is very efficient. When focused at the waveguide input cross section, power coupling from  $TEM_{01}$  to  $TM_{01}$  reaches a maximum of 97% at  $w_0/R = 0.56$ , and from  $TEM_{00}$  to  $EH_{11}$  is 98% at  $w_0/R = 0.64$ , where  $w_0$  is the Gaussian beam waist. Despite the fact that the modes are leaky the guiding can be quite effective with rather long  $1/e$  power attenuation length  $L_{attn} = \gamma_g^2 R/2\chi$ .

## 2 ACCELERATION IN VACUUM

According to Eq.(2), phase velocity of the  $TM_{01}$  mode is larger than the speed of light,  $c$

$$v_p = \frac{\omega}{\beta_{01}} = \frac{c}{1 - 1/2\gamma_g^2}. \quad (8)$$

We define an acceleration phase slippage length over which a relativistic electron, while gaining energy, slips a full  $\pi$  phase with respect to the acceleration wave

$$L_a = \frac{\lambda}{1/\gamma_g^2 + 1/\gamma^2}. \quad (9)$$

Over this distance, energy gain of the electron on-axis is

$$\Delta W_a = eE_a \int_0^{L_a} \sin(\pi z/L_a) dz = eE_a L_a T_a, \quad (10)$$

where  $T_a = 2/\pi$  is a reduction factor due to a  $\pi$  phase slippage during acceleration. Here we have neglected the small attenuation of the acceleration field over a distance  $L_a$ . In parallel, let's also define a deceleration length,  $L_d$ , over which the electron slips another  $\pi$  phase while losing energy  $\Delta W_d = eE_a L_d T_d$ , where  $T_d$  can be different from  $T_a$  if  $L_d/L_a \neq 1$ . The average acceleration gradient during a period of  $2\pi$  phase slippage is then given by

$$G = \frac{\Delta W_a - \Delta W_d}{L_a + L_d} = G_a \frac{1 - (L_d/L_a)(T_d/T_a)}{1 + L_d/L_a}, \quad (11)$$

where  $G_a = \Delta W_a/L_a = eE_a T_a$ . To have net acceleration, the ratio  $L_d/L_a$  should be made small. This can be done by introducing a magnetic field during the half period of deceleration. The effect of magnetic field is to reduce longitudinal velocity of the electron such that it slips faster, thus taking less time or shorter distance,  $L_d$ .

For simplicity, we assume the field is sinusoidal with a period  $\lambda_w$ ,  $B_y = B_0 \cos(2\pi z/\lambda_w)$ . The length  $L_d$  for a  $\pi$  phase slippage is then defined by

$$\left[ \frac{1}{\gamma_g^2} + \frac{1}{\gamma^2} + \frac{a_w^2}{\gamma^2} \right] \frac{\pi L_d}{\lambda} - \frac{a_w^2 \lambda_w}{2\gamma^2 \lambda} \sin \left[ \frac{4\pi L_d}{\lambda_w} \right] = \pi, \quad (12)$$

where  $a_w = eB_0 \lambda_w / 2\pi \sqrt{2} mc$ . If we set  $\lambda_w = L_d$  then

$$L_d = \frac{\lambda}{1/\gamma_g^2 + 1/\gamma^2 + a_w^2/\gamma^2}, \quad (13)$$

and  $a_w$  is now determined by

$$a_w = \sqrt[3]{Q_1 + \sqrt{Q_1^2 + Q_2^3}} + \sqrt[3]{Q_1 - \sqrt{Q_1^2 + Q_2^3}}, \quad (14)$$

with  $Q_1 = eB_0 \lambda \gamma^2 / 4\pi \sqrt{2} mc$  and  $Q_2 = [1 + (\gamma/\gamma_g)^2]/3$ . Due to longitudinal oscillation,  $T_d$  is different from  $T_a$

$$T_d = \frac{1}{\pi} \int_0^\pi \sin[\theta - \kappa \sin(4\theta)] d\theta, \quad (15)$$

where  $\kappa = (1 - L_d/L_a)/4$ . The value of  $T_d$  varies in the range  $\{1.84 \leftrightarrow 2\}/\pi$  for  $L_d/L_a$  in the range  $\{0 \leftrightarrow 1\}$ . We have assumed the electron is decelerated by the on-axis value of  $E_z$ , but as the electron is deflected off-axis, it will see a weaker field. The maximum transverse orbital offset in the wiggler field is  $\Delta X_{max} = \sqrt{2} a_w \lambda_w / \pi \gamma$ .

Due to magnetic deflection, electron will radiate and lose energy. Energy loss per wiggler period is

$$\Delta W_s = \frac{8\pi^2 mc^2}{3} \left( \frac{r_e}{\lambda_w} \right) a_w^2 \gamma^2, \quad (16)$$

where  $r_e$  is the classical radius of electron. The maximum possible energy that can be accelerated with this method can be determined by the condition:  $\Delta W_a > \Delta W_d + \Delta W_s$ .

Transverse forces due to EM wave do not cancel to order of  $1/\gamma^2$  in a waveguide mode, thus giving rise to either focusing or defocusing depending on acceleration phase,  $\phi_a$ , which varies constantly due to slippage. The corresponding beta function is

$$\beta_t = \gamma_g \lambda \sqrt{(\gamma mc^2 / \pi e \lambda E_a \sin \phi_a) / [1 - (\gamma_g/\gamma)^2]}, \quad (17)$$

the term grouped in the first bracket on the right is the ratio of electron energy to its energy gain per wavelength.

### 3 ACCELERATION IN GASES

The phase velocity of the  $TM_{01}$  mode in gases is given by

$$v_p = \frac{\omega}{\beta_{01}} = \frac{c}{1 - 1/2\gamma_g^2 + \delta\nu_1}. \quad (18)$$

The corresponding phase slippage length is then

$$L_{slip} = \frac{\lambda}{|1/\gamma_g^2 + 1/\gamma^2 - 2\delta\nu_1|}. \quad (19)$$

The phase matching condition is obtained by making the denominator zero, thus  $\delta\nu_1 = 1/2\gamma_g^2 + 1/2\gamma^2$ . This condition suggests an alternative way to maintain phase matching as  $\gamma$  increases during acceleration: instead of varying  $\delta\nu_1$ , one may change  $\gamma_g$  by tapering waveguide radius. The beta function in gases is smaller than that in vacuum by a factor of  $\sqrt{2}$  in the limit  $\gamma_g/\gamma \ll 1$ .

### 4 ACCELERATION IN PLASMAS

Wave equation for laser field propagation in weakly relativistic plasmas under cold fluid condition is given by

$$\left[ \nabla^2 - \frac{1}{c^2} \frac{\partial^2}{\partial t^2} \right] \mathbf{E}_l = \frac{\omega_p^2}{c^2} \left[ 1 + \frac{\delta n}{n_0} - \frac{a^2}{2} \right] \mathbf{E}_l. \quad (20)$$

The plasma density modulation,  $\delta n/n_0$ , driven by the ponderomotive potential of a laser pulse,  $a^2 = \langle |e\mathbf{E}_l/mc\omega|^2 \rangle$ , will generate a wakefield,  $\mathbf{E}_w = -\nabla\Phi$ , where the wake potential,  $\Phi$ , is determined by

$$\left[ \frac{\partial^2}{\partial t^2} + \omega_p^2 \right] \Phi = \omega_p^2 \frac{mc^2}{e} \frac{a^2}{2}. \quad (21)$$

To close the loop,  $\delta n/n_0$  is given by Poisson's equation:  $\nabla^2 \Phi = (e/\varepsilon_0) \delta n/n_0$ , where  $\omega_p = \sqrt{e^2 n_0 / \varepsilon_0 m}$  and  $\varepsilon_0 = 1/Z_0 c$ . The approach here parallels to that in reference [2].

Under the condition  $a^2 \ll 1$ , we will have  $\delta n/n_0 \ll 1$ . As a result, the second and third term on the right of Eq.(20) can be dropped and the wave equation is then decoupled from the plasma equations. The only effect of plasma on laser propagation is through an index of refraction  $\nu_1 = 1 - \omega_p^2/2\omega^2$ . Consider a capillary tube filled with a uniform plasma of density  $n_0$ , a laser pulse propagating through the waveguide will excite a wakefield with phase velocity

equals the group velocity of the pulse. For  $EH_{11}$  mode, the group velocity is given by

$$v_g = \frac{d\omega}{d\beta_{11}} = \frac{c}{1 + 1/2\gamma_g^2 + 1/2\gamma_p^2}, \quad (22)$$

where  $\gamma_p = \omega/\omega_p \gg 1$ . Introducing a variable  $\zeta = z - v_g t$ , Eq.(21) can be solved as

$$\Phi = -(k_p m c^2 / e) \int_{\zeta}^{\infty} d\zeta' \sin[k_p (\zeta - \zeta')] \frac{a^2}{2}, \quad (23)$$

where  $k_p = \omega_p / v_g$ . For a Gaussian pulse of  $EH_{11}$  mode

$$a^2(\rho, \zeta) = \frac{a_0^2}{2} J_0^2(U_{11}\rho) e^{-\zeta^2/2\sigma_z^2 - \zeta/L_{attn}}, \quad (24)$$

where  $\rho = r/R$ , the wake potential behind the pulse is

$$\Phi = -\Phi_0 J_0^2(U_{11}\rho) e^{-z/L_{attn}} \sin(k_p z - \omega_p t), \quad (25)$$

$$\Phi_0 = (\sqrt{2\pi} m c^2 / 4e) a_0^2 k_p \sigma_z e^{-(k_p \sigma_z)^2/2}. \quad (26)$$

Longitudinal wakefield is then given by

$$E_{wz} = E_a J_0^2(U_{11}\rho) e^{-z/L_{attn}} \cos(k_p z - \omega_p t), \quad (27)$$

and transverse wakefield by

$$E_{wr} = -2(\gamma_p/\gamma_g) E_a J_0(U_{11}\rho) J_1(U_{11}\rho) e^{-z/L_{attn}} \sin(k_p z - \omega_p t), \quad (28)$$

where the peak acceleration field,  $E_a = \Phi_0 k_p$ , is maximized if the laser pulse length is chosen according to the condition  $k_p \sigma_z = 1$ . From here on, we will use this optimal condition wherever it is relevant.

There are several characteristic length parameters for laser wakefield acceleration. First, the slippage length is

$$L_{slip} = \frac{\lambda_p}{|1/\gamma_g^2 + 1/\gamma_p^2 - 1/\gamma^2|}. \quad (29)$$

Next, the pump depletion length,  $L_{pump}$ , is defined by the condition  $W_l = W_w$ , where  $W_l$  is the initial energy of the laser pulse given by

$$W_l = \frac{\sqrt{2\pi} J_1^2(U_{11}) R^2 \lambda_p E_0^2}{4Z_0 c}, \quad (30)$$

and  $W_w$  is the energy in the wakefield the laser pulse left behind as it propagates a distance  $L_{pump}$ , given by

$$W_w = (\pi m c / 4e)^2 [\varepsilon_0 / \text{Exp}(1)] a_0^4 \omega_p^2 R^2 L_{pump} [I_z + (\gamma_p/\gamma_g)^2 I_r]. \quad (31)$$

The two terms above correspond to energy in the longitudinal and transverse wakefield, respectively, and the defined integrals have the value  $I_z = \int_0^1 d\rho \rho J_0^4(U_{11}\rho) = 0.0762$ ,  $I_r = \frac{1}{4} \int_0^1 d\rho \rho J_0^2(U_{11}\rho) J_1^2(U_{11}\rho) = 0.00635$ . We then have

$$L_{pump} = [4\sqrt{2\pi} J_1^2(U_{11}) \text{Exp}(1) / \pi^2] \frac{\lambda_p}{a_0^2 (I_z/\gamma_p^2 + I_r/\gamma_g^2)}. \quad (32)$$

In calculating  $W_w$  we have left out the attenuation factor  $\exp(-z/L_{attn})$  since it is characterized by a separate quantity,  $L_{attn}$ . Given group velocity dispersion, Eq.(22), a pulse will double its length over a propagation distance

$$L_{disp} = \frac{\sqrt{3} \gamma_p^2 \lambda}{\pi(1/\gamma_g^2 + 1/\gamma_p^2)}. \quad (33)$$

Finally, the beta function due to the transverse wakefield is

$$\beta_t = (2/U_{11}) [\text{Exp}(1)/2\pi]^{1/4} \sqrt{\frac{\gamma}{\sin \phi_a}} \frac{R}{a_0}. \quad (34)$$

## 5 EXAMPLES

In the following examples we will use  $\lambda = 1\mu\text{m}$ ,  $\nu_2 = 1.5$ , assume an initial electron energy of 1 GeV, set  $\sin \phi_a = 1$ , and neglect the small difference between  $T_a$  and  $T_d$ .

**Table 1.** Laser Acceleration in Vacuum.

$P$ [TW]	100	$E_a$ [GV/m]	3.7
$R/\lambda$	250	$E_s$ [GV/m]	3.0
$\gamma_g$	410	$G$ [GV/m]	1.1
$B_0$ [T]	1.5	$\Delta W_a$ [GeV]	0.38
$a_w$	6.2	$\Delta W_d$ [GeV]	0.15
$L_{attn}$ [m]	10	$\Delta W_s$ [eV]	88
$\beta_t$ [cm]	12	$L_a$ [cm]	16
$\Delta X_{max}/R$	0.35	$L_d$ [cm]	6.2

Acceleration in gases requires  $\delta\nu_1 = 3.1 \times 10^{-6}$ . Energy gain in plasmas is defined by  $\Delta W_a = e E_a L_{slip} T_a$ .

**Table 2.** Laser Acceleration in Plasmas.

$P$ [TW]	20	$E_a$ [GV/m]	0.94
$W_l$ [J]	2.7	$E_s$ [GV/m]	1.7
$a_0$	0.28	$\Delta W_a$ [GeV]	0.56
$\gamma_p$	100	$L_{slip}$ [m]	0.94
$n_0$ [ $10^{17}/\text{cm}^3$ ]	1.1	$L_{pump}$ [m]	126
$R/\lambda$	150	$L_{disp}$ [m]	52
$\gamma_g$	392	$L_{attn}$ [m]	7.9
$\sigma_z$ [ $\mu\text{m}$ ]	16	$\beta_t$ [cm]	1.6

## 6 CONCLUSIONS

I have introduced the concepts and techniques that will significantly advance the development of laser acceleration. This work was supported by the U.S. Department of Energy under contract No.DE-AC03-76SF00098.

## 7 REFERENCES

- [1] E. Marcatili and R. Schmeltzer, *B.S.T.J.*, **43**(1964)1783.
- [2] L. Gorbunov and V. Kirsanov, *Sov. Phys. JETP*, **66**(1987)290.