Numerical Studies of Wake Excitation in Plasma Channels*

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Abstract

The wake fields produced by an intense, short laser pulse propagating in a plasma channel with an arbitrary density profile are investigated. Plasma channels, viewed as accelerating structures, have many desirable features that are not shared by a homogeneous plasma. Moreover, they are becoming experimentally realizable. As part of an overall program to analyze plasma channels as accelerating structures, a new fluid simulation code has been developed with the primary purpose of producing fast tools to explore parameter space for both theoretical investigation of accelerator performance as well as the modeling and design of experiments. This code has flexible physics content, for example, the laser can either be fully resolved temporally or treated as ponderomotive force. An important feature, from the accelerator design point of view, is the capability to study beam propagation dynamics. We present preliminary results consisting of a detailed analysis of the transverse structure of the wake for a wide range of channel profiles.

1 INTRODUCTION

As accelerating structures, plasmas have the desirable ability to support extremely large gradients without experiencing the electrical breakdown which limits the gradient in conventional structures. Since this concept was first introduced, numerous configurations for particle acceleration in plasmas with uniform density have been proposed. (See Esarey *et al.* [1] for a comprehensive review of plasma accelerators concepts.) In the case of laser driven plasma accelerators, the usable accelerating length is determined by the distance over which the laser pulse maintains a high intensity. That is, the efficient use of the laser energy is limited by diffraction of the drive pulse. At very high intensities, relativistic optical guiding and self-channeling serve to limit diffraction and thereby extend the interaction length of the device. However, these mechanisms are inherently nonlinear; one would prefer not to rely on nonlinear processes for device operation.

Guiding in a pre-formed density channel gives the option to operate in a linear regime since such guiding is not dependent on the laser intensity. The original theoretical investigation of guiding in channels, which considered the case of parabolic channels, is due to Sprangle and co-workers[2]. The field structure in a parabolic channel is similar to that in a homogeneous plasma *i.e.* the fields have electrostatic character and the transverse field profile is determined by the driver profile. In contrast, the fields in a hollow channel are electromagnetic with the accelerating gradient being transversely uniform *independent* of the transverse profile of the driver and to lowest order, the focusing fields are weak and linear. Since the original experiment work by Milchberg [3] numerous results in channels have been reported by many labs worldwide. Now that the experimental techniques for the controlled creation of channels is emerging, it is timely to begin systematic studies of the accelerating characteristics of plasma channels.

A consequence of the central advantage of a plasma accelerating structure, namely the ability to support gradients that would result in electrical breakdown in a metallic structure, is that the "wall" of the structure can exhibit a complex dynamics that must be faithfully modeled in order to determine the electrical properties of the structure.

2 CHARACTERIZING PLASMA CHANNELS

An optimized design of a plasma based accelerating structure requires the investigation of a considerable parameter space. By characterizing the structures in terms of various figures-of-merit, one can explore this parameter space in a systematic and controlled way using these characteristics as guide-posts. Here we outline the beginnings of such a search. For preliminary studies, we have found two quantitative characteristics to be quite informative [4, 5]: Q and $[R/Q]$. Parameterizing the time dependence of the accelerating field as $E_z \sim \exp\left[-(i\,\omega+\gamma)(t-z/c)\right]$, we define the quality factor of the cavity, Q, by

$$Q = \frac{\omega}{2\gamma} \tag{1}$$

which determines the number of electron bunches that can be accelerated. This is of interest for both reasons of efficiency as well for constraints imposed by applications (*e.g.* interaction region physics issues in a collider).

Following conventional resonator theory, we define the figure of merit $[R/Q]$ as

$$\left[\frac{R}{Q}\right] \equiv \frac{E_z(0)^2}{\omega_m\,\mathcal{U}_m} \sim k_p\,Z_0, \tag{2}$$

where $E_z(0)$ is the peak accelerating gradient on axis, ω_m is the mode frequency, $\mathcal{U}_m$ is the mode energy per unit length, k_p is the plasma wavenumber, and Z_0 is the impedance of free space. (The last relation follows from dimensional analysis.) For the fundamental mode, $[R/Q]$ characterizes the energy spread imparted to the accelerated beam, whereas for the higher-order modes, $[R/Q]$ characterizes beams instabilities [5]. In more pragmatic terms,

* Supported by the U. S. DoE under grant No. PDDEFG-03-95ER-40936.

$[R/Q]$ can be viewed as a measure of the gradient achieved ($E_z(0)$) for the for the energy invested in the mode ($\mathcal{U}_m$).

3 THE BASIC MODEL

We model the plasma as relativistic, cold, fluid electrons with a neutralizing, immobile ionic background. From here, we make the following approximations:

- quasi-static approximation where all field quantities are assumed to depend only upon $\xi = t - z/c$;
- the higher frequency elements of the laser-plasma interaction are modeled ponderomotively;
- slab geometry.

Given these approximations, the fluid momentum balance equations become

$$\left(1 - \frac{p_z}{\gamma\, m\, c}\right)\frac{\partial p_x}{\partial \xi} + \frac{p_x}{\gamma\, m}\,\frac{\partial p_x}{\partial x} = q\left(E_x - \frac{p_z}{\gamma\, m\, c}\,B_y - mc^2\frac{\partial \gamma_L}{\partial x}\right) \quad (3)$$

$$\left(1 - \frac{p_z}{\gamma\, m\, c}\right)\frac{\partial p_z}{\partial \xi} + \frac{p_x}{\gamma\, m}\,\frac{\partial p_z}{\partial x} = q\left(E_z + \frac{p_z}{\gamma\, m\, c}\,B_y + mc^2\frac{\partial \gamma_L}{\partial \xi}\right), \quad (4)$$

where $\gamma_L(x,\xi) = \sqrt{1+a^2}$ and $a(x,\xi)$ is the dimensionless vector potential of the incident laser defined in terms of the dimensionful vector potential by

$$\mathbf{A}_L = \sqrt{2}\,\frac{mc^2}{q}\,a(x,\xi)\cos(\omega_0\,\xi). \quad (5)$$

The electromagnetic fields are determined by Maxwell's equations coupled to the fluid current density ($qn_e\mathbf{v}$) and charge density (qn_e) *viz.*

$$\frac{\partial B_y}{\partial \xi} - \frac{\partial E_x}{\partial \xi} - c\frac{\partial E_z}{\partial x} = 0\,, \quad (6)$$

$$\frac{\partial E_x}{\partial \xi} - \frac{\partial B_y}{\partial \xi} + \frac{4\pi q}{mc}\,\frac{n_e\,p_x}{\gamma} = 0\,, \quad (7)$$

$$\frac{\partial E_z}{\partial \xi} - c\frac{\partial B_y}{\partial x} + \frac{4\pi q}{mc}\,\frac{n_e p_z}{\gamma} = 0\,, \quad (8)$$

$$4\pi q(n_e - n_i) - \frac{\partial E_x}{\partial x} + \frac{1}{c}\,\frac{\partial E_z}{\partial \xi} = 0, \quad (9)$$

where n_i is the fixed background ion density. Note that through Ampere's law, Poisson's equation and the continuity equation are equivalent and thus only one of the latter is required. Computationally, it is advantageous to use Poisson's equation in place of the continuity equation. We solve (3), (4), and (6)–(9) numerically using the Crank–Nicholson method. Although this method is implicit, it has three significant advantages: it is (linearly) unconditionally stable, exhibits no amplitude dissipation and is second-order in both x and ξ. These characteristics allow for a large ratio of the ξ to x step sizes keeping execution time down even for runs with fine spatial resolution.

4 HOLLOW CHANNELS

In the ideal hollow channel where

$$\omega_p(x) = \begin{cases} 0 & x \le b, \\ \omega_{p_0} & x > b, \end{cases} \quad (10)$$

it is possible, in the context of linear theory, to obtain an analytical expression for the wake field [6]. In this case, one finds that the channel mode has infinite Q and oscillates at a frequency, $\omega_{\rm ch}$, which is less than ω_{p_0}. For the non-ideal case, *i.e.* where the channel walls are not infinity steep, matters are more complicated. The dielectric function, $\epsilon = 1 - \omega_p^2(x)/\omega^2$ is evidently spatially dependent and every $\omega < \omega_{p_0}$ is resonant with the local plasma frequency at some location in the wall. This resonant layer leads to absorption of the wake resulting in a low Q. This in turn means a large spread in frequency space about $\omega_{\rm ch}$, exciting much of the wall. To explore the effects of channel width and wall slope, we parameterize the ion density as shown in Fig. 1 and assume ponderomotive driver of the from

$$a^2(x,\xi) = a_0^2 \exp\left[-2\frac{x_c^2}{w_x^2}\right]\exp\left[\frac{-2(\xi-\xi_c)^2}{w_\xi^2}\right]. \quad (11)$$

As an additional simplification we used the linearized form of the fluid equations. (Nonlinear results will be presented elsewhere [7].) As an example of the effects of the resonant absorption, consider a a channel with $k_p b = 2$ and $\alpha = 17°$. The resulting wake field, shown in Fig. 2, has $Q \cong 7$. Resonant absorption transfers energy from the wake field to particle kinetic energy in the region of the channel wall. The velocity fields (currents) are organized in such a way that the corresponding electromagnetic fields are also spatially localized. This process results in the development of fine-scale spatial structure in the velocity and electric fields which requires high numerical resolution in the simulations.

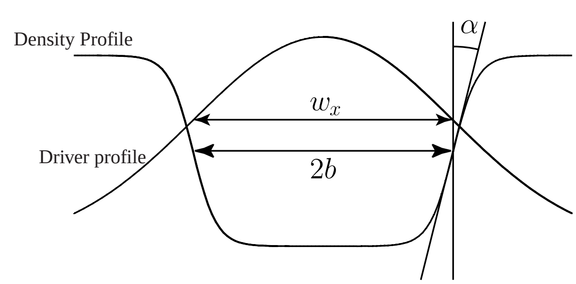

Figure 1: Density and transverse ponderomotive profiles.

In practice, since the laser driver extends into the bulk plasma, an electrostatic bulk mode will be excited in addition to the electromagnetic channel mode. Furthermore, it is not a simple matter to separate these modes; thus it is both convenient and reasonable to define an effective $[R/Q]$ as

$$\left[\frac{R}{Q}\right]_{\rm eff} = \frac{E_z(0)^2}{\omega_m\,\mathcal{U}_T}, \quad (12)$$

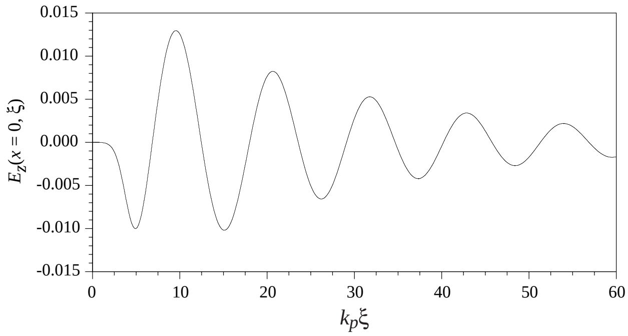

Figure 2: Longitudinal wake field on axis for a channel with $k_p b = 2$, $\alpha = 17°$, $\omega_{p_0}\xi_c = 4$, and $\omega_{p_0} w_\xi = 1.5$.

where $\mathcal{U}_T$ is the *total* energy in the plasma. As a result of replacing $\mathcal{U}_m$ with $\mathcal{U}_T$, the energy spent exciting the unwanted bulk mode will be reflected in a reduction in $[R/Q]_{\text{eff}}$. This effect can be seen in Fig. 3, where we plot $[R/Q]_{\text{eff}}$ *versus* driver width, w_x, keeping a_0^2 and $k_p\, b$ constant. (We normalize $[R/Q]_{\text{eff}}$ to $k_p^2\, Z_0$.)

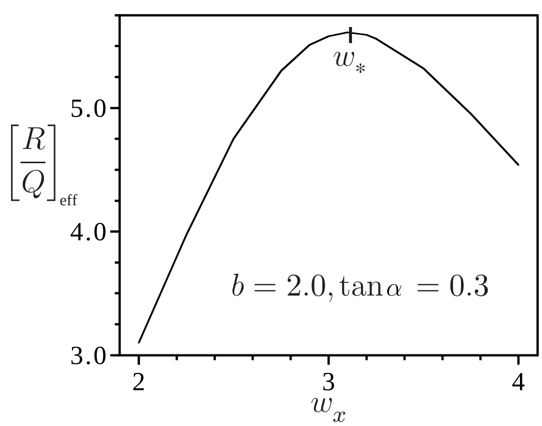

Figure 3: Effect of transverse driver width upon $[R/Q]_{\text{eff}}$. The optimal value of w_x is denoted by w_*.

The physical interpretation of this result is straightforward. The channel mode is supported by surface currents in the channel walls so a very narrow driver excites the wall only a small amount, yielding a low $E_z(0)$. Conversely, a wide driver excites the wall significantly but the exponential "wings" also excite a large bulk mode which contributes to $\mathcal{U}_T$ but not $E_z(0)$. Hence there is a optimal driver width, w_*, which balances these competing effects. The effect of the slope of the channel walls which is shown in Fig. 4. Clearly, Q is strongly dependent on this slope. As the walls are made ever less steep, the size of the resonant region grows allowing faster transfer of energy from the wake into the wall region, yielding a lower Q.

5 CONCLUSIONS

We have begun the systematic study of the accelerating properties of plasma channels by considering Q and $[R/Q]$. These figures-of-merit allow for a well defined optimization of a plasma based accelerating structures. Our results are clearly preliminary; we have considered only some of the relevant parameters. In particular we have ignored the constraint on the driver width imposed by the guiding condition which is surely to require an operating point that differs from the optimal width determined from $[R/Q]$. Additionally, laboratory channels are unlikely to be completely hollow. In such channels, the base density supports an electrostatic mode in addition to the electromagnetic mode altering the desirable beam transport properties of the hollow channel and also limiting (*via* wave breaking) the accelerating gradient that can be supported. This suggests that transverse pulse shaping (*i.e.* a higher-order gaussian spatial mode) may prove important in the design of successful hollow channel accelerating structures.

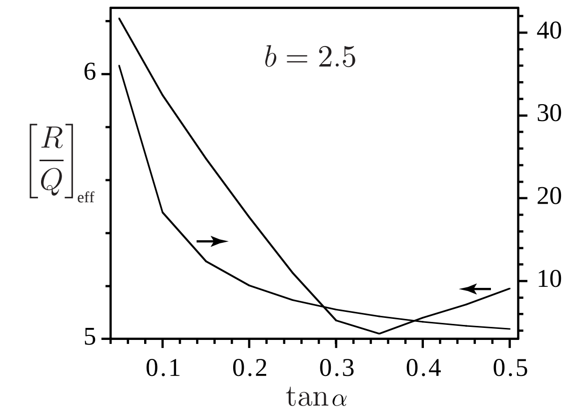

Figure 4: The effect of the wall inclination on Q and $[R/Q]_{\text{eff}}$ with optimal driver width.

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