SYNCHROTRON RADIATION IN INHOMOGENEOUS MAGNETIC FIELDS

Yu.A.Bashmakov,
P.N.Lebedev Phys. Inst., Moscow, Russia

Abstract

The radiation of electrons in magnets with linear gaps is investigated. Typical spatial distribution of the magnetic field is discussed, the characteristic parameterization of this distribution is given. The law of the electron motion in such field and the form of the electromagnetic field pulses arising at this motion is considered. Results of the numerical calculation of the angular and the spectral distributions are given and the comparison with experimental data is carried out.

1 INTRODUCTION

The importance of the study the relativistic charged particle radiation from magnet with straight sections increase in connection with more wide usage for electromagnetic generation of the special devices installed in straight sections of electron storage rings - synchrotron radiation sources [1] - [3]. At the present paper the formation of radiation and its spectrum are studied for the real edge fields.

2 THE DISTRIBUTION OF THE FINGING MAGNETIC FIELD

The field at the edge of the real magnet changes smoothly from zero to its maximum value H_m. It is convenient to describe the shape of this distribution by the dimensionless function $F(s) = H(s)/H_m$, where $H(s)$ is the value of the guiding field on the particle trajectory, the displacement s along the trajectory is convenient to measure in the units of height of the magnetic gap h and it is taken from "virtual boundary" of the field. Its position is defined by equality of two integrals: first one is the integral of the real field distribution and second one is the integral of the unity step function dropping at the virtual boundary. We parameterize the $F(s)$ dependence using the fact that magnetic field distribution has distinguish point - bending point z_0 (axis z coincides with the straight section axis, and the center of the coordinate system is placed in the straight section center [2]). The geometric bound of the magnet normal to the straight section axis places at $z = z_b$. For definiteness consider edge placed at positive z ($z_b > 0$). Then the law of changing of the magnetic field along straight section axis may be represented in the form

$$F(u) = F_0[1 + \tanh(B_1 u_-)], u_- \leq 0, \quad (1)$$

$$F(u) = F_0[1 + (1 - F_0)\tanh(B_2 u_-)], u_- > 0, \quad (2)$$

where $u_- = u - u_0$, $u = z/h$, $B_1 = D_m/F_0$, $B_2 = D_m(1 - F_0)^{-1}$, $D_m = (dF/du)_m$ is the maximum value of the derivative of the magnetic field, F_0 is the value of the field at the bending point u_0 . By means of the selection of three parameters u_0, F_0 and D_m one can achieve a rather good description of the magnetic field for any specific apparatus. It is obvious, that dependence (2) gives true asymptotic behavior of the field both inside and outside the magnet. The bending point as a rule is inside the magnet gap $(z_0 > z_b)$. Transition to the field with sharp dropping can be carry out in (1) : it is enough to direct z_0 to z_b and D_m to infinity. For the "symmetric" field distribution [4]. one has $F = 0.5$, and $B = 2D_m$. The numerical modeling is fulfilled for the Electron Synchrotron "Pakhra". The magnetic field at the edges of this synchrotron is good described by the dependence (2) if set $u_0 - u_b = 0.833$, $F_0 = 0.757$, $D_m = 0.509$. Out of the magnet at the distance of 15 cm from its edge ($u - u_b = -1.666$, the magnet gap height $h = 9$ cm) the field falls down up to $F = 0.00284$, inside the magnet at the same distance from the edge $(u - u_b = 1.666)$ $F = 0.986$.

3 PARTICLES MOTION

The angle θ_t forming by the particle velocity vector v with straight section axis is determined in the following way

$$\theta_t(t) = (h/R_0)\Phi(u), \quad \Phi(u) = \int_0^u F(u)du, \quad (3)$$

where $hdu = \beta cdt$ is path elements passing by a particle, $R_0 = E/(eH_m)$ radius of curvature of trajectory in the magnetic field H_m, E is energy of particle. Substituting (1) into (2) one obtains the law of the particle motion in the specific magnetic field [1]. The value $\Phi_0 = F_0 \ln 2/B_1$ determines the characteristic bending angle accumulated by the particle at its approach to the bending point z_0 . For $u > u_0 + q\,(q \approx 2 - 3)$ the time variation in the bending angle is adequately described by the linear function $\Phi(u) = \Phi_0 + u - u_0$, which is inherent to the particle motion in uniform magnetic field H_m in circle of radius R_0. It is easy to find the position of the virtual boundary of the magnetic field: $u_v = u_0 - (2F_0 - 1)ln2/D_m$. For the symmetric field distribution $(2F_0 = 1)$ the virtual boundary registers with the bending point. For the most of electron and proton synchrotrons Φ_0 is of the same order of magnitude as unit and, therefore, the characteristic bending angle θ_t, being in the order of magnitude as h/R_0, very different for electron and proton synchrotrons. For the synchrotron "Pakhra", for example, $\Phi_0 = 0.771$, $\theta_{t0} = 0.0166$, then for the CERN Proton Synchrotron on 400 GeV $h = 5$cm, $R_0 = 2200$ m, and $\theta_{t0} \sim 2.27 \times 10^{-5}$. In the central region of the transient domain it is possible to expand $\Phi(u)$ into a power series and at the bound of this region to perform an expansion on small value $\exp(-2B_{1,2}|u-u_0|) < 1$ [1]. Using the expansion one can find $\int_0^t \theta_t\,dt,\ \int_0^t \theta_t^2\,dt$ integrals that are part of the equation of motion.

4 THE DOMAIN OF THE RADIATION GENERATION

The general expressions for the spectral-angular distribution of the relativistic charged particle radiation from magnet with straight sections are obtained in [2]. The radiation spectral characteristics are defined by the form of the pre-exponent factor in $B^{\pm}_{\omega,i}$ in equation (7) of reference [2]. The general analysis of the expression was given in Ref [2]: the position of its zeros and peaks are determined. The investigation of this dependence for several electron energy at the "Pakhra" synchrotron for the particular field (1) was performed in [1]. The radiation pulses for different electron energies were studied. These pulses correspond to $B^{\pm}_{\omega,i}$ and propagate along the straight section axis ($\theta = 0$). The form of each of these bipolar pulses is close to asymmetric about its center (zero): maximum and minimum are equally spaced from the pulse center and its modulus are approximately equal too. At the given energy range the change of the distance between maximum and minimum can be neglected. The center of the pulse corresponds to the bending angle $\theta_t = \gamma^{-1}$ [1] and at energy growth it shifts distinctly into the straight section center in the weak field region. This reduces the "effective" straight section length [5]. The integral of this $B^{\pm}_{0,i}$ pulse is equal to zero within a good accuracy. For precise determination of this integral it is necessary to make a specific assumptions on the azimuthal extend θ_b of the magnetic sectors.
At the problem in question ($\theta_b \gg 1$) from (7) in [2] one obtains radiation distributions with the transverse polarization vectors $\mathbf{e_1}$ and $\mathbf{e_2}$. For the radiation pulses propagating in the orbital plane ($\cos\varphi = \pm 1$) at an angle $\theta = 2 \times 10^{-3}$ with respect to the straight section axis in complete agreement with [2] they mutual symmetry is broken and $B_{0,i} \neq 0$.
For chosen observed angle and electron energy $\vartheta^2 \cos 2\phi > 1$ and, therefore, the total radiation pulse is mainly formed inside one of the magnets and has at $\vartheta_{t1,2}$ two close placed zeros [1]. At deviation from the straight section axis the main pulse becomes narrower and takes a form approaching to that of the standard pulse of the synchrotron radiation. At the same time the accompanying pulse degenerates. In spite of the essential difference of shapes of these pulses the integrals on its according to (4) have equal magnitude (also see [7]). Note that the equality to zero of $B^{\pm}_{0,i}$ can be used as a criterion of formation of the synchrotron radiation spectrum. It follows that in the orbital plane this condition is rigorously satisfied only for $\vartheta \geq 1$. It was shown that the region of the radiation formation has along the straight section axis characteristic dimension about 50 cm. that is about $5h$. It is obvious that in this region both the magnetic field and hence radius of curvature of the particle trajectory change highly. Let us go to discussion the calculation of the $B^{\pm}_{\omega,i}$ integral which define the spectral radiation properties [2]. In the exponent index of these integrals side by side with the $(1+\vartheta^2)s/2\gamma^2$ term presents the $\int_0^z \theta^2\,ds/2$ term displaced slow down of the longitudinal motion and comes into force since the deflection angle becomes $\theta_t \approx \gamma^{-1}$. Note that in the theory of the bremsstrahlung of high-energy particles the multiple scattering in a medium causes the appearance of this type term [7].
For synchrotron radiation in homogeneous magnetic field similar term $\int_0^s \theta_t^2\,ds/2 = s^3/(6R_0)$, where $\theta_t = s/R_0$, was explicitly isolated in [8]. It was shown that spectral distribution of the radiation can be express in terms of the Airy integrals [9]. These integrals are like to considered ones, that is why the $B^{\pm}_{\omega,i}$ integration technique was developed at the Airy integrals.

5 SPECTRAL-ANGULAR DISTRIBUTIONS

For the synchrotron "Pakhra" there were calculated for the propagating along straight section axis radiation dependencies on the electron energy and the angular radiation distributions at the fixed wave length $\lambda = 5000\,\mathring{A}$. The dependence of the radiation with $\mathbf{e}_1$ polarization on electron energy is shown in Fig. 1. It has characteristic oscillatory

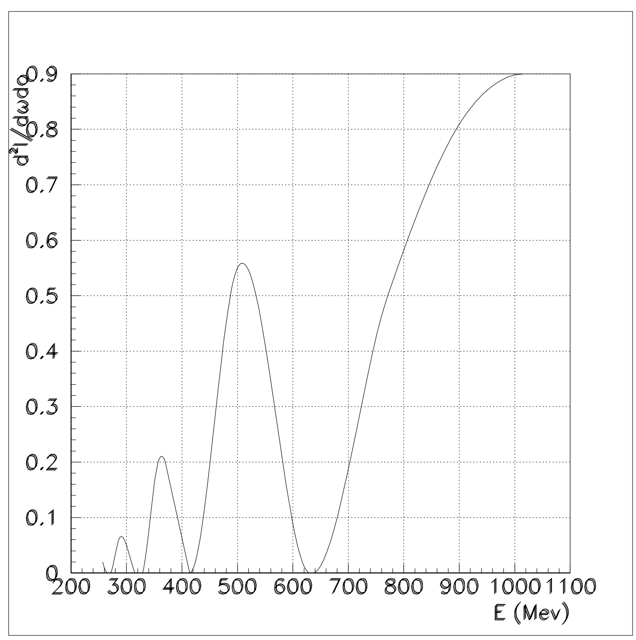

Figure 1: Radiation intensity dependence versus electron energy at $\lambda = 5000\,\mathring{A},\ \theta = 0$

structure and at the minimum the intensity falls up to zero. The angular distributions for vertical plane are exhibited in Fig. 2 (the electron energy is at the minimum : $E = 395$ MeV) The radiation along the axis is negligible. There is the side maximum at $\theta = 0.7$ mrad next intensity falls fast.

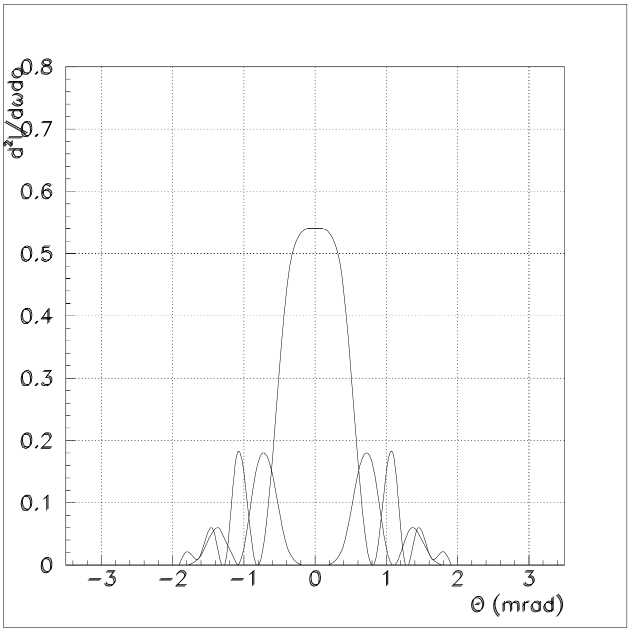

Figure 2: Angular radiation distributions at $\lambda = 5000\mathring{A}$: in vertical plane.

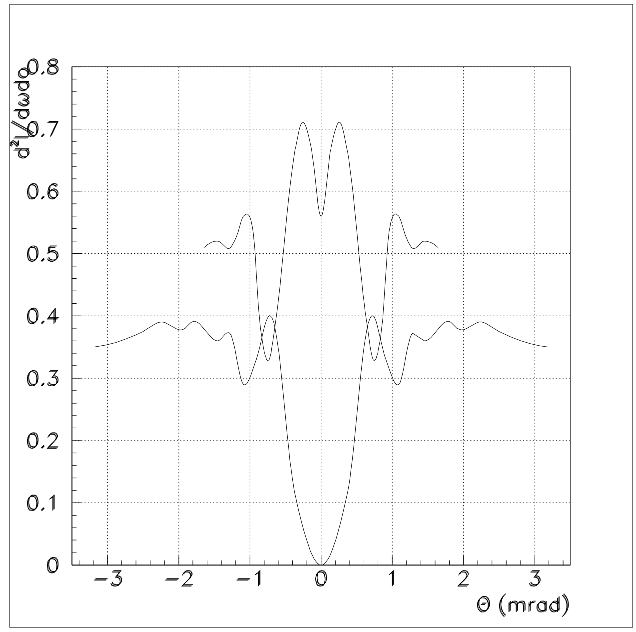

Figure 3: Angular radiation distributions at $\lambda = 5000\mathring{A}$: in horizontal plane.

The height of the second maximum is one-fourth as large as the first one and the third maximum is one-fourth as large as the second one. The width of the first maximum at half-height is $\Delta\theta_1 = 0.32$ mrad. The period of following oscillations reduces quickly. The results of our calculations differ from computation making for sharp discontinuity of the field by the more fast descent of the side maximums intensities. As energy increases up to 475 MeV the powerful maximum appears at the center. The relation of the side maximum intensities does not change. At energy growth its shift is up in angle. The central maximum now excedes the first side maximum by a factor of two and one-half. It levels out in the center. On the whole the oscillatory nature of this distribution slightly decreases in comparison with the previous one.
Angular distribution of the radiation propagating in the horizontal plane were found for given electron energy Fig. 3. These distributions differ essentially from the corresponding vertical distributions. Actually its have two or three evident maximums with nonzero intensities. For the rather large angles ($\theta \geq 2$ mrad) the intensity falls monotonously down.
All listed peculiarities of the angular distributions are well shown in photos given in Ref.[6]. The ring structures observed in these photos are responsible for the close proximity of locations in angular radiation distributions of maxima and minima for two mutually perpendicular planes. The fact that the magnitude of angular spread of the electron beam is the same order as the characteristic width of the maxima of the angular distribution of the radiation gives rise to smoothing of the oscillations in observed distributions.

6 CONCLUSIONS

It should be noted that now the edge magnetic field radiation is used for electron beam diagnostic at synchrotnons and storage rings [10].

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