Intensity Effects on Inverse-Bremsstrahlung Electron Acceleration*

R. Pakter
MIT - PSFC, Cambridge, MA, USA

Abstract

The effects of beam intensity on the laser field on Inverse-Bremsstrahlung Electron Acceleration are investigated. A self-consistent Hamiltonian formalism that takes into account both particles and wave dynamics is developed. It is shown that efficient acceleration is achieved for high-density beams. However, for such high densities, beam plasma effects impose a limitation on energy gain. A method is proposed in order to remove the limiting effects.

1 INTRODUCTION

Among different methods proposed for laser-particle acceleration a promising branch is the one where particles and electromagnetic fields interact directly without the aid of dielectrics or plasmas, because of the difficulties to control instabilities and other damaging effects generated by the presence of such media. In particular, Kawatana and coworkers introduced the concept of Inverse-Bremsstrahlung Electron Acceleration [1], where a *small* electrostatic field applied perpendicular to a propagating electromagnetic wave breaks the symmetry in the oscillatory wave-particle interaction. They showed that with properly chosen values for the applied electrostatic field strength, net energy gain is obtained in one cycle of the wave. By analyzing the nonlinear equations involved in the single-particle-wave interaction, Hussein and Pato [2] demonstrated that by alternating the direction of the applied electrostatic field at appropriate positions, the acceleration is extended for more than one wave cycle, leading to high energy gain. They called this scheme as Nonlinear Amplification of Inverse-Bremsstrahlung Electron Acceleration (NAIBEA). In this paper, we investigate beam current effects in the NAIBEA scheme.

2 MODEL DESCRIPTION

We consider a beam of electrons of charge $-e$ and mass m interacting with an applied plane electromagnetic wave propagating in the x-direction and an applied electrostatic field pointing along the y-direction. The vector potential that describes the electromagnetic wave is written as

$$\frac{e\mathbf{A}}{mc^2} = -\frac{1}{2}A\, e^{i\varphi}\,\hat{\mathbf{e}}_y + c.c., \tag{1}$$

where c is the speed of light *in vacuo*, $\varphi \equiv \omega t - kx$, with ω being the wave frequency and $k = \omega/c$ the wave number, A is the complex wave amplitude, and $c.c.$ stands for complex conjugate. The wave electric and magnetic fields are then given by $\mathbf{E}_{wave} = E_y\hat{\mathbf{e}}_y$ and $\mathbf{B}_{wave} = B_z\hat{\mathbf{e}}_z$, with $E_y = B_z = iE\, e^{i\varphi}/2 + c.c.$ and $E = mc\omega A/e$. Normalizing space to $1/k$, time to $1/\omega$, energy to mc^2, momentum to mc and vector potential to e/mc^2, the dynamics of the i^{th} electron in the beam is described by the following particle Hamiltonian

$$H_i = \gamma_i + E_{app}y_i, \tag{2}$$

$$\gamma_i = \{1 + P_{xi}^2 + [P_{yi} + A_y]^2 + P_{zi}^2\}^{1/2}. \tag{3}$$

Here, γ_i is the relativistic mass factor, $\mathbf{P}_i = \mathbf{p}_i - \mathbf{A}$ is the canonical momentum, with $\mathbf{p}_i$ being the mechanical momentum, and E_{app} is the normalized applied electrostatic field in the y-direction. The energy equation for the particle is obtained from Eq. (2) as

$$\dot{H}_i = -v_{yi}E_y, \tag{4}$$

where the dot stands for derivative with respect to t and $\mathbf{v}_i = \mathbf{p}_i/\gamma_i$ is the normalized (to c) particle velocity. The NAIBEA scheme consists of alternating the sign of a properly chosen E_{app} at the positions where the phase φ satisfies $\varphi = (2n+1)\pi/2$, $n = 1, 2, \cdots$. With this alternation, one shows that the right-hand-side of Eq. (4) is always positive leading to continuous particle energization [2].

To self-consistently take into account the effects generated by beam current on the electromagnetic fields we apply a formalism which is similar to that employed in Ref. [3]. A slow-time evolution equation for A is readily derived from Maxwell equations as

$$\dot{A} = \frac{4\pi i e^2 k}{mc^2 VT}\int J_y\, e^{-i(t'-x')}d^3r'dt', \tag{5}$$

where a Fourier transform over the fast (primed) variables has been performed introducing the volume V and period T and

$$J_y = -\sum_{i=1}^{N} v_{yi}(t)\,\delta[\mathbf{r} - \mathbf{r}_i(t)] \tag{6}$$

is the y-component of the electron current density, which has been normalized to eck^3. Here, N is the total number of particles in the system, and $\mathbf{r}_i(t)$ is the instantaneous displacement of the i^{th} particle. Using the polar representation for the wave amplitude $A = \sqrt{\rho}\, e^{i\sigma}$, Eq. (6), and the relation $v_{yi} = P_{yi} + A_y/\gamma_i$, we can re-write Eq. (5) in the form [4]

$$\dot{\sigma} = -\frac{\delta}{\sqrt{\rho}N}\sum_{i=1}^{N}\frac{[P_{yi} - \sqrt{\rho}\cos\varphi']}{\gamma_i}\cos\varphi', \tag{7}$$

$$\dot{\rho} = -\frac{2\delta\sqrt{\rho}}{N}\sum_{i=1}^{N}\frac{[P_{yi} - \sqrt{\rho}\cos\varphi']}{\gamma_i}\sin\varphi', \tag{8}$$

where

$$\delta \equiv \omega_b^2/\omega^2, \tag{9}$$

* Work supported by CAPES, Brazil and in part by U.S. Department of Energy under Grant No. DE-FG02-95ER-40919.

with $\omega_b^2 = 4\pi e^2 n_e/m$ being the beam plasma frequency squared and n_e the average electron density, $\varphi' = t - x_i + \sigma$, and use has been made of the conditions $v_{xi} \approx 1$ and $|\dot{P}_{yi}| = |E_{app}| \ll 1$.

An interesting point is that rescaling the wave dynamical quantities according to $\sigma = 2\bar{\sigma}/N$ and $\rho = \delta\bar{\rho}$ one concludes that all relevant dynamical equations for both particles and fields can be derived from one generalized Hamiltonian given by

$$H = \sum_{i=1}^{N} H_i = \sum_{i=1}^{N} [\gamma_i + E_{app} y_i], \tag{10}$$

$$\gamma_i = \{1 + P_{xi}^2 + [P_{yi} - \sqrt{\delta\bar{\rho}}\cos(t - x_i + 2\bar{\sigma}/N)]^2 + P_{zi}^2\}^{1/2}.$$

In the above Hamiltonian formalism the equations of motion for the wave quantities are given by

$$\dot{\bar{\sigma}} = \frac{\partial H}{\partial \bar{\rho}}, \; \dot{\bar{\rho}} = -\frac{\partial H}{\partial \bar{\sigma}}, \tag{11}$$

where $\bar{\sigma}$ and $\bar{\rho}$ play the role of canonically conjugated coordinate and momentum, respectively.

It readily follows from the generalized Hamiltonian in Eq. (10) that the energy exchange between particles and electromagnetic wave obeys a conservation law of the form

$$\frac{N\bar{\rho}}{2} + \sum_{i=1}^{N} (\gamma_i + E_{app} y_i) = \text{const.} \tag{12}$$

Note that $mc^2(N\bar{\rho}/2) = u_{wave} V_d$ is the total electromagnetic energy stored in the wave, where $u_{wave} = |E|^2/8\pi$ is the wave energy density and $V_d = V/k^3$ is the dimensional volume.

3 RESULTS OF THE ANALYSIS

In order to analyze the self-consistent interaction in a NAIBEA scheme, we numerically integrate the set of equations derived from the Hamiltonian in Eq. (10). We model the interaction considering a cold beam of N electrons per wavelength of the laser field, homogeneously distributed along the x-direction. We consider a specific example discussed in previous papers [1,2], namely, a 10 μm wavelength laser with electric field amplitude $|E| = 1.636 \times 10^9$ V/cm, which corresponds to an intensity of 3.5×10^{15} W/cm^2. The strength of the applied electrostatic field is $|E_{app}| = 4.28 \times 10^{-5}|E|$. The electrons are injected with an energy corresponding to $\gamma = 106.8$ at an angle of 0.608^o with respect to the x-axis. For this case, the single-particle (not self-consistent) analysis, based on Eq. (2), reveals that an electron initially at $x(0) = 0$ attain a final energy corresponding to $\gamma = 850$ after 96 cm of interaction when one inversion in the sign of E_{app} is performed. The optimal position for the electrostatic field reversion (i.e., when $\varphi = 3\pi/2$) is found to be 32.8 cm from the injection point.

Now we investigate what happens when the wave dynamics is taken into account. We consider two distinct cases, a low-density beam with $\delta = 5 \times 10^{-8}$ and a high-density beam with $\delta = 10^{-3}$. In Fig. 1 we show the results obtained for the self-consistent interaction with $N = 50$ particles per wavelength when one inversion in E_{app} is performed at the optimal position determined by the single-particle analysis. The number of particles in the simulation is chosen to obtain convergent (independent of N) results for the wave dynamical quantities. To compare self-consistent results with single-particle results, one particle is chosen among the N particles as a *tag* particle whose energy is monitored during the acceleration. The *tag* particle is launched exactly with $x(0) = 0$ (which is the initial condition used in the single-particle analysis). The figure presents the amplitude (a) and phase (b) of the wave, and the energy (in terms of γ) of the *tag* particle (c) as a function of the dimensional interaction distance $s = x/k$ for both the low-density case (dashed curves) and the high-density case (solid curves).

For the low-density case with $\delta = 5 \times 10^{-8}$ (dashed curves) the wave is essentially unaffected by the presence of particles, with $\bar{\rho}$ and $\bar{\sigma}$ keeping their values unchanged throughout the interaction, as seen in Figs. 1(a) and 1(b). Hence, the acceleration shown in Fig. 1(c) agrees with that found in the single-particle analysis where a maximum energy corresponding to $\gamma = 850$ is attained at $s = 96.0$ cm. Despite the large particle energization, it should be pointed out that the acceleration process for low densities is clearly *inefficient*, since little energy is transferred from the wave to the particle beam.

For the high-density case with $\delta = 10^{-3}$ (solid curves), however, the acceleration process is dramatically affected by the wave dynamics. Figure 1(a) shows that the wave is severely damped as it interacts with the particle beam, transferring up to 70% of its initial energy to the beam. As a result of wave depletion, the instantaneous rate of energy change given by Eq. (4) is reduced, and the maximum energy obtained by the *tag* particle is decreased to $\gamma = 350$ [see Fig. 1(c)]. Although the final energy in the high-density case is much lower than that in the low-density case, it still represents a good acceleration with gradients on the order of hundreds of MeV/m.

By examining the high-density case in more detail, one readily finds another reason for the limited particle acceleration, besides the wave depletion. Figure 1(b) shows that beam plasma effects cause the wave phase velocity to increase, which is indicated by a nearly monotonic increase in $\bar{\sigma}$. Because phase synchronous is required in the NAIBEA scheme, even small changes in $\bar{\sigma}$ can drive particles and wave out of phase, eventually changing the sign of $-v_{yi}E_y$ in Eq. (4) and ceasing the acceleration process.

To overcome the limitation on particle acceleration imposed by the beam-plasma-induced phase shift, we notice, from the generalized Hamiltonian in Eq. (10), that the effective wave phase seen by the particles is $\varphi' = \varphi + 2\bar{\sigma}/N$ instead of φ. Thus, by changing the sign of E_{app} according to $\varphi' = (2n+1)\pi/2$, $n = 1, 2, \cdots$, we can compensate self-consistent variations of the wave phase, thereby prolonging the acceleration process.

To test the efficacy of the compensation procedure, we consider the high-density beam example presented in Fig. 1. Integrating the self-consistent set of equations, we readily obtain the interaction distance s for which $\varphi' = 3\pi/2$ is satisfied: $s = 28.0$ cm. In Fig. 2, the wave amplitude $\bar{\rho}$ (solid curve) and the *tag* particle energy γ (dashed curve) are shown as a function of s for the case where E_{app} is changed at the optimized position $s = 28.0$ cm. Comparing these results with the previous results in Figs. 1(a) and 1(c), solid curves, one observes apparent improvements in the acceleration process with a 20% increase in the total energy delivered by the wave to the particle beam, as well as a 30% increase in the energy attained by the *tag* particle.

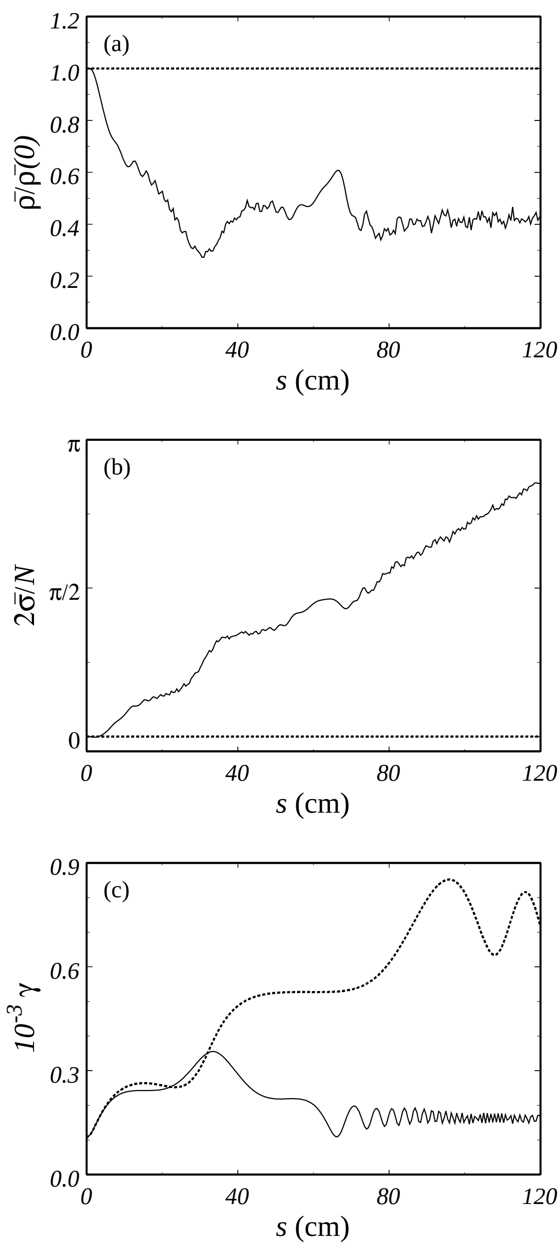

Figure 1: Results for low (dashed curves) and high (solid curves) beam densities.

4 CONCLUSION

We have investigated the effects of beam intensity on the laser field on the NAIBEA scheme. In particular, a self-consistent Hamiltonian formalism that takes into account both particles and wave dynamics has been developed. It was found that high particle gain and efficient energy exchange between wave and particles can be achieved simultaneously for high-density beams if beam plasma effects are judiciously taken into account.

5 REFERENCES

[1] S. Kawatana, T. Maruyama, H. Watanabe, and I. Takahoshi, Phys. Rev. Lett. **66**, 2072 (1991).

[2] M. S. Hussein and M. P. Pato, Phys. Rev. Lett. **68**, 1136 (1992).

[3] R. Pakter, R. S. Schneider, and F. B. Rizzato, Phys. Rev. E **49**, 1594 (1994).

[4] R. Pakter, Phys. Rev. E, *to appear* (1998).

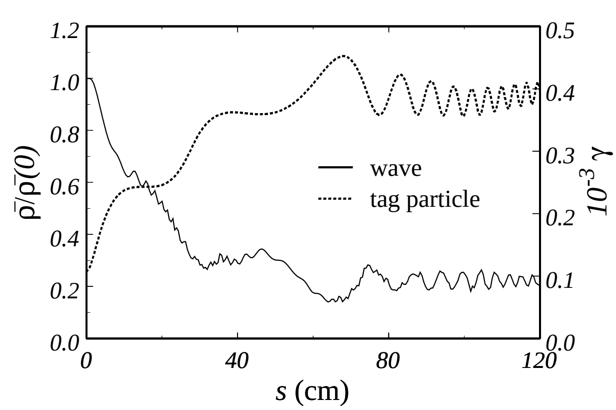

Figure 2: Results obtained when the inversion in E_{app} is performed at the optimized position.