ANALYSIS OF 3D HELICAL WIGGLER RADIATION FROM ELECTRON TRAJECTORY USING LIENARD-WIECHERT FIELDS

P. Gouard, J.-M. Lekston, CEA, Bruyeres-le-Chatel, France

Abstract

The new code SELF3D calculates the spontaneous emission from electrons following trajectories in the magnetic field of a helical wiggler using Liénard-Wiechert (LW) fields. These fields are exact solutions of the wave equation for a point charge. The spontaneous emission frequency spectrum is compared with experimental measurements at the CEA/CESTA Free Electron Laser (FEL). In the future, SELF3D will use a self-consistent treatment of electron beam dynamics and radiation field evolution in the magnetic wiggler. This approach is particularly well-suited to the investigation of self-amplified spontaneous emission (SASE) when no seed wave is injected at start-up.

1 INTRODUCTION

Experiments realised at Commissariat à l'Energie Atomique / Centre d'Etude Scientifiques et Techniques d'Aquitaine (C.E.A./C.E.S.T.A.) on bunching an intense electron beam reported the presence of two frequencies, one at 35 GHz and an other strong frequency at 2.95 GHz. These FEL experiments ran in the amplifier mode with a seed wave at 35 GHz. Measurements showed that a spontaneously generated low-frequency mode comes to dominate the high-frequency injected mode at long distances and late times [1][2].This low-frequency mode seems to be due to either spontaneous emission by the electron beam or a transfer between 35 GHz frequency and 3 GHz frequency by co-operative effects. Our task is to study the spontaneous emission of electron beam within the framework of experiments at C.E.A./C.E.S.T.A., with the aim of understanding the 3 GHz bunching. This approach was already tried by Tecimer and Elias [3] for spontaneous emission of a highly relativistic electron beam in free space. They used Liénard-Wiechert retarded potentials and made a self-consistent algorithm to calculate dynamics and fields. In the present paper we propose to extend this approach, well adapted for S.A.S.E. studies, to the case of an electron beam moving in a cylindrical waveguide with circular cross-section. In the second section, we present our calculation of the retarded electric field with Dirichlet boundary conditions. The calculation uses the dyadic Green's function. In section 3, the SOLITUDE[4] code and the SELF3D are presented. In section 4, we present preliminary numerical results obtained using the formalism developed in section 2. We calculate frequency distribution of the electric field, on axis, at this end of the waveguide, corresponding to an electron trajectory calculated using the code SOLITUDE (the code SELF3D is not already self-consistent)

2 THEORY[5][6]

2.1 dyadic green function

A dyadic Green's function is a dyad that relates a vector field to a vector current source. The use of dyadic Green's functions makes the formulation and the solutions of some electromagnetic problems more compact; the symbolic simplicity they offer makes their use attractive. However, only for a few simple geometries, like the circular cylindrical waveguide, can the dyadic Green's function be solved in closed form. By definition, the dyad G is solution of the vector wave equation with a point source:

$$\vec{\nabla} \wedge \vec{\nabla} \wedge \vec{E}(\vec{r},t) + \frac{1}{c^2}\frac{\partial^2}{\partial t^2}\vec{E}(\vec{r},t) = -\mu_0 \frac{\partial}{\partial t}\vec{J}(\vec{r},t)$$

$$\vec{\nabla} \wedge \vec{\nabla} \wedge \vec{\vec{G}}(\vec{r},\vec{r}\,',t-t') + \frac{1}{c^2}\frac{\partial^2}{\partial t^2}\vec{\vec{G}}(\vec{r},\vec{r}\,',t-t') =$$

$$\vec{\vec{I}}\,\delta(\vec{r}-\vec{r}\,')\,\delta(t-t') \qquad 1$$

Using Green's theorem one can prove that

$$\vec{E}(\vec{r}\,',t') = -\mu_0 \int dv \int dt\, \vec{J}(\vec{r},t).\vec{\vec{G}}(r,r',t-t') + Einc. \qquad 2$$

where **J**(**r**,t) is the current source and Einc is related to initials conditions on the electric field **E**(**r**,t). For the rest of the development, we take the Fourier transform of each equation.

2.2 Vector wave function

The derivation of the dyadic Green's function for a cylindrical waveguide follows a well defined procedure : find an expansion in cylindrical vector wave function. A vector wave function, by definition, is an eigenvector of the homogeneous vector Helmhotz wave equation. We use three distinct vectors, denoted **L**, **M**, **N**, which are linearly independent, and which form a basis for an arbityry vector field.

We have:

$$\vec{L}(\vec{r}) = \vec{\nabla}\psi(\vec{r})$$

$$\vec{M}(\vec{r}) = \vec{\nabla} \wedge (\psi(\vec{r})\hat{e}_z)$$

$$\vec{N}(\vec{r}) = \frac{1}{k}\vec{\nabla} \wedge \vec{\nabla} \wedge (\psi(\vec{r})\hat{e}_z)$$

$$\text{then } (\vec{\nabla}^2 + k^2)\psi(\vec{r}) = 0$$

We can define two eigenfunctions which satisfy the Dirichlet boundary condition at r = r_g (waveguide radius), corresponding to the cylindrical wall of the waveguide. They are

$$\psi_{nm}(\vec{r}) = J_n(k_r r)\,{}^{\cos}_{\sin}(n\theta)e^{ik_z z} \text{ and } k^2 = k_r^2 + k_z^2$$

Where $J_n(k_r r)$ is the Bessel function of the first kind

And

$\vec{n} \wedge \vec{M} = 0$ on S related to TE modes

then $k_r = q_{nm}$ for $\frac{\partial}{\partial r} J_n(q_{nm} r_g) = 0$

$\vec{n} \wedge \vec{N} = 0$ and $n \wedge \vec{L} = 0$ on S related to TM modes

then $k_r = k_{nm}$ for $J_n(k_{nm} r_g) = 0$

One can prove, with standard scalar product, that **M**, **N**, **L** are orthogonals but not normalised

2.3 Derivation of the dyadic Green's function

After normalisation of the vector wave function, we now apply the Ohm-Rayleigh method to derive the dyadic Green's function for the cylindrical waveguide. According to this method, we first seek an eigenfunction expansion for the source function using the vector wave functions introduced in the previous section.

Taking into account the vector wave equation (1), we follow the same procedure with the dyadic Green's function. We find.

$$\vec{\vec{G}}(\vec{r},\vec{r}',\omega) = \int dk_z \sum_{nm} \frac{1}{(1+\delta_0)2\pi^2}\left(\frac{\vec{M}_{nm}(\vec{r})\vec{M}^*_{nm}(\vec{r}')}{q^2_{nm} I_{q_{nm}}(k_z^2 + q^2_{nm} - \frac{\omega^2}{c^2})}\right.$$
$$\left. + \frac{\vec{N}_{nm}(\vec{r})\vec{N}^*_{nm}(\vec{r}')}{k^2_{nm} I_{k_{nm}}(k_z^2 + k^2_{nm} - \frac{\omega^2}{c^2})} - \frac{\vec{L}_{nm}(\vec{r})\vec{L}^*_{nm}(\vec{r}')}{I_{k_{nm}}(k_z^2 + k^2_{nm})\frac{\omega^2}{c^2}}\right) \quad 3$$

By imposing the condition of radiation, one may carry out the integration over k_z. But one must first extract the singularity in the last term which violates the Jordan lemma.

3.4 Frequency analysis of electrical field

We now determine the expression that gives us the frequency distribution of the electrical field at a point. Taking the Fourier transform of (2) with (3), we find

With

$\vec{J}(\vec{r},t) = e\vec{V}(t)\delta(\vec{r} - \vec{r}_0(t))$

$\vec{r}_0(t)$ is the electron's trajectory

and $\vec{V}(t)$ electron's velocity

$$\vec{E}(\vec{r}',\omega) = i\omega e\mu_0 \int dt\, \vec{V}(t).\vec{\vec{G}}(\vec{r},\vec{r}_0(t),\omega)e^{i\omega t}$$

We calculate the frequency distribution of radial electrical field on the axis at end of the wiggler in the TE11 mode. these conditions simplify the relation, finally, we have:

$$\vec{E}(Z,\omega) = Cst.\,\omega.$$
$$\int dt\left(\frac{J_1(q_{11}r_0(t))}{r_0(t)}\cos(\theta_0(t))\dot{r}_0(t)\right.$$
$$\left. - \frac{\partial}{\partial r} J_1(q_{11}r_0(t))\sin(\theta_0(t))r_0(t)\dot{\theta}_0(t)\right).$$
$$\frac{e^{i\left(\sqrt{\frac{\omega^2}{c^2} - q_{11}^2}\,|Z - Z_0(t)| + \omega t\right)}}{\sqrt{\frac{\omega^2}{c^2} - q_{11}^2}} \quad 4$$

This is the expression that we use in simulations.

3 SIMULATIONS

we generate, using the code SOLITUDE, the trajectories of electrons that we use to calculate frequency distribution of the spontaneous emission. This code is a two-frequency, non-linear three-dimension simulation code designed for the purpose of studying FEL amplifier experiments. The electromagnetic waves to be amplified propagate in a cylindrical waveguide and are expanded in TE and TM modes. With the S.V.A.P. (Slowly Varying Amplitude and Phase) approximation, we solve Maxwell's equations by averaging over a wave period λ and obtain the amplitude and the phase evolution of the first two TE and TM modes. Electron trajectories are computed by integrating the Lorenz force equation. The code takes into account of the adiabatic six-period wiggler entrance. The coupled system of non-linear differential equations obtained is solved by using a fifth-order Runge-Kutta method (the variable step Messon method).At the start of the wiggler, the electron beam position and emittance are chosen to agree with experimental measurements. Results of simulation made with SOLITUDE are in generally good agreement with such measurements.

SELF3D integrates the expression (4) according of the time along an electron's trajectory deduced from SOLITUDE.

4 RESULTS AND COMMENTS

We made three different simulations to study effects of the seed wave in the development of 2.95 GHz. In the first we introduced a large power (≈kW) at 35 GHz and a weak (≈W) at 3 GHz. The first simulation thus corresponds to the amplifier mode experiments. Results are in a good agreement with SOLITUDE results and experimental measurements

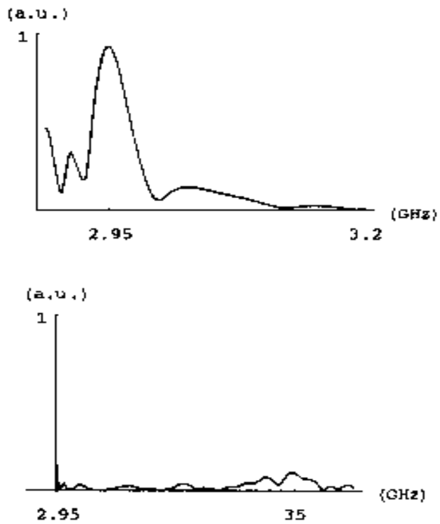

Fig 1 First simulation results

On the Fig 1, the simulation results show two peaks, one broad at 35GHz and a strong sharp one at 2.95GHz

In the second simulation no power was injected initially at 3 Ghz, this mode was not allowed to occur in the SOLITUDE calculation.

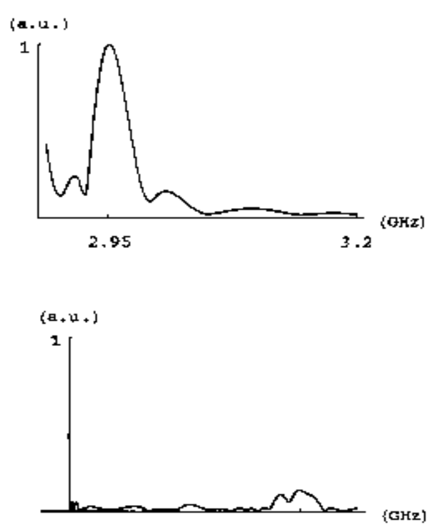

Fig 2 Second simulation results

This simulation also finds two peaks at the same position and with similar shapes, as shown in Fig. 2

Finally, we injected only low power at both frequencies in order to simulate the SASE mode.

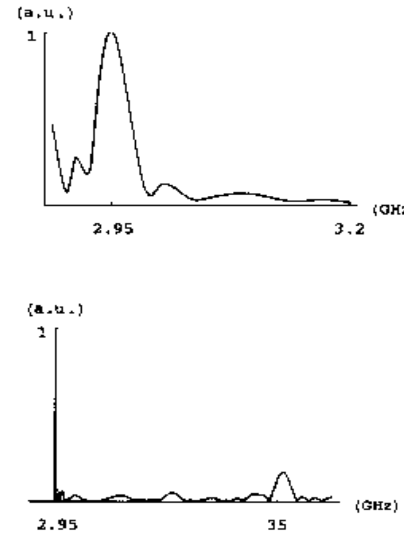

Fig 3 Third simulation results

Once again we have a strong sharp peak at 2.95GHz and a weak broad peak at 35 Ghz, as may be seen in Fig. 3

The two last simulations show that the low frequency is due to the spontaneous emission by the electrons, because no cooperative effect was introduce in SOLITUDE calculations.

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