PASER: Particle Acceleration by Stimulated Emission of Radiation

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Abstract

It is shown that a bunch of electrons which move in an active medium can be accelerated. The acceleration is proportional to the population inversion and the number of electrons in the bunch. A model for evaluation of the emittance variation in the interaction is presented.

1 INTRODUCTION

When an electron moves along a vacuum channel in a dielectric material it may cause radiation to be emitted provided that its velocity is greater than the phase velocity of an electromagnetic plane wave in the medium. This is the so–called Cerenkov radiation. What a remote observer measures as electromagnetic energy comes at the expense of the particle's kinetic energy or in other words, the particle is decelerated. For a better understanding of the deceleration force, one has to examine the field distribution in the vicinity of the particle. Ignoring for a moment the presence of the dielectric, a point charge generates in its rest frame of reference an electrostatic field which transforms in the laboratory frame into an infinite spectrum of evanescent waves. As these waves hit the discontinuity between the vacuum channel and the dielectric, a so called secondary field is generated. This is the reaction of the medium to the presence of the charged particle. It is the action of this secondary field which decelerates the electron. If instead of a passive dielectric medium, an active medium is used, the action of this secondary field may cause the particle to accelerate [1-2]. Thus energy stored in the medium can be transferred to the moving electron. Sinct the mechanism resembles the inverse of a laser, we call it PASER which stands for Particle Acceleration by Stimulated Emission of Radiation.

In the past, there were two schemes in which active medium was suggested in order to support the acceleration process [3, 4]. In both cases the active medium facilitates the generation of solitons extending this way the interaction region. To illustrate the role of the active medium let us consider the Wake-Field Acceleration scheme. It has been shown that high gradients may develop in the plasma however, the interaction with electrons alters the propagation characteristics of the medium and the overlap of the electrons with the laser beam is limited. Fisher and Tajima [4] have shown that an active medium can be used in order to preserve the radiation pulse-shape, energy and velocity. The scheme relies partially on the self-induced transparency theory which was first developed by McCall and Hahn [5] in 1969. Fisher and Tajima envision their system to use the outer shell electrons to form the plasma and the inner shell electron resonant transition as the constituent of the active medium. As previously indicated, in this study it will be shown that the active medium can be utilized not only to preserve the laser pulse shape but also to directly accelerate the electrons; in fact no plasma is required in the PASER scheme.

2 BASIC FORMULATION

Consider a point charge (q) moving at a constant velocity, v, whose the current density is described by

$$J_z(r,z,t) = -qv\frac{1}{2\pi r}\,\delta(r)\,\delta(z-vt)\,. \qquad (1)$$

This charge moves in a vacuum channel bored in an otherwise infinite medium characterized by a dielectric coefficient ϵ_r. The only non-zero field *on axis* which acts on the particle is the longitudinal electric field and only the waves "reflected" from the radial discontinuity contribute to the force which acts on the particle. Consequently, the normalized gradient reads

$$\mathcal{E} \equiv E_z(r=0, z=vt, t)\left(\frac{q}{4\pi\epsilon_0 R^2}\right)^{-1} = \frac{2}{\pi}\int_0^\infty dx \frac{\mathrm{Im}[\zeta(x)]}{\{\mathrm{I}_0(x)+\mathrm{Re}[\zeta(x)]\mathrm{I}_1(x)\}^2+\{\mathrm{Im}[\zeta(x)]\,\mathrm{I}_1(x)\}^2}\,, \qquad (2)$$

where

$$\zeta(x) = \frac{1}{\epsilon_r(x)}\sqrt{1-\beta^2\epsilon_r(x)}\frac{\mathrm{K}_0[x\gamma\sqrt{1-\beta^2\epsilon_r(x)}]}{\mathrm{K}_1[x\gamma\sqrt{1-\beta^2\epsilon_r(x)}]}\,; \qquad (3)$$

$x \equiv \omega R/c\gamma\beta$ is the normalized frequency variable. The detailed calculation is presented in Ref.2. We shall use this expression to evaluate the force which acts on a charge moving in an active medium.

3 FORCE IN ACTIVE MEDIUM

The interaction of a moving "macro-particle" with a stationary two-state quantum system which consists of either atoms or molecules is considered here within the framework of the macroscopic (and scalar) dielectric coefficient. This coefficient is given by

$$\epsilon_r(\omega) = 1 - \chi \frac{(\omega-\omega_0)T_2 + j}{1+\xi^2+(\omega-\omega_0)^2 T_2^2} \tag{4}$$

and it is tacitly assumed that the transients at the microscopic level are negligible. The macro-particle moves along a vacuum channel "bored" in an otherwise infinite dielectric medium. $\omega_0/2\pi$ is the resonant frequency of the medium, T_2 is the spectral line width, $\chi = \mu^2 \Delta N T_2/\epsilon_0 \hbar$ is the normalized population inversion (and it is negative in this case); μ is the atom's dipole moment, $\Delta N \equiv N_1 - N_2$ is the density of the population difference - subscript 1 represents the lower energy state and subscript 2 the higher one. Changes in the population difference due to energy transfer is considered here through the saturation term $\xi^2 = (E/E_{\rm sat})^2$; E is the amplitude of the acting electric field and the saturation field is given by $E_{\rm sat} = \hbar/\mu\sqrt{\tau T_2}$ where τ is the characteristic time in which the population reaches its equilibrium state. Note that the background dielectric coefficient is assumed to be unity excluding this way the possibility of generation of Cerenkov radiation.

For the analytic evaluation of the integral it is convenient to further simplify the model which describes the medium. Examining the dielectric coefficient in (4) we observe that its real part is unity at resonance and off resonance $[{\rm Re}(\epsilon_r - 1)]$, is anti-symmetric relative to resonace and vanishes far away from this point. Consequently, we approximate the dielectric coefficient with one whose real part is unity at all frequencies and its imaginary part is constant in a window of frequencies around resonance. The width of this window is determined by the line width and is determined such that the area of this window is identical with that calculated from (4). Explicitly $\epsilon_r(\omega) \simeq 1 - j\bar{\sigma}(\omega)$ and

$$\bar{\sigma}(\omega) \equiv \bar{\sigma}_0 \begin{cases} 1 & \text{for } |\omega-\omega_0|T_2 < \pi/2\,, \\ 0 & \text{for } |\omega-\omega_0|T_2 > \pi/2\,, \end{cases} \tag{5}$$

where $\bar{\sigma}_0 = \chi/(1+\zeta^2)$. With this definition it is now instructive to use the definition of the skin depth which in the framework of out notation reads $\delta = \sqrt{2/|\bar{\sigma}_0|(\omega_0/c)^2}$. With this definition we get

$$\mathcal{E} \simeq {\rm sgn}\,(\bar{\sigma})\,\frac{1}{\omega_0 T_2} \begin{cases} 4\delta/R & \text{for } \delta \ll R\,, \\ 4(R/\delta)^2 & \text{for } \delta \gg R\,. \end{cases} \tag{6}$$

Let us consider a medium whose $\lambda_0 = 1\mu$m, $\omega_0 T_2 \sim 100$ and $n = 1$. For effective interaction the radius of the channel has to be on the order of the radiation wavelength: in this case we chose $R = 1\mu$m. The other parameters are $N = 10^8$(number of electrons in the bunch), $\bar{\sigma}_0 = -0.1$ and $n = 1$. Line *(i)* in Fig. 1 illustrates the gradient which acts on the particle as a function of its initial energy. For energies on the order of a few tens of MeV the gradient a bunch of 10^8 particles experiences is on the order of GV/m. One can therefore envision a system in which a bunch of electrons is injected into a chamber filled with active gas. At these energies the mean free path, even at high pressure, is much longer than the practical length of a typical acceleration module (1-10m). In spite of this promising gradient there still remains the problem of competition with Cerenkov effect.

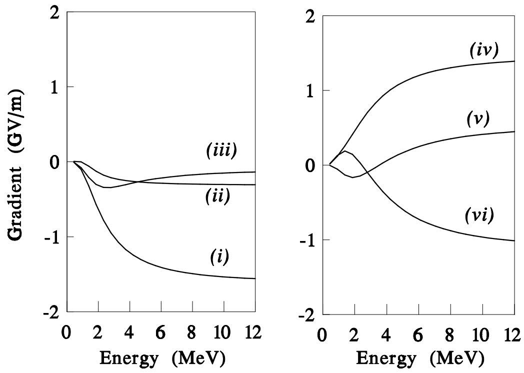

Figure 1: The gradient which develops in the active medium, as a function of the initial energy of the bunch.

In order to examine the feasibility of the method in a realistic medium we shall consider a gas whose molecules are assumed to have three resonances (4 level system) and only in one or at the most two, the population is inverted. The remainder serve as a model for the background refraction coefficient of the medium. Explicitly, the dielectric coefficient of the medium is assumed to have the form:

$$\begin{aligned} \epsilon_r(\omega) &= 1 - \sum_{i=0}^{2} \chi_i \frac{(\omega-\omega_{0,i})T_{2,i} + j}{1+\xi^2+(\omega-\omega_{0,i})^2 T_{2,i}^2}\,, \\ &\simeq 1 - \sum_{i=0}^{2} \bar{\sigma}_i(\omega)\,, \end{aligned} \tag{7}$$

where

$$\bar{\sigma}_i(\omega) \equiv \bar{\sigma}_{0,i} \begin{cases} 1 & \text{for } |\omega-\omega_{0,i}|T_{2,i} < \pi/2\,, \\ 0 & \text{for } |\omega-\omega_{0,i}|T_{2,i} > \pi/2\,, \end{cases} \tag{8}$$

and $\bar{\sigma}_{0,i} = \chi_i/(1+\xi^2)$. The parameters are as in the previous case with the exception of the following the three resonances are chosen at $\lambda_0 = 0.8\mu$m, $\lambda_1 = 1.0\mu$m and $\lambda_2 = 1.3\mu$m. The line width in each case satisfies $\omega_i T_{2,i} = 100$. From the cases we examined we chose to present here six:

	χ_0	χ_1	χ_2
(i)	0.00	-0.10	0.00
(ii)	0.00	-0.10	-0.05
(iii)	0.05	-0.10	0.00
(iv)	0.05	-0.10	0.10
(v)	0.06	-0.10	0.01
(vi)	-0.10	-0.02	0.10

These six cases are illustrated in Fig. 1. Curves *(ii)* and *(iii)* in the left frame illustrates the deceleration process associated with two resonances whose population is not inverted.

The effect of shorter wavelength is, as expected, more pronounced and in this case the acceleration at high energies is very small. In the right frame we observe that if the population inversion is not sufficiently high comparing to the other two transitions then the bunch is actually decelerated—see *(iv)*. Even if the population inversion of one transition is larger than the other two, acceleration is not guranteed at high energies as revealed by *(v)*. However, proper excitation of the material may ensure acceleration at high energies although at low energies the Cerenkov process is dominant—as revealed by *(vi)*.

4 EMITTANCE CONSIDERATIONS

In order to examine the potential of this scheme we shall consider a bunch of electrons which are uniformly distributed in a fraction of one wavelength. To each particle we attribute an index ν; we assume N such particles in one period of the wave. If only the ν's particle was present, it generates at its location a wake which is $E_{0,\nu}$; the velocity of the individual particle is denoted by v_ν. We assume that on axis the longitudinal electric field is

$$E_z = -\sum_{\nu=1}^{N} E_{0,\nu} \cos\left[\omega_r\left(t - \frac{z}{v_\nu}\right)\right] \times e^{\omega_i (t - z/v_\nu)} h\left(t - \frac{z}{v_\nu}\right) , \qquad (9)$$

where $h(\xi)$ is the step function. Note that the fact that the particles move in an active medium is expressed by the exponential grow in the amplitude of the field - with the coefficient ω_i which is is well known is proportional to the population inversion. Close to the axis, according to Gauss, law the radial electric field is given by $E_r \simeq -(r/2)\partial E_z/\partial z$ and from Ampere law $H_\phi \simeq (r/2)\epsilon_0 \partial E_z/\partial t$. The amplitudes $E_{0,\nu}$ are determined based on (2) and (6). We have simulated the dynamics of a bunch of particles assuming this field distribution. The typical amount of charge per unit length is $3 \times 10^{-6} C/m$. We further assume that the active medium has its resonance at $\lambda = 1\mu m$ and the particles distribution is on the order of 0.1λ. The dipole moment in the material is given by $\mu \simeq 1.6 \times 10^{-29} Cm$ and the atoms/molecules density is $10^{25} m^{-3}$. The average gradient at the location of the bunch is $600 MV/m$; $\omega_i/\omega_r \simeq 0.01$. The left frame of Fig. 2 illustrates how the particle is accelerated from 25MeV to more than 400MeV in less than 16cm. This acceleration is accompanied by a slight increase in the emittance from 36.5 to 38.5 $\mathrm{mm} \times \mathrm{mrad}$. If we repeat the same exercise at microwave frequency, say 20GHz, $\omega_i/\omega_r \simeq 0.1$ and then $E_0 \simeq 1 MV/m$ and in 40 cm the particle is accelerated by 0.5MeV but the emittance increases from 37 to 58 $\mathrm{mm} \times \mathrm{mrad}$ at the output as illustrated in the right frame of Fig. 2. These results indicate that acceleration of electrons with active medium is feasible and in the optical regime no substantial increase in the emittance is anticipated.

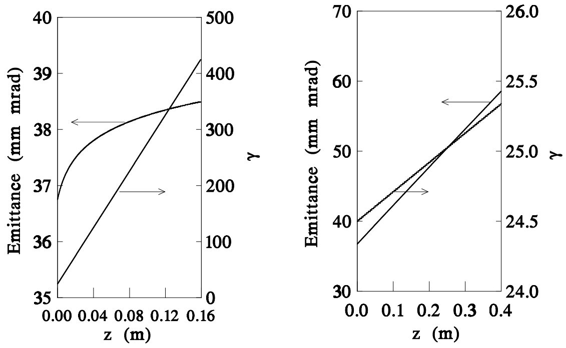

Figure 2: Emittance and energy variations along the interaction region when the resonance of the medium is in the optical range (left frame) and microwave range (right frame)

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