

COMPENSATING THE UNEQUAL BUNCH SPACING IN THE NLC DAMPING RINGS *

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1 INTRODUCTION AND SUMMARY

The damping rings of the Next Linear Collider (NLC), at any given time, will contain four trains of 90 bunches each. Within each train the bunches populate adjacent buckets and between trains there is a gap that extends over 43 buckets. A consequence of an uneven filling scheme is that within each train the synchronous phase will vary from bunch to bunch. In the NLC after extraction the beam enters the bunch compressor and then the X-band linac. The phase variation in the ring, if uncompensated, will lead to a phase variation in the X-band linac which, in turn, will result in an unacceptable spread in the final energy of the individual bunches of a train. The synchronous phase variation, however, can be compensated, either in the damping ring itself or in the bunch compressor that follows. The subject of this paper is compensation in the damping ring. (Compensation in the compressor is discussed in Ref. [1].)

In this report we begin by finding the synchronous phase variation in damping rings with bunch trains and gaps of arbitrary length. These results are then applied to the parameters of the NLC damping rings. Finally, we study two methods of compensating this phase variation: in the first method two passive subharmonic cavities are employed, and in the second the klystron output is varied as a function of time. We find that, for the NLC, a nominal phase variation of 6° within a train can be reduced by almost an order of magnitude by either method of compensation, with the cost of the second method being an extra 10% in output power capability of the klystron. More details of this work will be found in Ref. [2]. Concerning beam loading in storage rings, the reader is referred to Ref. [3]; for rings with bunch trains and gaps, see also Ref. [4].

2 VARIATION IN BUNCH PHASE

Consider a storage ring filled with N_t equal trains of N bunches each. Let us suppose that all initial transients have died out and that the longitudinal bunch positions have settled down to their asymptotic values. We know from the symmetry of the problem that the position of a given bunch within a train must be the same for all trains, and we can therefore limit our consideration to one train and the gap following it. Let us, in this section, ignore the beam loading effects of modes other than the fundamental. Then the cavity voltage can be represented by a phasor rotating at the fundamental frequency. The total cavity voltage associated with bunch n , \tilde{V}_c^n (we use a tilde to denote phasor

quantities), is given by the sum

$$\tilde{V}_c^n = \tilde{V}_b^n + \tilde{V}_g^n \quad , \quad (1)$$

with \tilde{V}_b^n the beam induced voltage and \tilde{V}_g^n the generator voltage associated with bunch n . Our phasor clock steps in increments of time $\Delta t = T_0/n_h = 2\pi/\omega_{rf}$, with T_0 the revolution time, n_h the harmonic number, and ω_{rf} the nominal rf frequency. The generator voltage is normally independent of n and set to

$$\tilde{V}_g = V_{c0} \exp(i\phi_0) - \tilde{V}_{b0} \quad , \quad (2)$$

with V_{c0} the nominal peak cavity voltage and ϕ_0 the nominal synchronous phase, defined by $\phi_0 = \cos^{-1}[(U_0 + U_{hml})/eV_{c0}]$, where U_0 is the per turn synchrotron radiation energy loss and U_{hml} is the higher mode losses. The phasor \tilde{V}_{b0} is the beam induced voltage assuming the current is evenly distributed around the ring[3]:

$$\tilde{V}_{b0} = -\frac{2I_0R}{1+\beta} \cos\psi e^{i\psi} \quad , \quad (3)$$

where I_0 is the average beam current, R is the cavity shunt impedance, β the coupling parameter, and ψ the tuning angle. The tuning angle is defined by $\psi = \tan^{-1}(\Delta\omega T_F)$, where $\Delta\omega = \omega_0 - \omega_{rf}$ is the cavity detuning, ω_0 is the cavity resonant frequency; where $T_F = 2Q_L/\omega_{rf}$ is the cavity filling time and Q_L the loaded Q of the cavity.

The voltage increase that bunch n receives from the cavities is given by

$$\text{Re}[\tilde{V}_c^n \exp(i\Delta\theta_n)] = (U_0 + U_{hml})/e = V_{c0} \cos\phi_0 \quad , \quad (4)$$

with $\Delta\theta_n/\omega_{rf}$ the deviation in arrival time at the cavities of bunch n . Combining Eqs. (1), (2), and (4) we obtain

$$\cos\phi_0 - \cos(\phi_0 + \Delta\theta_n) = \text{Re}[(\tilde{V}_b^n - \tilde{V}_{b0}) \exp(i\Delta\theta_n)]/V_{c0} \quad (5)$$

The bunches shift in phase to compensate for differences in the real part of the beam induced voltage when they arrive at the cavities. If the phase shifts are small, as they are for the NLC damping rings, the phase difference between bunch n and bunch n' of a train becomes

$$\Delta\theta_{nn'} \approx \frac{\text{Re}(\tilde{V}_b^{n'} - \tilde{V}_b^n)}{V_{c0} \sin\phi_0} \quad . \quad (6)$$

The beam induced voltage at a clock time corresponding to the passage of bunch $n+1$ of a train is given by

$$\tilde{V}_b^{n+1} = -2kq \exp(-i\Delta\theta_{n+1}) + \tilde{V}_b^n \exp(i\Delta\bar{\omega}T_b) \quad (7)$$

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with k the loss factor ($= \frac{1}{5}\omega_{rf}R/Q$), q the charge per bunch, and T_b the nominal time between bunches. The parameter $\Delta\bar{\omega}$ is a complex quantity:

$$\Delta\bar{\omega} = \Delta\omega + i\frac{\omega_{rf}}{2Q_L} = \frac{\tan\psi + i}{T_F} \quad (8)$$

The second term in Eq. (7) gives the development of the beam loading phasor in the time interval between the n^{th} and the $(n+1)^{\text{st}}$ bunch, and the first term gives the contribution of the $(n+1)^{\text{st}}$ bunch just after its passage. From symmetry we know that the properties of bunch $N+1$ are the same as those for bunch 1; therefore we cross the gap and close the loop by setting

$$\tilde{V}_b^1 = -2kq \exp(-i\Delta\theta_1) + \tilde{V}_b^N \exp(i\Delta\bar{\omega}T_g) \quad (9)$$

with T_g the gap time interval.

Now let us assume, as is normally the case, that for all n the deviation in bunch position is small, $\Delta\theta_n \ll 1$. Then we can explicitly solve Eqs. (7) and (9) to give

$$\tilde{V}_b^n = -\frac{2kq}{1 - e^{i\bar{\theta}_b}} \left(1 - \frac{\sin\frac{1}{2}N_g\bar{\theta}_b}{\sin\frac{1}{2}(N+N_g)\bar{\theta}_b} e^{i(n-N/2)\bar{\theta}_b} \right) \quad (10)$$

with $\bar{\theta}_b = \Delta\bar{\omega}T_b$ the nominal (complex) phase change between bunches and $N_g = T_g/T_b - 1$ the number of ‘‘missing’’ bunches in the gap. The difference in induced voltage across a bunch train is given by

$$\tilde{V}_b^N - \tilde{V}_b^1 = -2kq \frac{\sin\frac{1}{2}N_g\bar{\theta}_b}{\sin\frac{1}{2}\bar{\theta}_b} \frac{\sin\frac{1}{2}(N-1)\bar{\theta}_b}{\sin\frac{1}{2}(N+N_g)\bar{\theta}_b} \quad (11)$$

Note that for N large this equation is symmetric with respect to N and N_g . Finally note that even in the case $\Delta\theta_n$ is not very small compared to 1 we can use Eq. (10) to obtain an initial solution, and then iterate Eqs. (5),(7), (9) to obtain the exact solution.

3 SHORT TRAINS AND SHORT GAPS

Let us denote a bunch train as being short if $|\Delta\bar{\omega}NT_b| \ll 1$; a gap as being short if $|\Delta\bar{\omega}N_gT_b| \ll 1$. According to these criteria, in the NLC damping rings both the train length and the gap length are short. In such a case Eq. (10) becomes

$$\tilde{V}_b^n \approx \frac{2kq}{i\bar{\theta}_b} \left(\frac{N}{N+N_g} \right) \left[1 - iN_g\bar{\theta}_b \left(\frac{n}{N} - \frac{1}{2} \right) \right] \quad (12)$$

Note that the leading order term is just equal to \tilde{V}_{b0} . From Eq. (12) we obtain

$$\tilde{V}_b^n - \tilde{V}_b^{n'} \approx -2kq \left[\frac{(n-n')N_g}{N+N_g} \right] \quad (13)$$

We see that the variation in \tilde{V}_b is to good approximation a real quantity, and that it varies linearly with bunch number, implying that the phase also varies linearly along the train [see Eq. (6)]. The total phase shift from the first to the last bunch of each train is given by

$$\Delta\theta_{1,N} \approx -\frac{2kq}{V_{c0} \sin\phi_0} \left[\frac{(N-1)N_g}{N+N_g} \right] \quad (14)$$

We note that our result is independent of ψ or β , and that, if N is not small, it is symmetric with respect to the length of the bunch train and the length of the gap. Note also that, if N is not small, Eq. (14) can be written in the form[5]

$$\Delta\theta_{1,N} \approx -\frac{2kI_0T_g}{V_{c0} \sin\phi_0} \quad (15)$$

This result is also valid when either, but not both, the bunch train length or the gap length is long.

4 THE NLC DAMPING RINGS

For the NLC damping rings the rf frequency is 714 MHz and the R/Q for the two rf cavities combined is 240 Ω ; therefore, $k = 0.54$ V/pC. We take $N = 90$, $N_g = 43$, $q = 2.5$ nC, and $T_b = 1.4$ ns; therefore, $I_0 = 1.2$ A. Also taking $V_{c0} = 1$ MV and $\phi_0 = 50^\circ$ we obtain from Eq. (14) $\Delta\theta_{1,N} = -5.8^\circ$. We have also numerically simulated this problem using turn-by-turn tracking[6], where we have continued until the steady state was reached. For the tracking the unloaded Q of the cavities was taken to be $Q_0 = 25,500$; for minimum reflected power $Q_L = 2200$ and $\psi = -44.5^\circ$. The phasor diagram of the resulting voltages is shown in Fig. 1; the shift in synchronous phase vs bunch number is shown in Fig. 2a. We see that, over each train, the variation is linear with bunch number, and that the total phase shift is -5.9° , in good agreement with our analytical results. Note that both the length of the gap and the length of the bunch train are short according to our criteria: $|\Delta\bar{\omega}T_g| = 0.1 \ll 1$ and $|\Delta\bar{\omega}NT_b| = 0.2 \ll 1$.

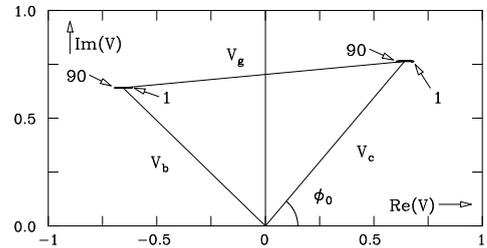


Figure 1: A diagram of the phasors \tilde{V}_b , \tilde{V}_g , and \tilde{V}_c , in units of MV, for the central particle in a train in the NLC damping rings. The variation from bunch 1 to 90 in a train is also indicated by the short arcs.

5 PHASE COMPENSATION

In Ref. [7] the authors find that a properly tuned, passive, lower harmonic cavity can be used to compensate the phase variation due to bunch trains. The frequency of this cavity must equal $\omega_{rf} - 2\pi mN_t/T_0$, with m an integer. Adding this cavity to our analysis we find, for a ring with short trains and short gaps, that this works by compensating the linear variation of $\text{Re}(\tilde{V}_b^n)$ along the train with a function varying as $\sin[2\pi m(n-N/2)/(N+N_g)]$. In the case of the SLC the gap length is only about half the train length, so the compensation does not work well. We will, therefore, also study the effect of using two lower harmonic cavities.

Fig. 2b gives the tracking results when an optimally tuned, passive cavity with frequency $\omega_{rf} - 8\pi/T_0$ is included in the damping ring. The parameters are: $R/Q = 34 \Omega$, $Q_0 = 3.43 \times 10^4$, $Q_L = 1.14 \times 10^4$, and $\psi = -85^\circ$.

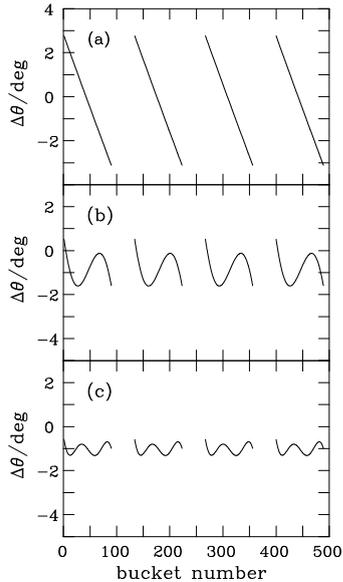


Figure 2: (a) The steady-state, bunch-to-bunch phase deviation vs bunch number for the NLC damping rings. Note that a negative value of phase is more toward toward the rf crest, and that zero represents the nominal bunch positions. Also shown are what can be achieved when (b) one or (c) two passive, lower harmonic cavities are added.

We see from the figure that the maximum phase deviation has been reduced to 2.1° , about a factor of 3 reduction in phase deviation. Adding a second, passive cavity, with frequency $\omega_{rf} - 16\pi/T_0$, $R/Q = 18 \Omega$, $Q_0 = 34,300$, $Q_L = 10,000$ and $\psi = -82^\circ$ gives the result shown in Fig. 2c. Now the maximum phase deviation has been reduced to 0.65° , which is a reduction factor of 9. Finally, note that the R/Q 's of the compensation cavities are very small, so they should not contribute to an instability.

Another method of compensation is to optimally vary the klystron output in both amplitude and phase as a function of time. This method requires extra power from the klystron. The amount required and the effectiveness of the compensation depend on the bandwidth or response time of the klystron output. To compensate perfectly \tilde{V}_g^n must be varied to match the negative of the variation in \tilde{V}_b^n . For the case of the NLC damping rings we obtain from Eq. (13) that this requires a total change—mostly in the real part of \tilde{V}_g^n —of $+78 \text{ kV}$ or, equivalently, a relative change of $+6\%$ (see Fig. 1). A plot of the desired variation is given in Fig. 3 (the solid lines). Let us assume that the amplitude and phase of \tilde{V}_g can be varied independently, and that when the rf amplitude or phase at the klystron input is switched, the output (change in \tilde{V}_g) responds according to $[1 - \exp(-t/t_d)]$, with t_d the voltage response time. We optimally switch at the beginning and end of each train to give initial amplitude and phase values of \tilde{V}_g of, respectively, 1.26 MV and 6.20° , and final values of 1.34 MV and 5.85° . Let us assume that t_d is 100 ns (a voltage response time that one can reasonably expect for a klystron at our frequency[8]). The resulting variation in amplitude,

for example, is then given by

$$|\tilde{V}_g^n|/\text{MV} = \begin{cases} 1.26 + 0.109[1 - \exp(-0.014(n-1))] \\ 1.34 - 0.180[1 - \exp(-0.014(n-1))] \end{cases} \quad (16)$$

where the upper (lower) line gives the voltage during the bunch train (gap) and n is the bucket number from the beginning of the current bunch or gap. The corresponding variation in $\text{Re}(\tilde{V}_g^n)$ is given by the dashed curve in Fig. 3. The difference between the solid and dashed curves equals the variation in $\text{Re}(\tilde{V}_b^n)$ from which, as before, we obtain the phase variation along a train. The results are shown in Fig. 4. The total phase variation is reduced to 0.74° . There is a small power cost compared to the nominal case of $|\tilde{V}_g^n|$ is kept constant at 1.3 MV . We see from Eq. (16) that an amplitude overhead of 5.3% , or equivalently, a power overhead of 10.6% is needed.

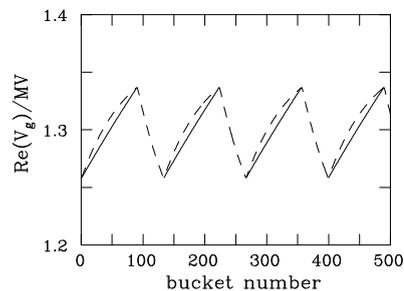


Figure 3: The variation of $\text{Re}(\tilde{V}_g)$ that eliminates the variation in bunch phase (the solid lines). The optimal variation that can be achieved, assuming a klystron voltage response time constant of 100 ns , is shown by the dashed line.

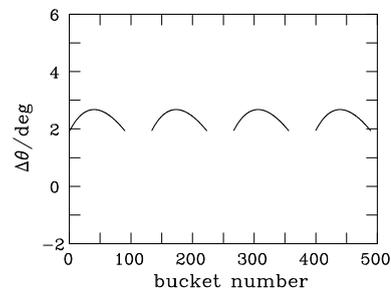


Figure 4: The phase variation that is achieved when \tilde{V}_g is varied optimally assuming a klystron voltage response time constant of 100 ns .

6 REFERENCES

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