# STUDIES FOR THE CLOSED ORBIT CORRECTION SYSTEM OF SOLEIL* 

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#### Abstract

The number and locations of dipolar correctors for the SOLEIL storage ring are discussed. The aim is then to calculate the corrector strengths and the residual closed orbit deviations due to standard magnetic errors. This is done in two different ways : from analytical formulas using the eigenvalues, eigenvectors and from correction of a sample of closed orbits using the singular value decomposition method.


## 1. INTRODUCTION

The SOLEIL project [1], is designed with a very small beam emittance to produce very high brilliance beams of synchrotron radiation in the VUV and soft X-ray regions. This requires relatively strong focusing, and magnet errors will therefore introduce large distortions in closed orbit (C.O.) which reduce the dynamic aperture. To keep the optimum SOLEIL performances it is therefore necessary to install a system of Beam Position Monitors (BPM) and dipolar correctors.

We have studied the C.O. correction system from a statistical point of view with two different methods: the first one, particularly interesting during the elaboration of a project uses analytical formulas involving eigenvalues and the second one the Singular Value Decomposition (SVD) technique to correct a sample of C.O. randomly chosen.

The number and the locations of the BPMs and correctors will be discussed. A minimum C.O. correction system (minimum number of correctors), is defined, from which any other solution with more correctors can be derived in order to decrease individual corrector strengths.

Finally, for some selected configurations, we will give the rms and maximum of corrector strengths and residual C.O. deviations after the correction of C.O. due to standard magnetic errors. That characterizes the effectiveness of the correction scheme.

## 2. CLOSED ORBIT CORRECTION METHODS USED

Two complementary correction methods are used. The first one starts with a least square minimization of the C.O. at the BPMs and makes use of eigenvalues [2]. This has been successfully applied to the Super-ACO storage ring [3]. To study a corrector scheme of a ring in project, a statistical analysis is necessary [4], then analytical formulas can be derived, given rms values of corrector strengths, residual C.O., as a function of initial C.O. and also of BPM and corrector errors [5]. As well known

[^0]formulas already give the rms C.O. deviations as a function of magnetic errors, the problem of C.O. correction can thus be completely treated analytically, from the C.O. errors generation to the correction.

The second method also deals with a least square minimization of C.O., but following the SVD theory [6], decomposes the corrector to BPM response matrix into a product of two unitary matrices and a diagonal matrix. The eigenvalues so obtained are the square root of the ones obtained by the first method, and the two techniques are very similar. To check the results of the first method, we decided to apply the SVD algorithm to correct a sample of 20 C.O. randomly generated by magnetic errors.

## 3. NUMBER AND LOCATION OF BPMs AND CORRECTORS

The SOLEIL lattice is composed of 4 superperiods, each one consisting of 4 cells. For the sake of simplicity, the betatron functions and phase advances are respectively shown in fig. 1 and fig. 2, for one cell only and in the case of the $2.7 \mathrm{~nm} . \mathrm{rad}$ lattice.

The BPMs must be placed at crucial points : as close as possible to the quadrupoles or the sextupoles, where misalignments are sources of orbit distortion or dynamic aperture reduction, and at the ends of each insertion straight section in order to provide local C.O. adjustment. Several solutions are possible [7], but the set of 7 BPMs per cell, as shown in fig. 2, has been adopted, in order to minimize their total number ( $7 \times 16=112$ ).

The most efficient correction is obtained when the correctors are located as near as possible to the sources generating the largest orbit deviation, i.e., the quadrupoles. Nevertheless, we discarded the possibility of incorporating the correctors inside the quadrupoles for several reasons [7] such as chromaticity change, good quality and reproducibility of quadrupole field. Taking into account the density of the magnetic elements of the lattice, the correctors will be included in the sextupoles. There are thus 7 possible correctors per cell and their positions, noted C 1 to C 7 , are shown in fig. 2 .

But is it necessary to use all these correctors? To answer, let us look at the optical properties of the lattice.

In the horizontal plane, the phase advance presents roughly 3 parts of nearly constant phase, that means that the correctors are not truly independent but they only form 3 really independent groups ( $\mathrm{C} 1, \mathrm{C} 2$ ) ; ( $\mathrm{C} 3, \mathrm{C} 4$, C5) ; (C6, C7). The minimum number of correctors is thus $3 \times 16=48$ chosen among these 3 groups at the same time. The $\beta_{\mathrm{x}}$ function tells us that $\mathrm{C} 1, \mathrm{C} 4, \mathrm{C} 7$ will be the most effective.

In the vertical plane, the phase advance is much smoother and only correctors located at the two different
ends of the cell will be really independent．The minimum number of correctors is thus $2 \times 16=32$ and the $\beta_{z}$ function imposes the choice of C2 and C6．

A careful study of a function measuring the efficiency of the correctors by using the SVD properties suggests the same correctors［7］．

Starting from this minimum configuration，any addition of other correctors will be helpful on condition that the right number of eigenvalues is used，that is［8］：
－In the worst case，the new correctors are not independent of the old ones，so the initial number of eigenvalues should be used，and the strength of correctors neighbouring to the new ones introduced will be lower． The residual orbit is only slightly reduced．
－In the best case，new independent parameters are introduced so that one can use more eigenvalues， corrector strengths and residual C．O．at the BPMs are lower．

## 4．RESULTS

The C．O．distortions are produced by the following errors，expressed with rms values ：quadrupole misalignment $\langle\mathrm{x}\rangle=\langle\mathrm{z}\rangle=1.10^{-4} \mathrm{~m}$ ，dipole misalignment $\langle\mathrm{s}\rangle=\langle\mathrm{z}\rangle=5.10^{-4} \mathrm{~m}$ ，dipole axial roll $\left\langle\theta_{\mathrm{s}}\right\rangle=2.10^{-4} \mathrm{rad}$ and dipole field error $\langle\mathrm{B} \ell\rangle /(\mathrm{B} \ell)_{\mathrm{o}}=5.10^{-4}$ ．

To summarize，we will only report here the results concerning the low emittance（ $2.7 \mathrm{~nm} . \mathrm{rad}$ ）lattice and that only for the minimum configuration and the one when all the correctors are used．

With this last configuration，one should pay attention to the degenerescence or near degenerescence problem［8］．Fig． 3 and 4 show the eigenvalues numbered following their decreasing values，respectively for the horizontal and the vertical plane when all BPMs and correctors are used．One can see that horizontally，there is a large discontinuity at $3 \times 16=48$ which is coherent with the number of effectively independent parameters mentioned above．Nevertheless，one can use up to $4 \times 16=64$ or even $5 \times 16=80$ eigenvalues knowing that the use of more eigenvalues gives more accurate correction but will require stronger corrector strengths． Beyond 80 eigenvalues，the increase in corrector strength is no longer worthwile because even if the residual C．O．
is reduced at the BPMs，it will be deteriorate elsewhere． In vertical，the largest discontinuity can be observed at $\mathrm{n}=32$ ，corresponding well to the minimum configuration and the largest number of eigenvalues one can use is $4 \times 16=64$ ．

Table 1 summarizes the results of the closed orbit correction calculation using the two methods．The corrector strengths and residual C．O．obtained by the analytic formulas are given in rms values．By the second method of correction on a sample of 20 different C．O． generated by the same magnetic rms errors truncated to 3 standard deviations，one can also calculate the same quantities．We present on the right hand side of Table 1， only the maximum values of corrector strengths and residual C．O．which correspond to approximatively 3 times the previous rms values．

Fig． 5 and 6 show respectively in the horizontal and vertical plane the rms residual C．O．along the machine， obtained by the first method，for the correction scheme where all the correctors and the 64 highest eigenvalues are used．Fig． 7 and 8 show the residual C．O．after the correction of one of the 20 different orbits using the second method with the same correction scheme．

When taking into account BPM alignment errors of $\langle x\rangle=\langle z\rangle=1.10^{-4} \mathrm{~m}$ ，only 48 eigenvalues will be usable and $\max <$ res．C．O．＞$=0.10 \mathrm{~mm}$ for the same corrector strengths．Indeed，following formulas（29）and（31）of［5］， if more eigenvalues are used，the residual C．O．without errors is further reduced，but the one with errors（due to BPM or even computer errors）will increase．

Corrector strength errors of $1 / 100$ ，that is $10^{-6} \mathrm{rad}$ ，do not change these results．

## 5．CONCLUSION

A minimum configuration making use of 48 correctors horizontally and 32 vertically with a strength of 0.5 mrad allows one to obtain satisfactory results．If we install a corrector in each sextupole，the corrector strengths can be lower by a factor of 2 on condition that the right number of eigenvalues is used．This configuration would be necessary to control the position and angle at each insertion straight section．

Table 1.

|  |  | FIRST METHOD |  |  |  |  | SECOND METHOD |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | Correctors used | Number of eigenvalues used | Mean of ＜corr．strengths＞ （mrad） | Max of ＜corr．strength＞ （mrad） | $\begin{gathered} \text { Max of } \\ \text { <res. } \text { CO at BPM> } \\ (\mathrm{mm}) \end{gathered}$ | $\begin{gathered} \text { Max of } \\ <\text { res. CO> } \\ (\mathrm{mm}) \end{gathered}$ | Max of cort．strength （mrad） | $\begin{gathered} \text { Max of } \\ \text { res. CO at BPM } \\ (\mathrm{mm}) \\ \hline \end{gathered}$ | Max of res． CO （mm） |
| $\begin{aligned} & \text { ت⿹\zh26灬 } \\ & \text { 울 } \\ & \text { 울 } \end{aligned}$ | $\begin{aligned} & 48 \\ & \mathrm{Cl}, \mathrm{C} 4, \mathrm{C} 7 \\ & \hline \end{aligned}$ | 48 | 0.12 | 0.14 | 0.07 | 0.09 | 0.45 | 0.18 | 0.23 |
|  | $\begin{aligned} & 112 \\ & \mathrm{Cl} \rightarrow \mathrm{C} 7 \end{aligned}$ | $\begin{aligned} & 48 \\ & 64 \\ & 80 \end{aligned}$ | $\begin{aligned} & 0.059 \\ & 0.075 \\ & 0.092 \\ & \hline \end{aligned}$ | $\begin{gathered} 0.081 \\ 0.098 \\ 0.11 \\ \hline \end{gathered}$ | $\begin{aligned} & 0.06 \\ & 0.03 \\ & 0.02 \\ & \hline \end{aligned}$ | $\begin{aligned} & 0.08 \\ & 0.07 \\ & 0.07 \end{aligned}$ | $\begin{gathered} 0.25 \\ 0.26 \\ 1 \\ \hline \end{gathered}$ | $\begin{gathered} 0.17 \\ 0.08 \\ 1 \\ \hline \end{gathered}$ | $\begin{gathered} 0.21 \\ 0.18 \\ \end{gathered}$ |
| $\begin{aligned} & \text { 毛 } \\ & > \end{aligned}$ | $\begin{aligned} & 32 \\ & \mathrm{C} 2, \mathrm{C} 6 \end{aligned}$ | 32 | 0.12 | 0.14 | 0.11 | 0.11 | 0.49 | 0.28 | 0.35 |
|  | $\left\{\begin{array}{l} 112 \\ \mathrm{Cl} \rightarrow \mathrm{C} 7 \end{array}\right.$ | $\begin{aligned} & 32 \\ & 48 \\ & 64 \end{aligned}$ | $\begin{aligned} & 0.042 \\ & 0.051 \\ & 0.059 \\ & \hline \end{aligned}$ | $\begin{aligned} & 0.053 \\ & 0.072 \\ & 0.084 \\ & \hline \end{aligned}$ | $\begin{aligned} & 0.11 \\ & 0.06 \\ & 0.03 \\ & \hline \end{aligned}$ | $\begin{aligned} & 0.11 \\ & 0.06 \\ & 0.05 \\ & \hline \end{aligned}$ | $\begin{aligned} & 0.19 \\ & 0.23 \\ & 0.26 \end{aligned}$ | $\begin{aligned} & 0.28 \\ & 0.15 \\ & 0.08 \end{aligned}$ | $\begin{aligned} & 0.29 \\ & 0.21 \\ & 0.13 \end{aligned}$ |

## 6. REFERENCES

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Figure 2





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