INSTABILITY DUE TO THE LONGITUDINAL HIGHER MODES IN THE ELECTRON STORAGE RING

Soon-Kwon Nam, Kangwon National University, Korea

ABSTRACT

The bunch length,the average rms energy spread as function of current and the thresholds current due to the potential well distortions are calculated by the simulation method using the long range wakefield. The growth rates and frequency shift with the longitudinal higher order modes are calculated by the eigenmode analysis of Sacherer' integral equation and compared to water bag model.

1 THE BUNCH LENGTH AND ENERGY SPREAD

The longitudinal single bunch collective motion of an electron beam in a storage ring is described by the Vlasov equation [1-2]

$$-\frac{\partial\Psi}{\partial\theta} = p\frac{\partial\Psi}{\partial q} + (-q + V(q,\theta))\frac{\partial\Psi}{\partial p} \qquad (1)$$

where $\Psi = \Psi(p,q,\theta)$ is the distribution function in the longitudinal phase space, $p \equiv (E_0\text{-}E)/E_0\,\sigma_\varepsilon$ the relative energy deviation, $q \equiv z/\sigma_z$ the longitudinal position, and $\theta \equiv \omega_s t$ the phase of the synchrotron motion. The charge of the bunch induces the longitudinal field V(q) with the longitudinal wake function W(q) as

$$V(q,\theta) = I\int_{-\infty}^{\infty} f(q',\theta)W(q'-q)dq' \qquad (2)$$

where $f(q,\theta) = \int_{-\infty}^{\infty}\Psi(p,q,\theta)dq$ and normalized and the parameter I represents the beam intensity as

$$I = \frac{Ne}{2\pi\nu_s\sigma_\varepsilon}\left(\frac{e}{E_0}\right) \qquad (3)$$

with N the number of the particles in the bunch, ν_s the synchrotron tune and E_0 the nominal beam energy. We used a tracking method[2] for simulating the effect of the wakefield on the longitudinal phase space of the beam. The long range wakefield as a function of the bunch coordinates z for the ring with cavities was calculated for the gaussian driving bunch with the bunch length of 1 mm. For the simulations we take the synchrotron tune of 0.0059, momentum spread of 7.15E-04, energy 2 GeV, bunch length of 0.0078 m, 5 azimuthal space harmonics and 60 mesh points. Fig. 1 shows the bunch length(σ_z/σ_{z0}) due to the potential well distortions(PWD) by the simulation method using the long range wakefield which is calculated by TBCI code[3]. From the simulation method applied to the current storage ring we find that the threshold to be 1.65 E+10 which is 4.699 mA.

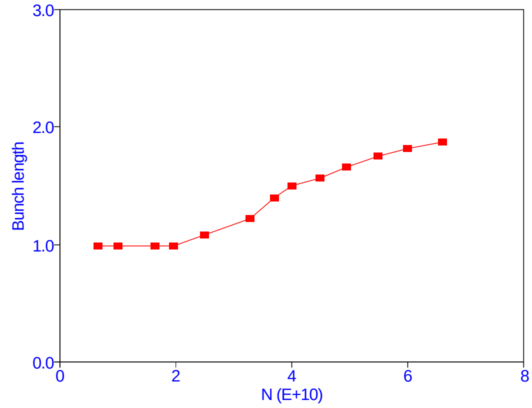

Figure: 1 The bunch length due to the PWD by the simulation method.

In Fig.2, we plot the average rms energy spread ($\sigma_\varepsilon/\sigma_{\varepsilon 0}$) due to the potential well distortions as function of current by the simulation method using the long range wakefield of our cavities.

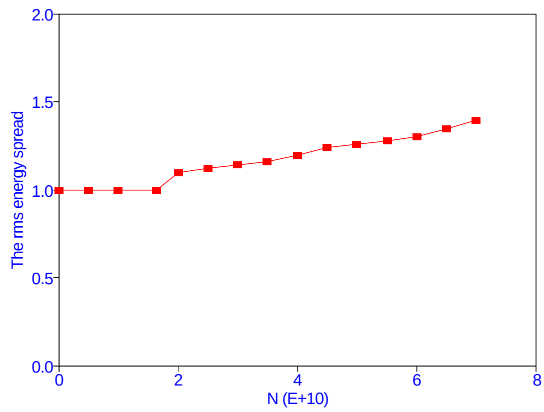

Figure: 2 The rms energy spread due to the PWD by the simulation method.

We find that the threshold with the potential well distortion is 1.65E+10 which is the value between 0.88 mA for without SPEAR and 6.24 mA for with SPEAR scaling. The first unstable mode appears at this current by the tracking calculation.

2 THE LONGITUDINAL GROWTH RATES AND TUNE SHIFT

We consider a beam with a single bunch executing a longitudinal oscillation of mode *m*. The method of analysis of longitudinal instabilities is that of Sacherer's [4] whose main result is an integral equation for the eigenmode and eigenfrequencies of the longitudinal bunch oscillations. Our method of analysis is based on Zotter's formalism[5] which is the eigenmode analysis of Sacherer' integral equation without mode coupling for a Gaussian beam and Chin's method[6]. Sacherer's integral equation can be reduced to the eigenvalue equation by expanding the radial mode function in orthogonal polynomials and the eigenvalue equation is written by

$$\det(M - \Delta\nu I) = 0 \qquad (4)$$

where the matrix M is the interaction matrix, and

$$\Delta\nu = \frac{\omega_m - m\omega_s}{\omega_s} \qquad (5)$$

is the relative complex tune shift and ω_m mode frequency, ω_s synchrotron angular frequency, *m* azimuthal mode number and ω_0 revolution angular frequency. The matrix M can be written by the finite sum formular[5] and the recurrence formular[6] as

$$M_{kl} = \frac{m\xi R_s}{M\sqrt{4Q^2-1}} E_{mk}E_{ml}[B_g(\nu_2) - B_g(\nu_1)] \qquad (6)$$

where,

$$E_{mk}E_{ml} = \frac{1}{2\pi\sigma^2\sqrt{(m+k)!k!(m+l)!l!}} \qquad (7)$$

$$g = m + k + l \qquad (8)$$

We solved the eigenvalue equation (4) numerically with all the matrix elements for a narrowband impedance. The results are shown in Fig.3 and Fig.4 for the radial mode *k*=0,1 and azimuthal mode *m* = 1,2. Fig.3 shows the growth rates and bunch length in our storage ring. Fig.4 shows the tune shift and bunch length for the several modes.

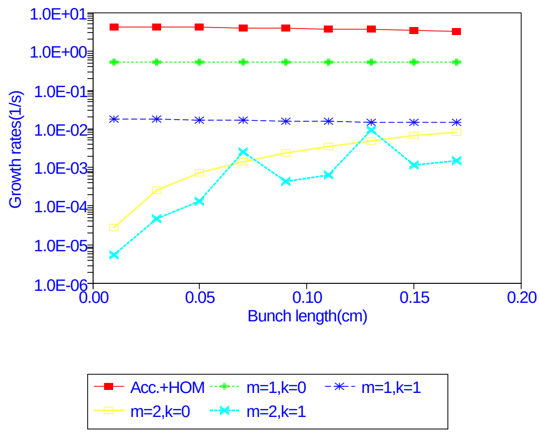

Figure: 3 Growth rates and bunch length for the several modes.

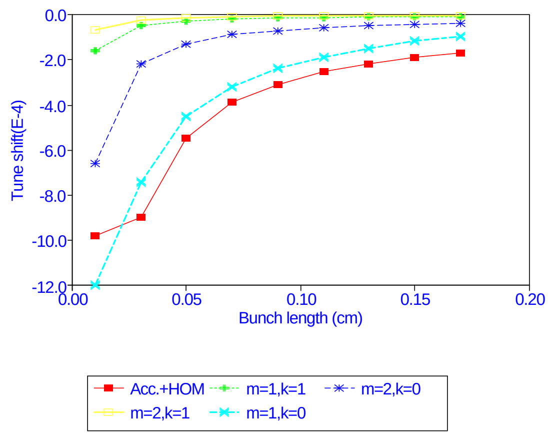

Figure: 4 Tune shift and bunch length for the several modes.

We calculated total 15 modes including fundamental one in the longitudinal case of our designed RF cavity with the length of 40 cm, diameter 46.035 cm, peak voltage 1.5MV and frequency 499.113 MHz. Table 2 summarizes the higher order modes to be troublesome except for the fundamental mode amoung 15 modes.

Table 1. Higher order modes causing beam instabilities.

Freq.(MHz)	Mode type	R/Q(ohm)	Q- value
499.113	TM010	58.899	35716
843.999	TM011	5.565	30279
1114.140	TM020	0.014	34027
1463.590	TM021	0.193	32396

We have also calculated the coupled bunch instability growth rates for the parasitic modes of the designed RF cavity. The growth rates are small enough compared to the radiation damping times of 4.5 *ms* longitudinally.

We can divide the matrix M in eq.(4) into the contribution from the accelerating mode and that of higher modes of the cavity. Thus

$$M = M_{acc.} + M_{h} \tag{9}$$

If eigenvectors have about same values, we can calculate eigenvalues of M using the impedances computed for the our RF cavity. The results are shown in Fig.3-4. The growth rates and tune shifts for the sum of accelerating mode and higher modes are compared with several modes in Fig.3-4.

The integral equation for the eigenmodes and eigenfrequencies can also be solved if the unpurturbed beam has a uniform distribution inside an ellipse in the longitudinal phase space. We now consider the water-bag model[7] to get the growth rates and tune shifts. The results are compared to the other cases in Table 2 -3.

Table 2 Growth rates and tune shifts for dipole mode

Bunch length [m]	Tune shift(E-04) m=1,k=0 / m=1,k=1	Growth rates(1/s) m=1,k=0 / m=1,k=1
0.003	-9.0 / -0.54	0.528 / 0.017
0.007	-3.97 / -0.23	0.527 / 0.016
0.011	-2.58 / -0.15	0.526 / 0.015

Table 3 Growth rates for quadrupole mode.

Bunch length [m]	Growth rates(1/s) rec. formula(m=2)	Growth rates(1/s) Water bag(m=2)
0.003	1.07E-06	0.511E-07
0.007	5.40E-06	0.345E-06
0.011	6.90E-06	0.687E-06

3 CONCLUSIONS

We have studied the longitudinal instability for the bunch length,the average rms energy spread as function of current and the thresholds current due to the potential well distortions by the simulation method using the long range wakefield. The growth rates and frequency shift with the longitudinal higher order modes are calculated by the eigenmode analysis of Sacherer' integral equation and compared to water bag model. The tune shifts of azimuthal modes decrease as the bunch lengthens, while the growth rates of quadrupole mode are increasing functions of the bunch length in the region of our bunch length of 0.0078 *m*. But the growth rates, even in any cases, are small enough compared to the radiation damping times of 4.5 ms in our storage ring.

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