COOLED BEAM INSTABILITIES DRIVEN BY A RING PERIODICITY PERTURBATION

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Abstract

Cooling process of an ion beam from diluted to crystalline state has been simulated in a ring with periodicity 8 and betatron phase advance per period smaller then 90 degrees, as chosen for the design of the CSR project ring. The consequences of a small lattice periodicity perturbation (quadrupole errors, e-cooling solenoid....) has been studied and interpreted by means of envelope modes theory.

1 INTRODUCTION

In these years, after the development of new ion beam cooling techniques, growing interest has been devoted to the possibility of reaching ordered beam states or crystalline beams [1]. Several workshops have been held on this topic, and some specific proposal for the experimental search for crystalline beams have been written. In particular at LNL (Laboratori Nazionali di Legnaro) the construction of specific ion storage ring has recently been proposed (CSR project) [2]. Two are main aspects of the beam physics related to crystalline beams: the crystalline state structure and the evolution from ordinary to ordered beam. The first aspect consists mainly in the determination of the geometry in which the cooled ions organize themselves (minimizing the potential energy), and the study of the coherent modes of these crystals.

The evolution of the beam during the cooling process involvs different phenomena such as coherent instabilities and intrabeam scattering. In this paper we consider this second part of the problem; our simulations show the relevance of the envelope instability, and are compared with the results of calculation based on KV distribution. Moreover we consider the effect of the periodicity breaking determined by the presence of the electron cooling and show that is not in general negligible.

2 ENVELOPE INSTABILITY

We have considered one period of CSR. The current circulating in the ring so to achieve the various crystalline structures is relatively low. For example for Li^{7+} (that can be laser cooled) 10^8 particles accumulated at β=0.07 correspond to complicated multishell structures. At injection (1π mm mrad normalized emittance) this corresponds to a Laslett tune shift of just 10^{-3}.

During the cooling process from this situation to a crystalline beam the tune has to go to zero and various parameter regions have to be crossed, where space charge and intra beam scattering (IBS) become important. Beams with significant space charge tune shift can suffer coherent instabilities, such as the envelope instability, with a growth time much shorter than IBS. It has been shown that this instability is avoided if the zero current betatron phase advance per period σ_0 is smaller than 90^0 [3][4].

In the case of CSR there is a cooling that can compete with the instability growth rate. In Fig. 1 is shown the result of the simulations for Li (parameters mentioned before); the minimum cooling rate necessary to reach $\Gamma=(Ze)^2/akT=1$ is plotted [5] : Ze is the charge of the ion, a is the Wigner-Seitz radius and kT the beam temperature . Simulations are done with PARMT using the Poisson solver routine.

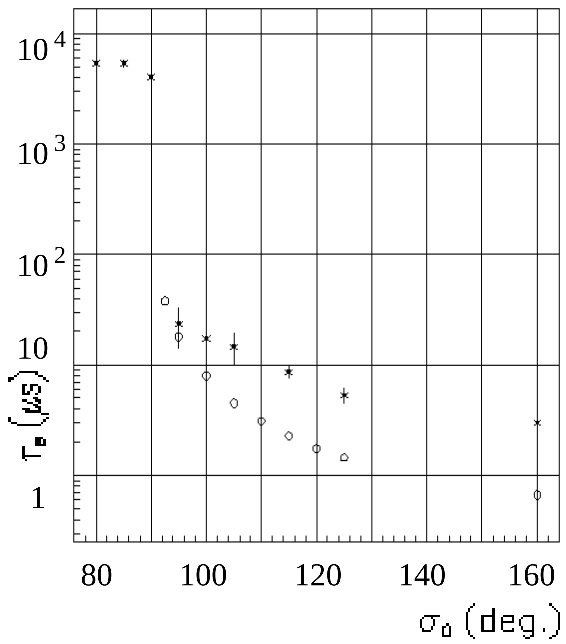

Figure 1: e-folding blow-up time τ_e in function of initial Phase advance σ_0. Square boxes are from PARMT, empty circles from K-V calculation.

As foreseen by the theory, there is a steep jump around 90^0. This interpretation has been checked with the help of the code KV [6]. This code, assuming a Kapchinskiy Vladimirskiy distribution, calculates for the given lattice the envelopes modes frequencies as function of the depressed tune σ. When the envelope modes hit a parametric resonance, the growth rate λ is calculated. As an example in Fig. 2 the growth rates corresponding to

$\sigma_0=120^0$ are plotted, with four resonances and a maximum rate at $\sigma\approx75^o$.

In multiparticle simulations we cool the beam from $\sigma=\sigma_0$ to $\sigma=0$ and all these resonances have to be crossed. Therefore we can compare the maximum growth rate of the instability with the minimum successful cooling rate, as shown in Fig.1.

For $\sigma_0<90^o$ envelope instability is avoided: therefore no K-V blow-up times are shown whereas PARMT results are related to other weaker heating mechanism.

For $\sigma_0>90^o$ K-V instabilities growth time are smaller then the PARMT ones, especially when the instability is stronger. Actually, K-V calculates the linear growth in the absence of any Landau Damping and growth rate mitigation due to the non linearity of the forces.

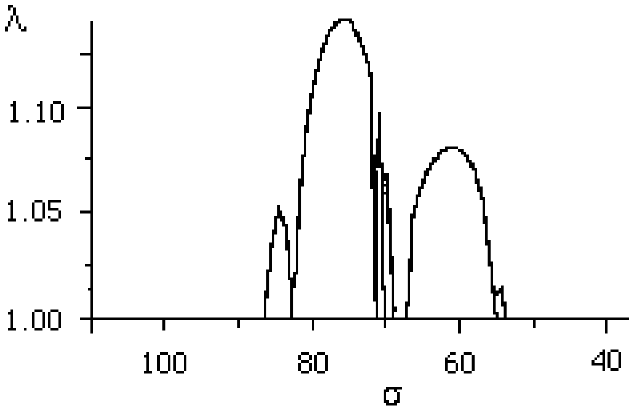

Figure 2 Growth rate λ in function of depressed phase advance σ. Initial phase advance is $\sigma_0=120^0$

3 PERIODICITY PERTURBATION

A ring with periodicity 8 and phase advances 83.3^0 and 53.3^0 per cell has been selected for CSR, avoiding the parametric resonances, except for machine errors. We consider the effect of a simple thin lens, representing the electron cooler. The electrons indeed have a focusing effect that can be considered as a thin lens of focal length:

$$\frac{1}{f} = \frac{2\pi Z n r_p l_c}{A\beta^2},$$

where l_c=1m is the length of the cooler, $n=10^8$ cm^{-3} is the electron density. For our Li beam f=36 m. The focusing strength of the cooler is almost doubled by the focusing of the solenoid [7].

The periodicity breaking due to this effect is overwhelming respect to gradient errors in the quadrupoles. The effective periodicity of the lattice is lowered to one, and instabilities related to parametric resonances can occur .

In Fig. 3 are shown the cooling curves (1/Γ vs number of turns) for different f, for a vertical zig-zag in CSR ring (cooling time τ=65μs) . The simulations are performed with the Coulomb routine of PARMT: IBS and ordered beam particle to particle interaction are therefore correctly calculated. The parameter f increases regularly by 10 m from one curve to the next one, while the Γ saturation zones are denser around three values, roughly 1, 100 and 2000. This effect, that suggests a resonant behavior, is relevant since the crystallization is prevented by our perturbation even with a very optimistic cooling.

The same calculation has been performed for a beam current corresponding to the simple 3D structure shown in fig. 4. The longitudinal temperature is defined as:

$$kT_z = mc^2(\gamma - 1)\left(\frac{\Delta p}{p} - \frac{\overline{p_z x}}{\overline{x^2}}\, x\right)^2$$

considering that in the ground state particles have equal revolution frequency and not linear velocity. The cooling curves calculated in this way (Fig. 5) have the same structure of those in Fig. 3, except for the Γ≈2000 zone that, probably due to the too strong cooling in the presence of a 3D structure [7], is not reached.

We have tried the following interpretation: the plateaux in the curves correspond to the equilibrium between cooling rate and growth rate (function of the order parameter Γ). In general, being the machine rather smooth, a Mathieu equation will be driven:

$$Y'' + \left(\nu^2 + 2G\cos m\vartheta\right)Y = 0$$

where the perturbation is the only θ dependent part, while $\nu=m/2+\varepsilon$, $G=R/2\pi f$, m integer, R ring average radius. The growth rate at resonance (ε=0) is G/m. At different beam temperature ν and Y can have different meanings:

- for Y single particle position ν is the (depressed) tune,
- for Y envelope dimension ν is the envelope tune,
- for Y crystal coherent mode amplitude ν is the tune.

By equating this growth rate with the cooling rate we get the threshold in terms of f:

$$f = \frac{1}{m}\frac{c\beta\tau}{\pi}.$$

For our parameters the fundamental mode is at f=400m; the agreement is rather poor since we could interpret the data with a fundamental f=70m and a second resonance f=35m. With the simple argument here presented we overestimate the growth rate by about a factor 6, in analogy wth what happened in the previous section. In other words in both cases the heating rate is mitigated respect to the instability (linear) growth rate by some mechanism, that can be different for space charge dominated, IBS dominated and crystallized beams. Investigations on this process of "thermalization" are in progress.

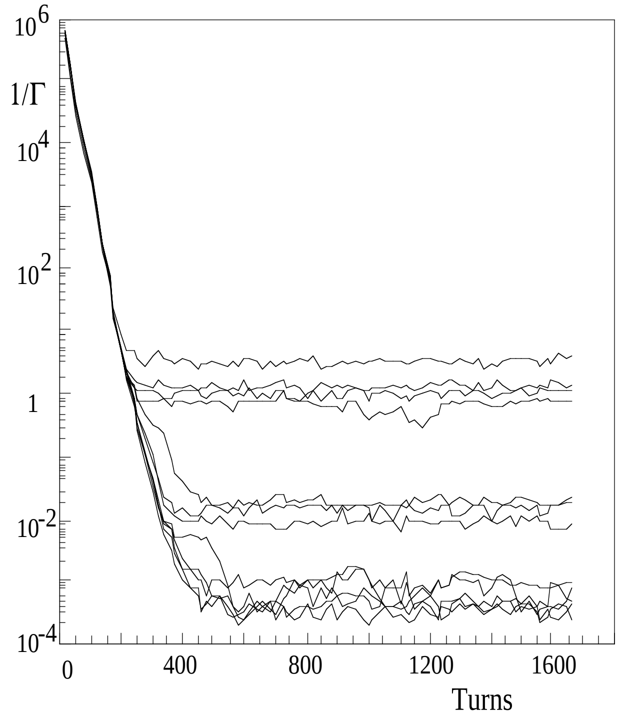

Figure 3 Cooling curves for a vertical zig-zag crystal and a thin lens perturbation. From the bottom to the top: form f=120m to 20m step 10 m.

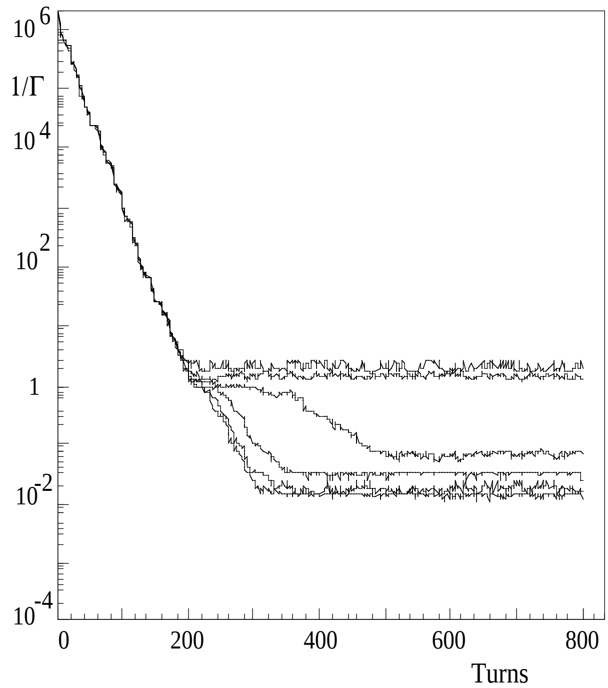

Figure 5 Cooling curves for a beam density corresponding to the structure in fig.4. From the bottom to the top: form f=120m to 20m step 20 m.

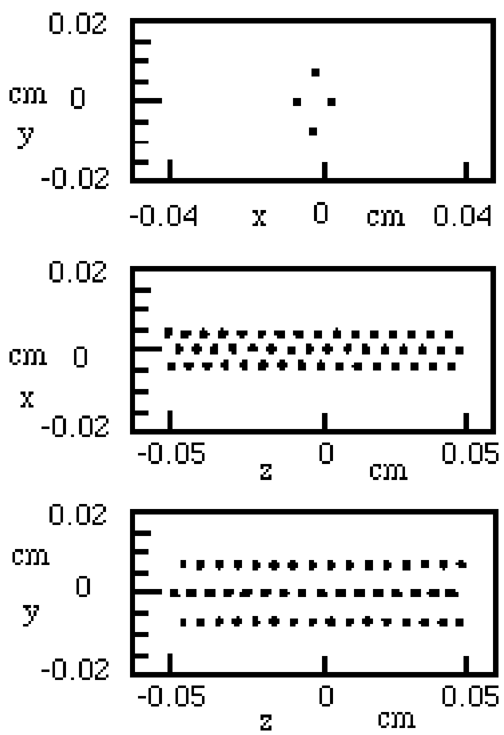

Figure 4 Ground state configuration for the particle density used in error studies.

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