BLACK-BOX PARAMETRIC ESTIMATION METHODS FOR LONGITUDINAL BEAM TRANSFER FUNCTION MEASUREMENTS

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Abstract

In the CERN PS Booster (PSB), longitudinal Beam Transfer Function (BTF) measurements are used to determine the energy spectrum of the linac beam injected and coasting on an extended flat bottom of the magnet cycle. The short duration (about 10 ms) of the measurement and coherent signals from density structure of the injected beam yielded strongly fluctuating results, and the system fell into disuse for a while.

BTF measurements can be degraded by different kinds of noise, which together with small excitations to avoid alteration of the beam, give a big variance of the results. Particularly in the PSB, this is seen as a base-line offset (BLO) of the energy distribution, whose variance is due to the noise and is not to be improved by longer data records. Different signal processing techniques are investigated to reduce the variance, where black-box parametric estimation methods are shown to have significantly lower BLO than other methods applied in the field.

1 INTRODUCTION

The energy spread of a coasting beam can be measured by examining the Schottky (energy) spectrum [1]. The PSB beam lacks this possibility due to residual structure from the linac and injection processes, causing much stronger coherent signals than the Schottky signal within the PSB cycle time 1.2 s [2]. The signal-to-noise ratio (SNR) is improved by using BTF measurements. The frequency distribution F_0, which is related to the energy distribution, is retrieved by integrating the real-part of the BTF

$$r_{\|}(\omega) = C\left(\mp\frac{\pi}{n}\frac{dF_0(\omega)}{d\omega_r} + j\mathrm{PV}\int \frac{\frac{dF_0(\Delta\omega_r)}{d\omega_r}}{\omega - n\omega_r}\, d\omega_r\right). \quad (1)$$

n is the harmonic of the revolution frequency ω_r [3].

Surrounding structures affect the beam, and can be modelled as a coupling impedance $Z_{\|}$ that has a non-linear effect on the BTF [4]. For low energy machines as the PSB, the coupling impedance mainly consists of a negative inductance due to space charge.

A BTF measurement always contains different kinds of noise that degrade the result. These can be thermal, quantisation and Schottky noise, but mainly consist of mentioned density structure of the PSB beam. The BTF estimate is a linear combination of measurement data, which can be regarded as a realisation of a stochastic process. Hence, an integration (summation in the discrete case) of the BTF is a random variable. The value of the integration far away from the main response we call the base-line offset (BLO):

$$\hat{F}_0(\infty) = \frac{n}{\pi C}\int_{-\infty}^{\infty} \mathrm{Re}\{\hat{r}_{\|}(\omega)\}\, d\omega \quad (2)$$

To decrease the effect of the noise and to get a smaller BLO variance, different signal processing methods have been investigated, both using simulated and measured data.

2 MEASUREMENT SET-UP

The measurements made at the PSB are carried out on a 50 MeV coasting proton beam which is kept stable for about 500 ms. The excitation signal is digitally produced which after up-mixing drives a cavity that has been detuned to avoid self-bunching of the beam. The cavity gap signal is returned to be used as a reference signal $u(t)$. The response of the beam $y(t)$ is recorded from a wall-current monitor. Both reference and response signals are quadrature down-mixed to save bandwidth (sampling rate) and thus are complex quantities. The buffers for saving sampled data are rather limited (1024 samples) compared to the time of stable beam: a bandwidth of 100 kHz, which is suitable for measurements around the sixth harmonic, gives a total measurement time of the order of 10 ms.

The model we use is a transfer function driven by an input signal and having noise $v(t)$ added to the response:

$$y(t) = r_{\|}(u(t)) + v(t). \quad (3)$$

3 SIGNAL PROCESSING METHODS

3.1 Non-parametric Methods

A common way of estimating a transfer function, is to divide the Discrete Fourier Transforms of the response and reference signals. This is called the Empirical Transfer Function Estimate (ETFE) in [5].

If the transfer function is a smooth function, the ETFE can be smoothed by weighted averaging with the variance of the estimate and a weighting function $W(\omega)$. The width of the weighting window controls the trade-off between variance and bias. For this smoothed ETFE we use a Hamming window in our measurements [6].

It is also possible to smooth the transfer function estimate by dividing the data record into R batches of length M and forming the averaged ETFE. We use $R = 4$ in the measurements.

Another smoothed estimate used for BTFs is the Time Gated Estimate (TGE). This transforms the ETFE into time domain and removes noise by gating the impulse response [4]. The TGE is the same as the smoothed ETFE when the estimate variance is constant and the same smoothing window is used. However, here we use a combination of Hamming and rectangular windows for the TGE [6].

3.2 Black-box Prediction Error Method (PEM)

Another approach to estimate the transfer function from digitised data is to assume the BTF to be a digital filter

$$G(q) = \frac{b_0 + b_1 q^{-1} + \cdots + b_{n_b} q^{-n_b}}{1 + a_1 q^{-1} + \cdots + a_{n_a} q^{-n_a}} = \frac{B(q)}{A(q)}. \quad (4)$$

q^{-1} is a time-delay operator such that $q^{-1}u(t) = u(t-1)$. Similarly, assuming a noise model to be a digital filter driven by a white noise process $e(t)$, we get a general model of the BTF measurement to be as

$$A(q)y(t) = \frac{B(q)}{F(q)}u(t) + \frac{C(q)}{D(q)}e(t). \quad (5)$$

There are usually much fewer parameters to be estimated than number of data, so there is a compression of data and variance decreases. The output can be predicted having a model (5). The model can then be optimised by minimising a scalar norm, which depends on the prediction errors [5].

A good model for BTF measurements is a Box-Jenkins model ($A(q) \equiv 1$ in equation (5)) with polynomial orders (n_b, n_c, n_d and n_f) of three.

3.3 Integration of Estimate

We saw earlier that the energy distribution is retrieved by integrating the real part of the BTF. For long records N, the integral in equation (2) can be approximated by the sum of sample values normalised with $2\pi/NT$.

All estimates are asymptotically uncorrelated at different frequencies [5]. The variance of the BLO is hence the sum of the variances of $\mathrm{Re}\{\hat{r}_N\}$ at each frequency. Further with the assumption that the SNR is constant for all frequencies, the BLO variance is inversely proportional to N and SNR:

$$\mathrm{Var}\,\hat{F}_N(\infty) \sim \frac{1}{2N}\frac{\Phi_v}{\Phi_u}. \quad (6)$$

Interestingly, the same assumptions for the averaged ETFE gives the same BLO variance. This is due to the fact that the gain in variance at each frequency, is lost in the number of frequency points.

When the TGE uses a normalised time gating window ($w_\kappa(0) = 1$), it can be shown that the BLO takes exactly the same value as when using the ETFE, due to circular convolution [6]. Hence, the BLO variance is also the same. Note however, that at lower frequencies, the variance of the distribution $\hat{F}_0(\omega)$ is smaller, when using the averaged ETFE or the TGE.

The smoothed ETFE is closely related to the TGE, so we expect them to have similar results. Computer simulations have also shown that the BLO variance decays with N. The same result was also found with simulations for the PEM estimate. Hence, equation (6) is valid for the smoothed ETFE and the PEM estimates too.

4 SIMULATIONS

Simulations have been carried out to see how well the different signal processing methods agree with a known BTF. The simulated BTF is disturbed with additive thermal (white) noise and Schottky noise [4]. The latter can also be seen as the structured noise that is present in the BTF measurements at the PSB, but with a larger magnitude.

The simulation uses corresponding physical data as for the measurement set-up in section 2, and signal processing parameters as in section 3. The reference signal $u(t)$ from an actual measurement at the PSB is used as input to the simulation model.

Regarding the stability diagram, the PEM is the far best model. The ETFE and the averaged ETFE are not suitable for such usage, since they are difficult to evaluate. However, the smoothed ETFE and the TGE give some information, see figure 1.

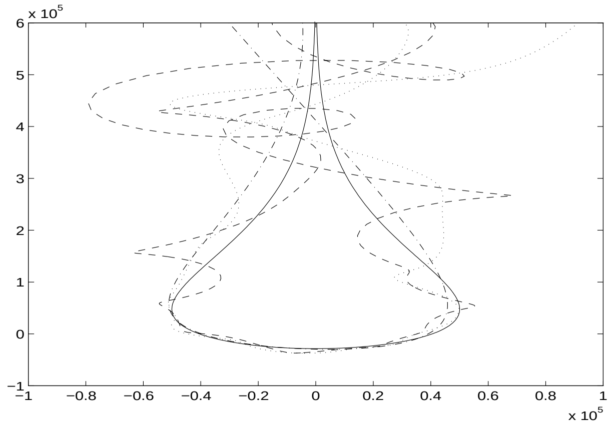

Figure 1: Stability diagrams. Simulated beam with normally distributed energy spread: Smoothed ETFE (dotted), TGE (dashed), PEM (dash-dotted) and noise-free BTF (solid).

All smoothing methods improve the ETFE to different extents, and at a fairly good SNR they all give good results concerning the integrated real part. However, the PEM gives a significantly smaller BLO, when the (simulated) measurement gives big deviations of base-line for the other investigated methods. The averaged ETFE can also increase the BLO, see figure 2.

For a less good SNR, the optimisation of the PEM does not reach the same agreement with the simulated BTF. However, the PEM still give better results than the other methods.

When the beam is subject to a strong coupling impedance, the stability diagram is (usually) moved closer

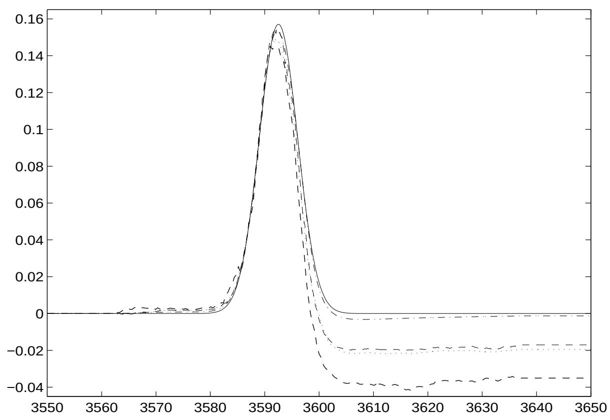

Figure 2: Integrated real parts of simulated BTF from beam with normally distributed energy spread: ETFE (dashed), averaged ETFE (thick dashed), smoothed ETFE (dotted), PEM (dash-dotted) and noise-free BTF (solid). (TGE is not plotted, but practically coincides with ETFE.)

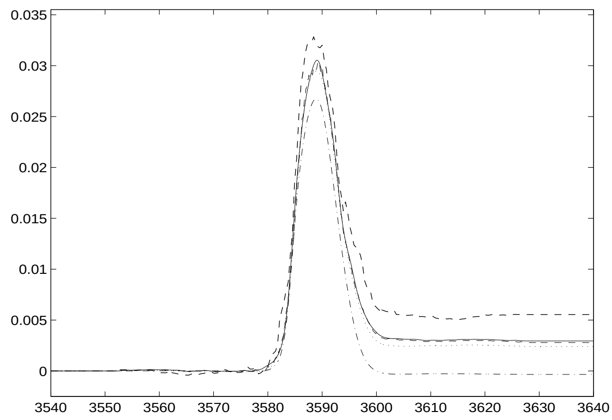

Figure 3: Integrated real parts of measured BTF: ETFE (dashed), averaged ETFE (thick dashed), smoothed ETFE (dotted), TGE (solid) and PEM (dash-dotted).

to the origin, whereby some samples may change sign. The stability diagram thus still has the same 'goodness', but when the corrected BTF is integrated, there are samples that give an erroneous contribution to the distribution. The PEM again gives the best result.

Finally, a beam with a quadratic distribution was simulated to see how well the methods apply to distributions with discontinuities. The PEM has bigger problems to apply to such distributions. Yet, the integration fits fairly well with the simulated distribution.

5 MEASUREMENTS ON BEAM

The initial noise structure of the beam decays exponentially with time. Hence, a good choice of time for the measurement reduces the noise level, since a change of the energy distribution cannot be observed during this noise degradation. A low intensity beam ($\leq 2 \cdot 10^{12}$ particles) also avoids density structures developing about 150 ms after injection.

If the beam is excited for a longer time, the energy distribution may be affected. This alteration is avoided with smaller excitation levels. Measurements have shown that a reduction of excitation power by 5 - 6 dB approximately doubles the time to beam alteration. Hence, the BLO variance does not decrease by extending measurement time, since gain in variance because of increased N (see section 3), is lost due to the necessarily lowered excitation. Figure 3 shows the distribution from a BTF measurement in the PSB with $\sim 7.58 \cdot 10^{11}$ particles and a well adjusted excitation level.

6 CONCLUSIONS

We have studied BTF measurements contaminated with different kinds of additive noise. Digital signal processing methods have been investigated to reduce their effect, where parametric estimation methods yielded substantially improved results, which has been demonstrated in BTF simulations, with a Box-Jenkins model of order 3 for all polynomials.

The integration of the real part of the BTF, representing the energy distribution of the particle beam, is a random variable, whose asymptotic value $\hat{F}_0(\infty)$ we have called the base-line offset (BLO). We have investigated its variance for the different BTF estimation methods, and found that it decreases with SNR and record length N.

BTF measurements have shown that the excitation signal must be carefully adjusted not to alter the beam, yet having the best SNR. The structures in the beam distribution at the PSB decays exponentially and reaches low levels after 100 ms, whereby the energy distribution has not changed. Beam intensities are kept low to stay below thresholds of coherent instabilities. Extending measurement duration has no improving effect on the BLO variance, as the excitation level has to be reduced in order to not alter the distribution. For more details, see [6].

7 REFERENCES

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