Transverse Feedback System with a Digital Filter and Additional Delay

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Abstract

Theory of resistive wall instability damping using a feedback system with a digital filter and delay is developed. A system of equations is obtained for description of beam motion. To solve equations the Z-transform method is used. The general solutions are analysed for feedback circuit with a digital filter and delay. The damping time is found for the feedback with an additional one revolution period delay and for the feedback with a single beam correction per two revolutions.

1 INTRODUCTION

Transverse feedback systems (TFS) are used in synchrotrons to damp the coherent transverse beam oscillations [1]. In these systems the kicker (DK) corrects the beam angle according to the beam deviation from the closed orbit in the pick-up (PU). A classical TFS consists of one PU and one DK per plane. A schematic diagram of this system is shown in Figure 1. The DK changes the

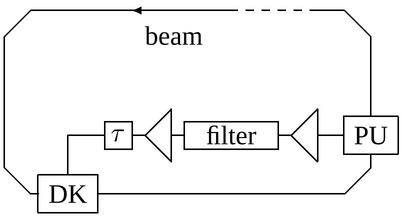

Figure 1: Schematic diagram of the TFS

angle of the same fraction of the beam that was measured by PU. The delay τ is adjusted to provide such a synchronization. The value of this delay is normally less than one turn and the kicker corrects the beam angle at every turn. But sometimes this value may be more than one revolution period. Such situation can occur for a synchrotron with a short length of circumference or for a feedback circuit with a special digital filter where procedures for signal transformations are realized with signal processors. This paper is based on studies of TFS for feedback circuit with a more than one revolution period delay.

2 THEORY

The description is based on the theory of multi-bunch resistive wall instability damping where a Z–transform method is used to obtain a general solution [2], [3]. This approach was effectively used to solve the problem of the beam dynamics in an accelerator with a digital feedback.

2.1 General Approach

Taking into account the results obtained in [2] the study of the transverse coherent motion bunch dynamic is started for independent bunches. In this case the bunch coupling, which occurs due to resistive wall instability, is neglected and the matrix method becomes suitable for the beam motion description.

Let the column matrix $\widehat{X}[n,s]$ determine the bunch state at the n-th turn at point s of the circumference C_0. The first element of this matrix equals the beam deviation $x[n,s]$ from the closed orbit and the second one is $x'[n,s]$. After a short DK the x' value of the beam is changed by $\Delta x'[n,s_K]$, while deviation remains the same as before the DK at point s_K^-. Hence, after DK at point s_K^+, the beam state is

$$\widehat{X}[n,s_K^+] = \widehat{X}[n,s_K^-] + \widehat{T}\Delta\widehat{X}[n,s_K],$$

where $\widehat{T}$ is the 2×2 matrix in which $T_{21}=1$ and the other elements are zero. The kick is determined with column matrix $\Delta\widehat{X}[n,s_K]$, where the first element equals $\Delta x'[n,s_K]$ and the second one has an arbitrary value.

Let us introduce the unperturbed revolution matrix $\widehat{M_0}$ from point s_P of the PU location to point s_P+C_0 and the transfer matrix $\widehat{M_1}$ from point s_K of the DK location to point s_P+C_0:

$$\begin{aligned}\widehat{M_0} &= \widehat{M}(s_P+C_0,s_P),\\ \widehat{M_1} &= \widehat{M}(s_P+C_0,s_K).\end{aligned} \qquad (1)$$

Then at the PU location at the $(n+1)$-th turn the beam state is

$$\widehat{X}[n+1,s_P] = \widehat{M_0}\widehat{X}[n,s_P] + \widehat{M_1}\widehat{T}\Delta\widehat{X}[n,s_K]. \qquad (2)$$

Let $\Delta x'[n,s_K]$ be proportional to the output voltage in the feedback circuit during n-th crossing of the kicker. The input voltage is assumed to be proportional to the beam deviation $x[m,s_P]$ in the pick-up. The kicker should change the angle of the same fraction of the beam that was measured by the PU. The delay $\tau=qT_0+\tau_l$ is adjusted to provide such a synchronization (q is integer, T_0 is the revolution period, τ_l is the time of the particle flight between PU and DK). For the studied problem the value of q is greater than zero and the kick at the n-th turn depends on the beam state at the previous turns ($m=n-q$).

2.2 Every Turn Correction

If the kicker corrects the beam angle at every turn, then:

$$\Delta\widehat{X}[n,s_K] = u[n-q]\frac{\mathbf{K}}{\sqrt{\beta_P\beta_K}}\widehat{X}[n-q,s_P], \qquad (3)$$

where β_P and β_K are the transverse betatron amplitude functions in the PU and DK locations, $\mathbf{K}$ is the gain of the feedback, and $u[n]$ is the discrete unit step function [4].

Substituting $\Delta\widehat{X}[n,s_K]$ from Eq.(3) in Eq.(2) we get

$$\begin{aligned}\widehat{X}[n+1,s_P] &= \widehat{M}_0\widehat{X}[n,s_P] + \\ &+u[n-q]\frac{\mathbf{K}}{\sqrt{\beta_P\beta_K}}\widehat{M}_1\widehat{T}\widehat{X}[n-q,s_P]. \qquad (4)\end{aligned}$$

Eq.(4) fully describes the beam dynamics in an accelerator with a feedback system considered. This equation is solved using Z-transform [4] for $\widehat{X}[n,s]$:

$$\begin{aligned}\widehat{\mathbf{X}}(z) &= \sum_{n=0}^{\infty}\widehat{X}[n,s]z^{-n}; \qquad (5)\\ \widehat{X}[n,s] &= \sum_k \mathrm{Rez}\left[\widehat{\mathbf{X}}(z_k)z_k^{n-1}\right].\end{aligned}$$

The motion of the particles will be stable if $|z_k| < 1$. The beam motion parameters are fully determined by the singular points z_k : the number of oscillations per turn $\{\mathrm{Re}Q_k\}$ equals $\arg(z_k)/2\pi$ and the damping time τ_D is

$$\frac{T_0}{\tau_D} = -\ln|z_k|. \qquad (6)$$

Using Z-transform for Eq.(4) we get

$$\widehat{\mathbf{X}}(z) = \frac{z\widehat{I}-\widehat{\mathbf{M}}^{-1}(z)\det\widehat{\mathbf{M}}(z)}{\det\left(z\widehat{I}-\widehat{\mathbf{M}}(z)\right)}z\widehat{X}[0,s_P], \qquad (7)$$

$$\widehat{\mathbf{M}}(z) = \widehat{M}_0 + \frac{z^{-q}\widetilde{\mathbf{K}}(z)}{\sqrt{\beta_P\beta_K}}\widehat{M}_1\widehat{T}, \qquad (8)$$

where $\widehat{I}$ is the unit matrix; $\widehat{X}[0,s_P]$ is the initial beam state matrix; $\widetilde{\mathbf{K}}(z)$ is the transfer function for a feedback circuit. It is known [4] that in radiotechnical sense the circuit is stable if all the singular points of $\widetilde{\mathbf{K}}(z)$ lie inside the circle $|z| < 1$. If this condition is fulfilled, the singular points z_k in (7) are found from the equation [3]:

$$\begin{aligned}&\det\left(z_k\widehat{I}-\widehat{\mathbf{M}}(z_k)\right) = \\ &= z_k^2 - z_k\mathrm{Tr}\widehat{\mathbf{M}}(z_k) + \det\widehat{\mathbf{M}}(z_k) = 0. \qquad (9)\end{aligned}$$

When instability occurs, Eq.(9) will have the same form but the betatron phase advances for elements of $\widehat{\mathbf{M}}(z)$ must be calculated with a complex value of $Q(z)$ both for coasting [5] and bunched [2] beams.

If the feedback circuit of a damper system has a digital filter then the transfer function $\widetilde{\mathbf{K}}(z)$ must include the filter system function. For example, $\widetilde{\mathbf{K}}(z)$ with an IIR–filter (see Fig.2) is

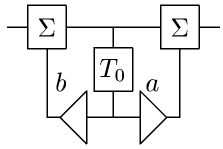

Figure 2: Schematic diagram of an IIR–filter

$$\widetilde{\mathbf{K}}(z) = |\mathbf{K}|\frac{z+a}{z-b}, \qquad (10)$$

where $|\mathbf{K}|$ is the gain factor of feedback circuit without any filter.

2.3 Single Correction per Two Turns

A feedback system with single correction per two turns can be of two types:

(a) DK corrects the beam angle at the same turn when PU measures beam deviation, and there are no corrections at the next turn;

(b) DK corrects the beam angle at the next turn after PU measuring of beam deviation.

The relation between two states for $\widehat{X}[m,s]$ is constructed due to the two turns periodicity of these corrections. In accordance with Eqs.(4, 3), the beam state $\widehat{X}[m+1,s]$ (after two turns) is

$$\begin{aligned}(a):\quad \widehat{X}[m+1,s_P] &= \widehat{M}_0\Big(\widehat{M}_0 + \\ &+\frac{\mathbf{K}}{\sqrt{\beta_P\beta_K}}\widehat{M}_1\widehat{T}\Big)\widehat{X}[m,s_P]; \qquad (11)\\ (b):\quad \widehat{X}[m+1,s_P] &= \Big(\widehat{M}_0^2 + \\ &+\frac{\mathbf{K}}{\sqrt{\beta_P\beta_K}}\widehat{M}_1\widehat{T}\Big)\widehat{X}[m,s_P]. \qquad (12)\end{aligned}$$

Z–transformations of Eqs.(11, 12) yield similar equations for $\widehat{\mathbf{X}}(z)$ as Eq.(7). The matrices for determining singular points z_k in Eq.(9) are

$$(a):\quad \widehat{\mathbf{M}}(z) = \widehat{M}_0^2 + \frac{\widetilde{\mathbf{K}}(z)}{\sqrt{\beta_P\beta_K}}\widehat{M}_0\widehat{M}_1\widehat{T}, \qquad (13)$$

$$(b):\quad \widehat{\mathbf{M}}(z) = \widehat{M}_0^2 + \frac{\widetilde{\mathbf{K}}(z)}{\sqrt{\beta_P\beta_K}}\widehat{M}_1\widehat{T}, \qquad (14)$$

where $\widetilde{\mathbf{K}}(z)$ has been determined in (10).

3 RESULTS

3.1 Every Turn Correction

For an every turn correction with an IIR–filter and an additional delay in the feedback path the equation (9) for z_k

with $\widehat{\mathbf{M}}(z)$ from Eq.(8) and $\widetilde{\mathbf{K}}(z)$ from Eq.(10) is

$$z^2 - \left(2\cos(2\pi Q) + \frac{|\mathbf{K}|}{z}\frac{z+a}{z-b}\sin(2\pi Q-\psi)\right) z + 1 - \frac{|\mathbf{K}|}{z}\frac{z+a}{z-b}\sin\psi = 0, \qquad (15)$$

where Q is the number of unperturbed betatron oscillations per revolution in transverse plane, and ψ is the betatron phase advance from PU to DK. Eq.(15) is the forth power equation, whose four roots determine the beam stability diagram. If $|\mathbf{K}| \ll 1$, then in linear approximation we obtain:

$$\begin{aligned} z_{1,2} &= \left(1 \mp i\frac{|\mathbf{K}|}{2}\exp\left(\mp i(2\pi Q+\psi)\right)\right)\exp(\pm i2\pi Q) \\ &\quad \pm i\frac{a+b}{2}|\mathbf{K}|\frac{b-\exp(\mp i2\pi Q)}{1-2b\cos(2\pi Q)+b^2}\exp(\mp i\psi)\ ; \\ z_3 &= b + |\mathbf{K}|\left(1+\frac{a}{b}\right)\frac{b\sin(2\pi Q-\psi)+\sin\psi}{1-2b\cos(2\pi Q)+b^2}\ ; \\ z_4 &= |\mathbf{K}|\frac{a}{b}\sin\psi\ . \end{aligned} \qquad (16)$$

Roots 1 and 2 correspond to the eigen frequencies with the number of oscillations per turn in the neighbourhood of ReQ. Roots 3 and 4 correspond to two new modes that are conditioned with the IIR–filter structure and the one turn additional delay. The conclusions [5] made for the filter parameters are valid here too. Thus, to provide the independence on $|\mathbf{K}|$ of the feedback action on the closed orbit displacement and for the best suppression of the revolution harmonics it is necessary to set $a = -1$. The other filter parameter b is chosen due to optimization on the maximum damping rate and the width of the stability region.

The best damping will be for the other PU and DK locations than without one turn delay. Indeed, the damping time τ_D without any filter ($a = b = 0$) is due to Eqs.(15, 6)

$$\frac{T_0}{\tau_D} = \frac{1}{2}|\mathbf{K}|\sin(2\pi Q+\psi) - 2\pi|\mathrm{Im}Q|. \qquad (17)$$

Hence, the best damping will be for the PU and DK locations such that

$$|\sin(2\pi Q+\psi)| = 1, \qquad (18)$$

i.e. if the sum of the phase advance ψ from PU to DK and the phase advance $2\pi Q$ for the turn is equal to an odd number of $\pi/2$ radians. This statement is quite clear because the PU sample and the DK correction are proceeded with one turn delay. It is necessary to emphasize that the damping rate value for optimal PU and DK locations (18) is the same as without additional delay.

3.2 Single Correction per Two Turns

All further results are shown for a feedback without a filter in order to simplify the final equations. Eq.(9) for z_k with $\widehat{\mathbf{M}}(z)$ from (13) and (14) is

$$\begin{aligned} (a):&\quad z^2 - (2\cos(4\pi Q) + |\mathbf{K}|\sin(4\pi Q-\psi))\,z + \\ &\quad +1 - |\mathbf{K}|\sin\psi = 0, \\ (b):&\quad z^2 - (2\cos(4\pi Q) + |\mathbf{K}|\sin(2\pi Q-\psi))\,z + \\ &\quad +1 - |\mathbf{K}|\sin(2\pi Q+\psi) = 0. \end{aligned}$$

Hence, the damping time

$$\frac{T_0}{\tau_d} = -\frac{1}{2}\ln|z_k|$$

in linear approximation ($|\mathbf{K}| \ll 1$) is

$$\begin{aligned} (a):&\quad \frac{T_0}{\tau_d} = \frac{1}{4}|\mathbf{K}|\sin\psi - 2\pi|\mathrm{Im}Q|; \\ (b):&\quad \frac{T_0}{\tau_d} = \frac{1}{4}|\mathbf{K}|\sin(2\pi Q+\psi) - 2\pi|\mathrm{Im}Q|. \end{aligned}$$

Therefore, the best damping will be for those PU and DK locations that take into account the phase advance from the PU sample to DK correction including the one turn delay. It is important to emphasize that the damping rate values for the feedback with the single correction per two turns are two times slower than the values for a feedback with every turn correction.

4 CONCLUSION

The consideration of damping regimes for a damper system with an additional one turn delay in the feedback path allows one to maintain that every turn correction is preferable. The damping rate value for this feedback with every turn correction is the same as for a damper system without additional delay.

5 REFERENCES

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