IMPACT OF FINAL-FOCUS GROUND MOTION ON NLC LUMINOSITY *

J. Irwin and F. Zimmermann, SLAC, Stanford, CA 94309, USA

Abstract

Vertical displacements of final-focus quadrupoles due to ground motion can cause the two beams of the Next Linear Collider (NLC) to miss each other at the interaction point (IP) and, in addition, will increase the IP spot size, and thus degrade the luminosity, by generating dispersion and skew coupling. The sensitivity of the final-focus optics to plane ground waves is strongly wavelength dependent, which is formally expressed in terms of a lattice-response function. In this paper, the rms beam-beam separation and the rms IP spot-size increase are estimated for the NLC final focus, using the measured ground-motion power spectrum, a realistic orbit-feedback response curve, and the appropriate lattice-response function. The luminosity loss due to ground motion is shown to be insignificant.

1 INTRODUCTION

Over the past few years, concerns were raised that quadrupole displacements due to ground motion may seriously reduce the luminosity of a linear collider [1]. This would be the case when the two colliding beams are steered off collision faster than orbit-feedback systems are able to correct the IP beam position. The design of the Next Linear Collider (NLC) [2] calls for a vertical IP spot size σ_y of 3–6 nm. The relative luminosity loss $\Delta L/L$ due to an rms beam-beam separation of size $\Delta \equiv y_{\rm right} - y_{\rm left}$ ($y_{\rm left}$ and $y_{\rm right}$ denote the respective vertical IP positions of the two beams) is approximately given by $\Delta L/L \approx \exp(-\Delta^2/(16\ \sigma_y^2))$; here we have taken into account the effect of disruption [3], i.e., the strong mutual attraction of the two beams during collision. According to simulations, the disruption reduces the sensitivity to vertical beam-beam offsets at least by a factor of 2, compared with that expected for rigid bunches. As an example, at the NLC an rms separation Δ of 1 nm would cause an average luminosity loss of 0.2–0.7%.

In this paper, we calculate the rms beam-beam separation due to ground motion and the resulting luminosity loss for the NLC. We will show that the measured strong correlation between ground-motion frequency and wavelength conspires with the insensitivity of the final-focus optics to long-wavelength perturbations so as to render the luminosity loss due to ground motion almost insignificant. Based on measurements in the SLAC linac [4], we can also derive an upper bound on the effect of any additional uncorrelated component of ground motion and we will demonstrate it to be similarly small. Finally, quadrupole displacements due to ground motion do not only steer the beams off collision, but can also increase the IP spot-size by generating dispersion and skew coupling. The tolerances on magnet displacements which are imposed by the spot-size increase are two or three orders of magnitude looser than those required to maintain collisions, and, therefore, if at all, this aspect of ground motion only becomes important on a longer time scale, *e.g.*, after minutes or hours. It will be addressed briefly towards the end of this report.

* Work supported by the Department of Energy, contract DE-AC03-76SF00515

2 GENERAL FORMALISM

If the vertical betatron phase advance from the entrance point e to the IP is a multiple of π, the offset of the two beams at the IP Δ due to an arbitrary vertical displacement $y(s)$ of the final-focus quadrupoles is

$$\Delta = -\sum_{i({\rm right})} k_i R_{34}^i y(s_{i,{\rm r}}) + R_{33}^e y(s_{e,{\rm r}}) + \sum_{i({\rm left})} k_i R_{34}^i y(s_{i,{\rm l}}) - R_{33}^e y(s_{e,{\rm l}}) \equiv \sum_j \mu_j y(s_j) \quad (1)$$

where R_{34}^i denotes the (3,4) R-matrix element from the ith magnet to the IP, R_{33}^e is the (3,3) R-matrix element from the entrance e of the final focus to the IP, and k_i the integrated strength of quadrupole i in units of m^{-1}. For simplicity, in the last line we have replaced the $\pm k_i R_{34}$ and $\pm R_{33}$ by dimensionless lattice parameters μ_j. Note that the subindex i counts elements on one side from the IP only, while the subindex j sums over both sides. If we square the sum in Eq. (1) we will find mixed expressions of the form $y(s_l)y(s_n) \equiv y(s_l)y(s_l + \Delta s_{nl})$, whose expectation value over position s and over time t is given by

$$< y(s_l)y(s_n) >_{s,t} = \lim_{S,T\to\infty} \frac{1}{ST} \int_{-\frac{S}{2}}^{\frac{S}{2}} \int_{-\frac{T}{2}}^{\frac{T}{2}} y^*(s_l,t) y(s_l + \Delta s_{nl}, t)\, ds\, dt = \int_0^\infty \frac{d\omega}{2\pi} \int_0^\infty \frac{dk}{2\pi} P(\omega,k) \cos(k \Delta s_{nl}) \quad (2)$$

where $\Delta s_{nl} \equiv s_n - s_l$, and the asterisk denotes the complex conjugate. The term $P(\omega, k)$ is the two-dimensional power spectrum (in terms of frequency and wave number) of the ground motion [5]. Inserting Eq. (2) into the square of Eq. (1), and including the frequency response $F(\omega)$ of an orbit feedback system, the rms beam-beam separation at the IP can generally be written as

$$< \Delta^2 >_{s,t} = \int_0^\infty \frac{dk}{2\pi} \int_0^\infty \frac{d\omega}{2\pi} P(\omega,k) G(k) F(\omega). \quad (3)$$

3 LATTICE RESPONSE

The function $G(k)$ in Eq. (3) is called the lattice response function. It equals the squared ratio of the IP beam-beam separation and the amplitude of a driving plane ground wave, and it can be expressed in terms of the lattice parameters μ_i and positions s_i as

$$G(k) = 4\left(\sum_i \mu_i \sin ks_i\right)^2, \quad (4)$$

where, again, the subindex i only sums over elements on one side from the IP. The lattice response function for the NLC final focus is shown in Fig. 1. For large wave numbers k (above 1 m^{-1}, which corresponds to a ground-wave frequency of about 75 Hz), there is no correlation between the motion of different quadrupoles and the response function $G(k)$ is about constant, equal to ten. Thus, for large wave numbers, the resulting beam separation at the IP is a factor of $\sqrt{10}$ larger than the amplitude of the driving ground wave. This asymptotic value is almost entirely determined by the last two quadrupoles in front of the IP (the final doublet), and it is independent of the rest of the lattice. The function $G(k)$ is calculated as if all quadrupoles move like the ground beneath their center, and the oscillations of $G(k)$ arise from the discrete distances between the centers of different magnets.

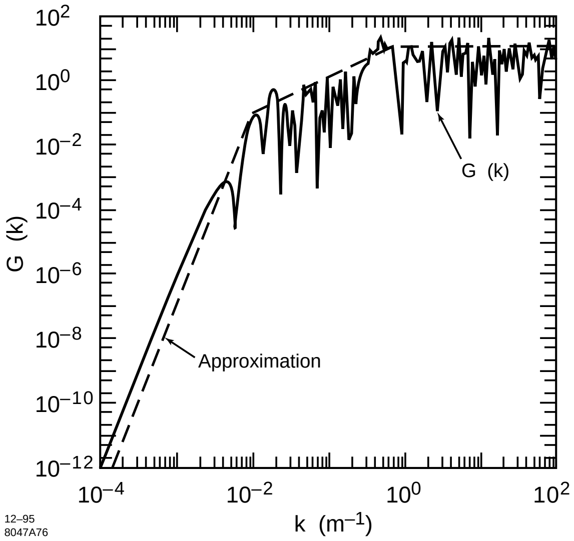

Figure 1: Lattice response function of the NLC final-focus system.

In contrast, for small wave numbers k, i.e., for $k < 0.01$ m^{-1} (or frequencies below 2 Hz), the function $G(k)$ increases as the sixth power of k. There are three reasons for this: a) a displacement of the entire final focus maintains the IP collision; b) similarly, a constant $y-s$ tilt also does not affect the beam-beam separation at the IP. Conditions a) and b) are equivalent to the following two sum rules:

$$-\sum_i k_i R_{34}^{i\to IP} + R_{33}^{e\to IP} \quad = \quad 1 \quad (5)$$

$$-\sum_i k_i R_{34}^{i\to IP} s_i + R_{33}^{e\to IP} s_e \quad = \quad 0. \quad (6)$$

These two equations imply at least a k^4 behavior at small k. In addition and c), the final-focus system consists of several paired optical $-I$-modules, whose steering effects exactly cancel each other for a quadratic (y versus s) perturbation. Consequently, the response at small k is expected to increase at least like k^6, as seen in Fig. 1. Since the ground-motion power spectrum is roughly proportional to $1/k^4$, the effect of plane-wave ground motion for small k (*i.e.,* below 2 Hz) is strongly suppressed. Finally, note that, in Eq. (6), the betatron phase advance from the final-focus entrance e to the IP was assumed to be a multiple of π, but this assumption is not essential.

4 GROUND MOTION

The 2-dimensional ground-motion power spectrum $P(\omega, k)$ in Eq. (3) is obtained from the equation

$$P(\omega,k) = 4P(\omega) \times \int_0^\infty (1 - R(\omega, L))\cos(kL)dL \quad (7)$$

where

$$P(\omega) \equiv \int_0^\infty \frac{dk}{2\pi} P(\omega, k) \quad (8)$$

represents the 1-dimensional power spectrum, and $R(\omega, \Delta s)$ describes the frequency-dependent correlation of ground motion between two locations a distance L apart. The function $R(\omega, L)$ has been measured in the SLAC linac tunnel [4] (Ref. [6] reports similar results from the LEP tunnel). It is well parametrized by the expression $R(\omega, L) \approx 1 - J_0(k(\omega)L)$ where J_0 denotes the zeroth order Bessel function, $k(\omega) \equiv \omega/v(\omega)$ with $v(\omega)$ [m s^{-1}] $\approx 450 + 1900\exp(-\omega/(4\pi))$ [4] and SI units are used throughout. The quantity $v(\omega)$ can be interpreted as the velocity of ground waves at frequency $f = \omega/(2\pi)$. Inserting the expression for $R(\omega, L)$ into Eq. (7) and performing the integration over distance L yields

$$P(\omega,k) = \begin{cases} \frac{4}{\sqrt{k(\omega)^2 - k^2}} P(\omega) & \text{if} \quad k(\omega) > k \\ 0 & \text{else} \end{cases} \quad (9)$$

Intriguingly, exactly this functional dependence is expected when the ground motion is composed of isotropic plane surface waves. According to measurements at various places [5], a reasonable approximation to $P(\omega)$ for a 'quiet' site (LEP tunnel, caves in Finland...) is $P(\omega)$[μm^2/Hz] $\approx 16 \times 10^{-3}/\omega^4$, where the angular frequency ω is given in units of s^{-1}, and only frequencies $\omega > 0$ are considered. Equation (9) allows to convert the lattice response function into frequency domain with the result

$$\tilde{G}(\omega) \equiv \int_0^{k(\omega)} \frac{dk}{2\pi} \frac{4G(k)}{\sqrt{k(\omega)^2 - k^2}}, \quad (10)$$

which may be used to rewrite the rms beam-beam separation, Eq. (3), as a single integral over frequency:

$$<\Delta^2>_{s,t} = \int_0^\infty \frac{d\omega}{2\pi} \tilde{G}(\omega) P(\omega) F(\omega), \quad (11)$$

Typical curves for the three functions P, $\tilde{G}$, and F are depicted in Fig. 2. The power density $P(f)$ shown approximates a 'quiet site' [5]; it decreases as $1/f^4$. The lattice response function $\tilde{G}(f)$ is obtained from the NLC final-focus response $G(k)$ of Fig. 1 using Eq. (10). As expected, it strongly suppresses the effect of low-frequency ground motion. The curve $F(f)$ represents a typical orbit-feedback response measured in the SLAC linac [7].

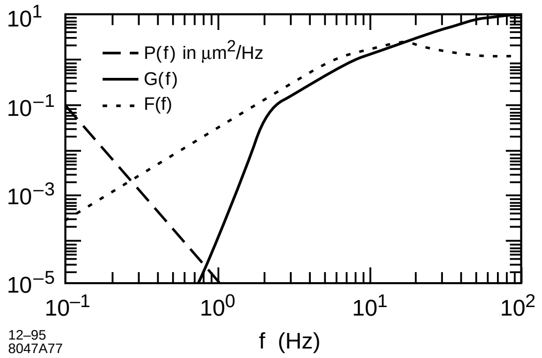

Figure 2: Three functions which determine the rms beam-beam separation due to plane-wave ground motion:
$F(f)$ — feedback response for the SLAC linac [7]; $P(f)$ — local power density; $\tilde{G}(f)$ — lattice response of the NLC final focus.
The integral over the product of these functions gives the square of the rms beam-beam separation.

Numerical integration of Eq. (11) using $P(\omega)$ for a quiet site yields an rms beam-beam separation of $0.2(0.3)$ nm with (without) orbit feedback, corresponding to a luminosity loss of about 0.02%. Above 6 Hz, the power spectrum measured in the SLAC linac tunnel is considerably higher than that of a quiet site, due to resonances of the linac-structure supports and due to cultural noise. For the actual SLAC spectrum, one calculates an rms beam-beam separation of 1.1–1.3 nm, or a luminosity loss of about 0.8%, which is still small.

Some authors have argued that at low frequencies a component of ground motion exists which cannot be cast into the above framework, and which is of diffusive character [8, 5]. They characterize this part of the ground motion by a so-called ATL law, according to which the change of the mean square relative displacement of two points is proportional to the distance between the points and to time. (An entirely different interpretation of ground motion as a systematic process was suggested in Ref. [9].) The two-dimensional spectral density describing the ATL law can be written $P_{ATL}(\omega,k) = 4A/(\omega^2 k^2)$ where $k > 0$, $\omega > 0$ is assumed, and $A \approx 10^{-8}$–10^{-5} μm^2 s^{-1} m^{-1} is an empirical constant, which depends on location and on time scale. SLAC measurements over periods of seconds and hours indicate $A < 6 \times 10^{-7}$ μm^2 s^{-1} m^{-1}. For the frequency range 0–0.01 Hz, where the ATL law, perhaps, might be applicable, the rms beam-beam separation Δ due to ATL-like ground motion can be calculated by numerical integration of (3) after inserting the expression for $P_{ATL}(\omega,k)$. For $A = 10^{-6}$ μm^2 m^{-1} s^{-1}, Δ is 15 pm. Assuming, as a worst case, that at frequencies above 0.01 Hz the size of uncorrelated motion is equal to the noise floor of the employed seismometers, the rms separation in the frequency range 0.01–6 Hz is estimated to be no larger than 242 pm. For a quiet site, the contribution from frequencies above 6 Hz is less than an additional 124 pm. In total then, the rms separation caused by uncorrelated or ATL-like ground motion does not exceed 0.3 nm, and the luminosity loss resulting from such motion is less than 0.05%.

5 SPOT SIZE

In complete analogy to Eq. (4), one can also introduce lattice-response functions describing the spot-size increase due to dispersion or skew coupling caused by magnet displacements. These functions are of the form

$$G_\delta(k) = \sum_{i,j} \mu_i^\delta \mu_j^\delta \cos(k\Delta s_{ij}) \qquad (12)$$

where the subindices i, j run over one side of the IP only, and the multipliers $\mu_{i,j}^\delta$ characterize the IP spot-size increase due to a displacement of quadrupole (or sextupole) i. For large k, the function $G_\delta(k)$ approaches an asymptotic value of about 10^{-3}, while for small k it increases as k^4. Similar considerations as in the previous section then show that plane ground waves do not sensibly affect the IP spot size. The effect of an ATL-like ground motion on the IP spot size can be determined by numerically integrating the product of $G_\delta(k)$, $F(\omega)$ and $P_{ATL}(\omega,k)$ over k and ω. For $A = 10^{-6}$ μm^2 s^{-1}, the result is an rms spot-size increase of less than 1 pm, to be added in quadrature.

6 CONCLUSIONS

Natural ground motion in the NLC is found to be an asset rather than a nuisance. The expected luminosity loss due to ground motion is less than 1% even on a not-so-quiet site, *e.g.*, in the SLAC-linac tunnel. Therefore, the ground (bedrock) can serve as a reference for stabilization. The NLC design luminosity will be achieved when magnet supports neither amplify nor damp the ground motion, but couple the magnets firmly to the ground beneath them.

7 REFERENCES

[1] Proc. of LC95, Tsukuba, *KEK*-**95-5** (1995).

[2] Zeroth Order Design Report for the Next Linear Collider, SLAC, unpublished draft (1996).

[3] P. Chen and K. Yokoya, CERN-US PAS 1990, 415 (1990).

[4] C. Adolphsen, private communication (1995).

[5] V.M. Juravlev, et al., 'Seismic Conditions in Finland and Stability Requirements for the Future Linear Collider' (1995).

[6] V.M. Juravlev, et al., CERN-SL/93-53 (1993).

[7] L. Hendrickson, private communication (1995).

[8] B. Baklakov, et al., INP Preprint 91-15 (1991).

[9] R. Pitthan, invited talk at 4th Int. Workshop on Acc. Alignment, Tsukuba, 1995, and SLAC-PUB-95-7043 (1995).