# DARK CURRENT SIMULATION IN HIGH GRADIENT ACCELERATING STRUCTURE 

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## Abstract

The dark current generated in high gradient disk-loaded accelerating structures is a major concern for the high energy linear colliders. In order to study whether it is really a problem, the dynamics of the field-emitted electrons was simulated numerically. In the simulation, electrons start from the wall of the accelerating structure with the initial velocity of zero. Their motion was calculated by integrating the equation of motion with Runge-Kutta method until they hit the wall or get out of the accelerating structure. For subsequent tracking, the SAD program was used, where accelerating structures and Q magnets were taken into account. The field-emitted electrons are expected to be largely deflected by the Q magnets and strike the vacuum chamber since their energy is much lower than the nominal one. In the case of $3 / 4 \pi$ accelerating structure for JLC C-band with the accelerating gradient of $50 \mathrm{MV} / \mathrm{m}$, the result showed that most of the electrons were unable to pass through the first eighteen Q magnets and that the number of remaining electrons was well below the acceptable level.

## 1 INTRODUCTION

The field emission in the accelerating structure is a major concern for proposed linear colliders aiming at the energy range above 1 TeV . A naive consideration on the capture condition of an electron at rest gives the threshold gradient[1]

$$
E_{\mathrm{th}}=\frac{\pi m c^{2}}{e \lambda}
$$

where $m$ is the mass of the electron and $\lambda$ is the wavelength in the accelerating structure. In order to achieve a high accelerating gradient with no associated dark current, higher RF frequency is preferable since the threshold gradient is inversely proportional to $\lambda$. The kinetic energy of the dark current, however, is very low compared with the nominal one. These low energy electrons would be largely deflected at the Q magnets unless they travel very close to its axis. Although there have been several numerical studies on this problem[2], they are limited to the motion of the electrons within one accelerating structure.

In this paper, the behaviour of field-emitted electrons is simulated for C -band constant impedance accelerating
structure with numerical integration of the equation of motion and SAD tracking code[3].

## 2 SIMULATION METHOD

### 2.1 In the First Accelerating Structure

The trajectory of the electrons in the accelerating structure was calculated by integrating the equation of motion. Since the accelerating structure considered is axisymmetric, the motion of the electrons is confined in the two dimensional plane, i.e., r-z plane. The space charge effect is not included in this calculation.

The accelerating structure is disk-loaded with constant impedance which is approximated by a series of simple pillbox cavities. The electromagnetic field in a unit cell was calculated with MAFIA eigenmode solver assuming that the phase advance per cell is $3 / 4 \pi$. The parameters of the accelerating structure is summarised in Table 1. The field in the whole structure is obtained by simply shifting the phase of this field properly for each cell. The equation of motion was integrated using Runge-Kutta method. The trajectory integration is terminated when the electron hits the cavity wall.

### 2.2 Subsequent Tracking by SAD

Once the electrons exit the accelerating structure, the simulation is continued using SAD tracking code. The configuration of the simulated accelerator line is shown in Fig. 1 together with the plot of $\sqrt{\beta}$, which is supposed to be the entrance of the main linac. The deflection of the electrons would be smaller there than at other downstream location. This is a series of normal FODO cells with two accelerating structures between Q magnets. Total number of the Q magnets is 18 . In the accelerating structure, only a uniform longitudinal field is present. The nominal

Table 1 Parameters of the simulated accelerating structure

| frequency | $f$ | 5.712 | GHz |
| :--- | :---: | ---: | :--- |
| accelerating gradient | $E_{\mathrm{ac}}$ | 50 | $\mathrm{MV} / \mathrm{m}$ |
| phase shift per cell |  | $3 \pi / 4$ |  |
| unit cell length |  | 19.68 | mm |
| aperture | $2 a$ | 12 | mm |
| cell diameter | $2 b$ | 41.4 | mm |
| disk thickness | $t$ | 4 | mm |
| total cell number |  | 90 |  |



Fig. 1 Accelerator line configuration used in the SAD tracking. The rectangles represent accelerating structures.
energy is 20 GeV at the left end. Electrons are considered to be lost when their radial coordinate exceeds 6 mm . An electron which starts exactly on the axis will remain on the axis since no alignment error of Q magnets is introduced in the tracking.

## 3 RESULT OF TRACKING

### 3.1 In the First Accelerating Structure

The length of the accelerating structure is 1.8 m . The electrons start at the surface of the wall of the first cell in the accelerating structure at the velocity of zero. Each time they cross an iris of the accelerator cell, their position $r$ and slope $r^{\prime}$ are recorded. This record corresponds to particles which start in the cells other than the first cell and reach the end of the accelerating structure.

Most of the electrons hit the cavity wall and only a small fraction is accelerated through the structure. An example of the trajectory is shown in Fig. 2. Their final $r$ and $r^{\prime}$ are small compared with other initial conditions. At first the particle goes near the axis of the accelerating structure and then accelerated along the axis.

There were a few trajectories which are very close to the axis. The final $r$ and $r^{\prime}$ in Fig. 3 are smaller than


Fig. 2 An example of the trajectory of an electron. The upper plot shows the first 10 cells and the lower shows the whole structure.


Fig. 3 An example of trajectories which indicates chaotic behaviour. The initial RF phase is changed by $0.1^{\circ}$ step in the lower plot.
those in Fig. 2. The particle "think twice" on the axis. The lower plot of Fig. 3 shows the neighbouring trajectories, changing the initial RF phase by $0.1^{\circ}$ step. They indicate a chaotic behaviour.

Fig. 4 is the plot of the final $r$ and $r^{\prime}$ with the initial position and RF phase changed by $50 \mu \mathrm{~m}$ and $2^{\circ}$ step, respectively. The plot is not symmetric about the $r$ axis since the electrons start only from positive $r$. A group of points which forms a smooth curve corresponds to the electrons sharing the same initial position in the starting cell and initial RF phase but with different starting cell. The electrons which come out with the energy larger than $90 \%$ of the full acceleration voltage ( 81 MV ), are plotted in Fig. 5. In this plot, the steps of the initial position and RF phase are reduced to $10 \mu \mathrm{~m}$ and $1^{\circ}$, respectively. The position $r$ and slope $r^{\prime}$ is strongly correlated. From this set of boundary conditions, all the electrons which come


Fig. 4 The distribution in the final phase space with initial position and RF phase changed by $50 \mu \mathrm{~m}$ and $2^{\circ}$ step, respectively.


Fig. 5 The distribution in the final phase space with initial position and RF phase changed by $10 \mu \mathrm{~m}$ and $1^{\circ}$ step, respectively. Only electrons with the final energy above 81 MeV are plotted.
out of the structure are passed to the subsequent SAD tracking.

### 3.2 Subsequent Tracking by SAD

The electrons which come out of the accelerating structure was used as the initial condition in the SAD tracking. Their total number was about $3.5 \times 10^{5}$. All of them exceeded the 6 mm aperture before the end of the tracking. However, this result do not necessarily leads to the conclusion that all the field-emitted electrons hit the chamber wall. The chaotic behaviour seen in Fig. 3 indicate that it is difficult to determine what initial condition gives the minimum distance from the origin in the $r-r^{\prime}$ plane from simulation.

In order to estimate the probability of survival, acceptance of the accelerator line in Fig. 1 was calculated. The $r-r^{\prime}$ plane were scanned with the initial energy of 90 MeV and the initial RF phase at the maximum acceleration. The dots in Fig. 6 are in the acceptance region. Although only the small area around the origin is shown here, the aperture is a thin strip approximately expressed as $r^{\prime}=-0.58 \times r$. The slope of this strip is negative because electrons must go near the centre of the first Q magnet, which is 2.2 m away from the starting point, otherwise they would largely deflected. This is also the case if the first Q magnet is focusing, then the strip is $r^{\prime}=-0.70 \times r$. The width of the strip in $r^{\prime}$ is $1.7 \times 10^{-18}$.

## 4 DISCUSSION

The probability $P_{\mathrm{s}}$ that the electrons survive the eighteen Q magnets can be estimated from the result of the simulation. First, let us assume that the thin acceptance strip in Fig. 6 extends out to the whole range of $r$. Then $P_{\mathrm{s}}$ is the probability that the points in Fig. 4 fall in the strip. For a rough upper limit estimation, let


Fig. 6 Aperture of the simulated accelerator line. Only a small area around the origin is shown.
us assume that all the points in Fig. 4 are concentrated on the strip of width $1 \times 10^{-4}$ which contain the acceptance strip. Then the upper limit on $P_{\mathrm{s}}$ is

$$
P_{s}<\frac{1.7 \times 10^{-18}}{1 \times 10^{-4}} \cong 2 \times 10^{-14}
$$

On the other hand, the total number $n_{\mathrm{td}}$ of field-emitted electrons within an RF pulse of length $t_{\mathrm{p}}$ is

$$
n_{\mathrm{td}}=\frac{I_{\text {peak }}}{e} t_{\mathrm{p}} n_{\mathrm{ac}}
$$

where $I_{\text {peak }}$ and $n_{\text {ac }}$ are the peak current of field-emitted electrons for one accelerating structure and the number of accelerating structures, respectively. Then the upper limit on the total number of field-emitted electrons arriving at IP for one RF pulse is

$$
n_{\mathrm{d}}<n_{\mathrm{td}} P_{\mathrm{s}}<1250\left(\frac{I_{\text {peak }}}{1 \mathrm{~A}}\right)\left(\frac{t_{\mathrm{p}}}{1 \mu \mathrm{~s}}\right)\left(\frac{n_{\mathrm{ac}}}{10000}\right) .
$$

When the assumed parameters in the above equation is moderate ones, field-emitted electrons will not be a problem.

For the next step, the inclusion of the secondary electron emission and elastic scattering will be necessary for more realistic simulation.

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