COHERENT STATES IN THE LONGITUDINAL DYNAMICS OF ELECTRON BEAMS IN PARTICLE ACCELERATORS

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Abstract

The possibility to generate and control coherent states for a charged particle bunch in a circular accelerating machine in the presence of radiation damping and quantum excitation is investigated in the framework of the Thermal Wave Model. It is proven that in correspondence of arbitrary time-variations of a radio-frequency (RF) strength, a Schrödinger-like equation predicts the final equilibrium state, toward which the system spontaneously goes, where a sort of *mesoscopical coherence* is reached. This state corresponds to an example of the Glauber-Klauder-Sudarshan (GKS) coherent states widely considered in quantum optics and optical fibers. In addition, it is shown that for suitable time-varying RF strength, coherent states are possible in each time (*locally-controlled coherence*) during the evolution toward the above asymptotic equilibrium, and the possibility that allow for carrying these analytical predictions in particle accelerators out is discussed.

1 INTRODUCTION

It has been recently pointed out [1, 2] that the longitudinal dynamics of an electron (positron) bunch in a linearized (harmonic-like) radio frequency (RF) potential well in the presence of radiation damping (RD) and quantum excitation (QE) is governed by a Schrödinger-like equation for a complex function, the beam wave function (BWF), whose squared modulus is proportional to the longitudinal density profile [1, 2]. In this *quantum-like* equation, Planck's constant is replaced with an arbitrary function of time. It has been shown that the physical meaning of this function is the *longitudinal beam emittance* and its time-variation accounts for both the dissipation due to RD and the quantum fluctuation given by QE [3]-[5]. This way, the above equation dynamically describes the bunch as a non-conservative system, and in the asymptotic limit it becomes a Schrödinger-like equation where the emittance is stabilized on its asymptotic minimum value (emittance damping). It has been shown that this model correctly recovers the results of the conventional description of particle accelerators. In addition, it has been observed that the above final equilibrium state of the system, toward which the system spontaneously goes, represents a sort of *macroscopical coherence*, where the *quantum-like uncertainty relation* is minimized [2]. It is very easy to see that, in the quantum-like language of the TWM, this state corresponds to an example of the Glauber-Klauder-Sudarshan (GKS) coherent states [6] widely considered in quantum optics [7], optical fibers [8], and stochastic mechanics [9]. Recently, the existence and the methods for generating these states has been also investigated for the transverse dynamics of charged particle beam in the accelerating machines in the TWM framework [10]. The basic hypothesis used in Ref.s [2] and [10] to generate GKS coherent states in particle accelerators was to consider constant the strength K of the linearized longitudinal or transverse potential well.

In this paper, we want to investigate on the possibility to generate and control coherent states for a charged particle bunch in a circular accelerating machine in the presence of RD and QE when the above strength is a time-dependent function. This is done on the basis of both the previous TWM approach given in [2] and the stochastic-mechanical approach given in [11]. The following steps are in order. First, we formulate our problem by specifying the assumptions taken into account and giving the basic equations of our problem. Then, we present the analytical results of the formulated problem, and show that, in correspondence of suitable time-variations of K, coherent states are analytically possible in each time (*locally-controlled coherence*) during the evolution toward the asymptotic equilibrium. In the quantum-like framework this final state plays the role of the ground state (the simplest GKS coherent state). Subsequently, we discuss on the possibility that allow us for carrying these analytical predictions out . Finally, some remarks and the conclusions are presented.

2 FORMULATION OF THE PROBLEM

We deal here with a relativistic electron (positron) bunch with nominal energy E_0 (synchronous particle energy) travelling in a circular accelerating machine of radius R. Let us consider that the bunch, during its periodic motion of nominal angular frequency ω_0, goes through a RF cavity where feels the RF-electric field. On the timescale larger than the revolution period $T_0 \equiv 2\pi/\omega_0$, we can describe the synchrotron oscillations in terms of the displacements x that an arbitrary particle executes in the bunch with respect to the synchronous particle. For small-amplitude oscillations in the configurational x-space, we can assume that the bunch is in a harmonic-like oscillator potential well of the form $(K/2)x^2$. Consequently, by taking also into account both RD and QE, the single-particle model assumed here consists in a damped harmonic oscillator in the presence of an effective stochastic force which accounts for the effect of QE. We point out that the emittance describes a stochastic effect but related to the temperature of the system, as well as plays the role analogous to a diffraction parameter, and in the framework of TWM this parameter is involved in the quantization rules to give a wave equation (Schrödinger-like equation). In fact, it has been pointed out that in TWM this longitudinal dynamics of a relativistic bunch with longitudinal velocity βc ($\beta \approx 1$) is governed by the following Schrödinger-like equation [2]

$$i\eta\widetilde{\epsilon}(s)\frac{\partial}{\partial s}\Psi(x,s) = -\frac{\eta^2\widetilde{\epsilon}^2(s)}{2}\frac{\partial^2}{\partial x^2}\Psi(x,s)+\frac{K}{2}x^2\Psi(x,s)\,, \quad (1)$$

where $s \equiv ct$ (t being the time), $\eta \equiv \gamma_0^{-2} - \gamma_T^{-2}$ (γ_0 and γ_T being the relativistic Lorentz factor $(1-\beta^2)^{-1/2}$ and the so-called *transition energy* which depends on the properties of the machine guide field; $1/\eta$ plays the role of an *effective mass* associated with the system), and $\widetilde{\epsilon}(s)$ is an arbitrary function of s, to be specified in correspondence of the particular stochastic effects considered, but satisfying the initial condition $\widetilde{\epsilon}(0) = \epsilon$. In particular, in [2] the time-varying function $\widetilde{\epsilon}(s)$ has been interpreted as the longitudinal emittance. This way the above Schrödinger-like equation where the Planck's constant is replaced with the function $\widetilde{\epsilon}(s)$ correctly describes the dissipative system under consideration (electron or positron bunch in the presence of RD and QE). In [2] the RF strength K was considered constant. Under this hypothesis, the appropriate *r.m.s. emittance* scaling law, due to the damping effect, has been naturally recovered, and the asymptotic equilibrium condition for the bunch length, due to the competition between QE and RD, has been found. We want now to consider the same problem but with the RF strength depending on s. Then, by solving for a complete set of solutions of the Schrödinger-like equation we look for the suitable conditions that select, from this set, the only solutions describing, also for $K(s)$, coherent states associated with the electron (positron) bunch dynamics.

3 SOLUTION

A complete set of normalized solutions of (1) with $K = K(s)$ is easily constructed from the one found for $K = constant$ given in Ref. [2]; it is very easy to see that now we have the following normalized solutions

$$\Psi_m(x,s) = \frac{\Psi_0(x,s)}{\sqrt{2^m n!}}H_m\left(\frac{x-x_0(s)}{\sqrt{2}\sigma(s)}\right)e^{\{i2m\phi(s)\}}\,, \quad (2)$$

with

$$\Psi_0(x,s) = \frac{\exp\left\{\Gamma(s)\left(x-x_0(s)\right)^2 + \frac{ip_0(s)}{\eta\widetilde{\epsilon}(s)}x + i\delta_0(s)\right\}}{\left\{2\pi\sigma^2(s)\right\}^{1/4}}\,, \quad (3)$$

where we have, more generally, introduced the shift $x_0(s)$ in the synchrotron coordinate and $\Gamma(s) \equiv -1/(4\sigma^2(s)) + i/(2\eta\widetilde{\epsilon}(s)\rho(s))$. Eq. (2) with (3) are solutions of (1) provided that the functions $x_0(s)$, $p_0(s)$, $\sigma(s)$, $\rho(s)$, $\delta_0(s)$, and $\phi(s)$ satisfy the following system of coupled differential equations

$$\frac{d^2x_0}{ds^2} - \left(\frac{1}{\widetilde{\epsilon}}\frac{d\widetilde{\epsilon}}{ds}\right)\frac{d\,x_0}{ds} + K(s)x_0 = 0\ , \quad (4)$$

$$p_0 = \frac{dx_0}{ds} = 0\ ,\ \ \frac{1}{\rho} = \frac{1}{\sigma}\frac{d\sigma}{ds}\ ,\ \ \frac{d\phi}{ds} = -\frac{\eta\widetilde{\epsilon}(s)}{4\sigma^2}\ , \quad (5)$$

$$\frac{d^2\sigma}{ds^2} - \left(\frac{1}{\widetilde{\epsilon}}\frac{d\widetilde{\epsilon}}{ds}\right)\frac{d\,\sigma}{ds} + K(s)\sigma - \frac{\eta^2\widetilde{\epsilon}^2(s)}{4\,\sigma^3} = 0\ , \quad (6)$$

$$\frac{d\delta_0}{ds} = -\frac{\eta\widetilde{\epsilon}(s)}{4\sigma^2} - \frac{1}{\eta\widetilde{\epsilon}(s)}\left[\frac{p_0^2}{2} - \frac{1}{2}K(s)x_0^2\right]\ , \quad (7)$$

Moreover, the longitudinal momentum spread can be also introduced by means of the following quantum-like definition:

$$\sigma_p^2(s) \equiv \langle(p-p_0(s))^2\rangle \equiv \widetilde{\epsilon}^2(s)\int_{-\infty}^{\infty}\left|\frac{\partial\Psi}{\partial x}\right|^2\,dx - p_0^2(s)\,. \quad (8)$$

Consequently, the following uncertainty relation holds:

$$\sigma_p^2(s)\sigma^2(s) = \frac{\widetilde{\epsilon}^2(s)}{4} + \frac{\sigma^2(s)}{\eta^2}\left(\frac{d\sigma}{ds}\right)^2 \geq \frac{\widetilde{\epsilon}^2(s)}{4}\ , \quad (9)$$

which shows that at each $\overline{s}$, namely at each time, the quantity $\widetilde{\epsilon}(\overline{s})/2$ could represent the minimum of the uncertainty product $\sigma_p(\overline{s})\sigma(\overline{s})$ that can be reached for $s = \overline{s}$ provided that, at the same time, the condition $(d\sigma/ds)_{s=\overline{s}} = 0$ is satisfied. This would be, of course, a *relative minimum uncertainty* because, due to the RD and QE, the emittance decreases asymptotically toward the limit $\epsilon_D = 55|\eta|r_e\gamma_0^5\lambda_e/(24\sqrt{3}\nu_s\gamma\rho_0^2)$ [2], where r_e is the classical electron radius, ρ_0 is the machine mean bending radius, ν_s is the synchrotron number, γ is the damping rate, and λ_e is the electron Compton wavelength. This means that $\epsilon_D/2$ corresponds to the absolute minimum uncertainty whose value, estimated for some present accelerating machines is typically $10^6 - 10^9$ times the Compton wavelength for electrons $\lambda_e \equiv \hbar/m_ec$. In other words, since the beam, by relaxing, goes to the absolute minimum uncertainty, $\epsilon_D/2$, the corresponding reachable steady state is an example of GKS coherent states [6]. It is worth noting that we deal here with a *mesoscopical property* of coherence. In fact, we do not analyze the microscopical details of the system, but describe it by a *coarse-grained* quantum-like model which retains the essentials of the physics (i. e., effective possibility of controlled coherence); this mesoscopical level of description is also testified by the numerical values of emittance, which are very large with respect to microscopical length scales, but valuably smaller than the macroscopical ones. Consequently, the physical meaning of the uncertainty relation (9) is completely different from the Heisenberg uncertainty relation because: (i) the time variation of $\tilde{\epsilon}$ has no correspondence with $\hbar$ which is only a (fundamental) constant; (ii) Eq. (9) describes the connection between σ and σ_p for a macroscopical system which obeys to the classical mechanics.

Nevertheless, the quantum-like uncertainty relation (9) would be naturally obtained in mesoscopic descriptions, such as the one used in stochastic mechanics, since in the Nelson-like description one can naturally introduce a time-varying diffusion coefficient $\nu \equiv \hbar/2m_e$ even if $\hbar$ is kept constant, because in principle some time-variation of the mass would be possible [11].

4 LOCALLY-CONTROLLED COHERENCE

Since (2) and (3) represent a complete set of orthonormal solutions of (1), in particular, we can look for a solution which at each s keeps the uncertainty at the relative minimum value. In fact, from (9) this minimum is reached for $d\sigma/ds = 0$, which implies that, by virtue of (6), $\sigma^4 = \tilde{\epsilon}^2(s)/4K(s)$. Consequently, if $\sigma = \sigma_0 = const$, $K(s)$ must vary as $\tilde{\epsilon}^2(s)$. In principle, this is possible by measuring the emittance reduction toward the asymptotic limit ϵ_D and to use this information in *real time* to suitably vary $K(s)$ according to $\tilde{\epsilon}^2(s)$.

Note that this possible way to fix the bunch length to a given value σ_0 and to have the relative minimum uncertainty is an example of the potential controlling method [12] for constructing, by acting externally, coherent states. This means that, starting from an instant $\overline{s}$ for which the equilibrium condition is satisfied, we have $\sigma(\overline{s}) = \sigma_0$. For a given K, after this time, $\tilde{\epsilon}^2(s)$ would change and consequently σ would change too. But, by adjusting K according to the measured $\tilde{\epsilon}^2(s)$, in principle we can satisfy the equilibrium condition instant by instant in such a way to recover $\sigma(s) = \sigma_0$ and reach and control the coherent state. Since s is a curvilinear coordinate, the correspondence between a controlled coherent state and each value $\overline{s}$ fixes a correspondence between this kind of coherent state and the position of the bunch center along the bunch orbit. For this reason, we can call these states *locally-controlled coherent states*, whose corresponding BWF is, at each $\overline{s}$, as one can expect, the fundamental mode of the set (2) and (3). Note that, according to (4), for a given bunch length σ_0, the mean value x_0 (coordinate of the bunch center in the co-moving frame) of the particle distribution associated with locally-controlled coherent states is a solution of the following classical motion equation:

$$\frac{d^2 x_0}{ds^2} - \left(\frac{1}{\tilde{\epsilon}}\frac{d\tilde{\epsilon}}{ds}\right)\frac{d\,x_0}{ds} + \frac{\eta^2\tilde{\epsilon}^2(s)}{4{\sigma_0}^4}x_0 = 0 \quad . \qquad (10)$$

5 CONCLUSIONS

The possibility to produce and control, in an accelerating machine, coherent states presented above in the framework of TWM can meet a suitable feasibility in a real accelerator. In fact, in the present accelerating machines, the techniques for beam controlling and beam monitoring are very well developed and widely used [13, 14]. In particular, feedback systems are used in order to control in real time the evolution of charged particle bunches [13]. In addition, methods for measuring the instantaneous emittance are also widely applied [14]. Consequently, there would be no obstacle *in principle* to conceive, just using the present technology, a network device which measures the longitudinal emittance with a suitable scansion, and changes in real time the strength K of the RF-cavity involved in the longitudinal bunch dynamics, according to the equilibrium condition for $\sigma = \sigma_0$. In a forthcoming paper, we will develop more deeply this idea in order to produce a technical design of the above feedback device.

6 REFERENCES

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