Microbunch Emittance Growth Due to Radiative Interaction [*]

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Abstract

In this article we study effects of cooperative electromagnetic radiation on transverse dynamics of short high-charge bunch in a bend.

1 INTRODUCTION

Longitudinal effects of microbunch cooperative radiative field were considered in Refs. [1, 2], and an **overtaking** tail-head interaction was analyzed. This interaction induces energy spread along the bunch which follows a curved trajectory. Apparently, this effect is accompanied by excitation of the effective transverse horizontal(radial) emittance due to dispersion in bend [1].

This paper is devoted to transverse microbunch dynamics under influence of two other cooperative radiation effects – centripetal force and collective focusing forces. All these forces grow when the bunch length decreases.

2 BASIC EQUATIONS

Let us derive particle dynamics equations for a relativistic ($\beta = v/c \approx 1$) bunch that follows curved trajectory with radius R. If a particle at the tail of the bunch radiates the electromagnetic fields, then the radiation propagates along the chord and "catches" another particle after *overtaking distance* [1] of $L_o = |AB| = \theta R = 2(3sR^2)^{1/3}$ (s is distance between particles).

Under influence of the radiation forces the particle motion nearby an equilibrium trajectory (for a given energy $\mathcal{E}$ and given momentum) can be described by the following equations:

$$x'' + (K^2 - n)x = K\frac{\Delta\mathcal{E}}{\mathcal{E}} + F_x/\mathcal{E} \qquad (1)$$

$$y'' + ny = \frac{F_y}{\mathcal{E}}, \quad \mathcal{E}' = e\vec{E}\vec{v}/v_z, \quad t' = (1 + Kx)/v_z.$$

Here $(') \equiv d/dz$, $K(z) = 1/R$ is the equilibrium orbit curvature, $n(z)$ is the external focusing quadrupole field index. We neglected terms of $y'\mathcal{E}'$, $x'\mathcal{E}'$ because no essential parametric damping is assumed.

The components of the Lorenz force $\vec{F} = e(\vec{E} + \vec{\beta} \times \vec{B})$ can be calculated via the electromagnetic potential $(A_o, A_x, A_y = 0, A_z)$ as

$$F_x = -\frac{\partial V_o}{\partial x} - e\frac{dA_x}{cdt} + e\frac{KA_z}{1 + Kx}, \qquad (2)$$

$$F_y = -\frac{\partial V_o}{\partial y}, \quad e\vec{E}\vec{\beta} = \frac{\partial}{\partial t}V_o - e\frac{dA_o}{dt}, \qquad (3)$$

where the interaction Hamiltonian is $V_o = e(A_o - \beta A_z)$.

To work out the perturbation of x and y motion under the effect of the microbunch field, we transform Eq.(1) into equations for complex amplitudes C_x and C_y accordingly to a standard form of unperturbed motion $y = C_y^* f_y + c.c., x = \Psi\frac{\Delta\mathcal{E}}{\mathcal{E}} + (C_x^* f_x + c.c.)$, here f_x , f_y are the complex solution (i.e. $f = u_1 + iu_2$, where u_1, u_2 are two independent solutions) of Eqs.(1) with zero right-hand part and with normalization $f'_{x,y}f^*_{x,y} - c.c. = 2i$. These Floquet functions relate to beta-functions as $|f_{x,y}|^2 = \beta_{x,y}$ and "Courant-Snyder invariant" ϵ relates to $C_{x,y}$ as $\epsilon_{x,y} = 2|C_{x,y}|^2$. Ψ is solution of (1) with right-hand part equal to K.

Now one can find the complex amplitudes (integrals of unperturbed motion) as functions of coordinates, velocities and energy:

$$2iC_y = yf'_y - y'f_y, \; 2iC_x = xf'_x - x'f_x + \eta\frac{\Delta\mathcal{E}}{\mathcal{E}}. \quad (4)$$

Taking into account equations of perturbed motion we obtain the time derivatives

$$2iC'_y = -f_y(F_y/\mathcal{E}) \qquad (5)$$

$$2iC'_x = -f_x(F_x/\mathcal{E}) + \eta\mathcal{E}'/\mathcal{E}. \qquad (6)$$

where $\eta(z) \equiv \int_{-\infty}^{z} f_x K dz$.

For displaced amplitudes $\hat{C}_x = C_x + e\frac{\eta A_o}{2i\mathcal{E}} - e\frac{f_x A_x}{2i\mathcal{E}}$, $\hat{C}_y = C_y$ and energy $\hat{\mathcal{E}} = \mathcal{E} + eA_o$ one can derive the final equations:

$$2i\mathcal{E}\hat{C}'_x = \left(\frac{\eta}{c}\frac{\partial}{\partial t} + f_x\frac{\partial}{\partial x}\right)V_o - ef'_x A_x, \qquad (7)$$

$$\hat{\mathcal{E}}' = \frac{1}{c}\frac{\partial}{\partial t}V_o(1 + Kx), \quad 2i\mathcal{E}\hat{C}'_y = f_y\frac{\partial}{\partial y}V_o. \qquad (8)$$

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3 TRANSVERSE MICROBUNCH RADIATIVE EFFECTS

To calculate the effective transverse forces and perturbation of the amplitudes, we assumed that the bunch length σ_s satisfies conditions of a "thin" bunch and absence of beam pipe shielding, [1], i.e. $\sigma_\perp\sqrt{\sigma_\perp/R} \ll \sigma_s \ll b\sqrt{b/R}$, where $\sigma_\perp$ is the transverse bunch size and b is the beam pipe size.

The electromagnetic potentials are given by the integrals as follows:

$$A_o = \int \frac{d^3 r_1}{c\tau}\rho(\vec{r_1}, t-\tau), \quad \tau \equiv \left|\vec{r} - \vec{r_1}\right|/c \tag{9}$$

$$\vec{A} = \int \frac{d^3 r_1 \vec{\beta_1}}{c\tau}\rho(\vec{r_1}, t-\tau), \tag{10}$$

here $\rho(\vec{r}, t)$ is the space charge density. The retarding distance $c\tau$ for a constant bending radius R can be presented in the form as $\left|\vec{r} - \vec{r_1}\right| \approx \left|\xi - \frac{\xi^3}{24R^2} + \xi\frac{x+x_1}{2R} + \frac{(x-x_1)^2+(y-y_1)^2}{2\xi}\right|, \xi \equiv z - z_1.$

The subject of [1] was to calculate the particle energy change, therefore, the interaction Hamiltonian V_0 was found at $x = x_1 = 0$, $y = y_1 = 0$. In this paper, we derive the effective transverse forces with linear accuracy on x and y. As in [1], we neglect the integration over $\xi < 0$, small ultra-relativistic terms $\propto \gamma^{-2}$ and transverse dispersion of τ in denominator of the integrand in (9) and (10). Then we get

$$V_o = Ne^2 \int\limits_0^\infty \frac{\xi d\xi}{2R^2}\left[1 - (\xi\frac{x}{2R} + \frac{x^2+y^2}{2\xi})\frac{1}{c}\frac{\partial}{\partial t} + \right.$$

$$\left.\frac{1}{2}\xi^2\frac{x^2}{4R^2}\frac{\partial^2}{c^2\partial t^2}\right]\lambda(s - \frac{\xi^3}{24R^2}), \tag{11}$$

$$A_x = \int\limits_0^\infty \frac{d\xi}{R}\left[1 + \xi\frac{x}{2R}\frac{\partial}{\partial s}\right]\lambda(s - \frac{\xi^3}{24R^2}),$$

where $\lambda(s) \equiv \lambda(z - \beta ct)$ is the linear charge distribution along the orbit $\int\lambda(s)ds = 1$. We assume here that charge density ρ is an even function of x_1 and y_1 and ignore the small terms of the order of $\sim \overline{x_1^2}, \overline{y_1^2}$. The linear term $\propto x$ in (11) is simply integrated, while the last term is reduced to be proportional to $\propto \partial/\partial t$ by integration in parts, then we obtain final expression for the Hamiltonian:

$$V_0 = U(s)(1+Kx) - F_0(s)x + \frac{1}{2}g(s)(3x^2+y^2), \tag{12}$$

where $F_0(s) = -\frac{2Ne^2}{R}\lambda(s)$,

$$U(s) = \frac{2Ne^2}{(3R^2)^{1/3}}\int\limits_0^\infty \frac{ds_1}{s_1^{1/3}}\lambda(s - s_1) \tag{13}$$

$$g(s) = \frac{Ne^2}{(3R^2)^{2/3}}\frac{\partial}{\partial s}\int\limits_0^\infty \frac{ds_1}{s_1^{2/3}}\lambda(s - s_1) \tag{14}$$

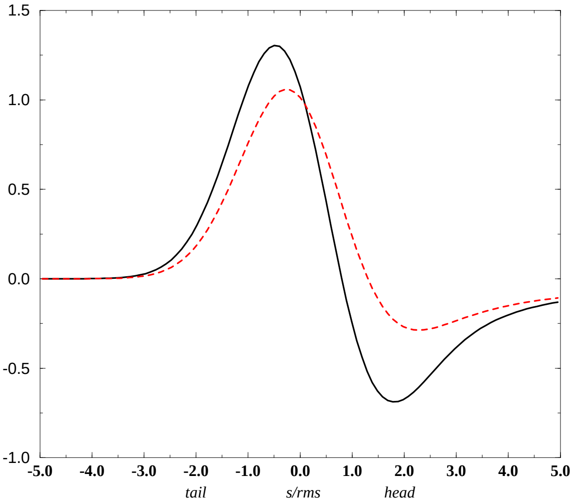

Figure 1: Overtake functions $I_0(s)$(dashed) and $I_1(s)$(solid) for Gaussian bunch.

The radial vector potential A_x contributes in C_x' with small terms of the order of $\sim (R^2\sigma_s)^{1/3}/\beta_x$ with respect to $F_0(s)$ and $g(s)$, and, therefore, they could be neglected in further consideration.

Then we have

$$2i\mathcal{E}\hat{C}_x' = -\eta\frac{\partial}{\partial s}U(s) - f_x F_0(s) + 3f_x g(s)x \tag{15}$$

$$2i\mathcal{E}\hat{C}_y' = f_y g(s)y, \quad \hat{\mathcal{E}}' = -\frac{\partial}{\partial s}U(s). \tag{16}$$

Therefore, comparing with initial Eqs.(5,6), one can see that: 1) $\partial U(s)/\partial s$ is longitudinal energy loss gradient originally found in [1, 2], 2) $F_0(s)$ is effective centripetal radial force, 3) terms with $g(s)$ describe focusing field distortions in both transverse planes. All these forces cause emittance growth.

For a bunch with Gaussian linear charge density distribution $\lambda(s) = (1/\sqrt{2\pi}\sigma_s)e^{-s^2/2\sigma^2}$ the energy loss gradient along the bunch is equal to [1] $\hat{\mathcal{E}}' = \frac{d\mathcal{E}}{cdt} = -\frac{2Ne^2}{\sqrt{2\pi}(3R^2\sigma_s^4)^{1/3}}I_0(s/\sigma_s)$, where the function $I_0(s/\sigma_s)$ is presented by dashed line in Fig.1. As it is qualitatively understood, the bunch head particles get some excess of energy while the tail and center part mostly loses the energy.

Transverse forces within the Gaussian bunch are given by formulae:

$$F_x(s) = -\frac{2Ne^2}{R}\lambda(s) - x\frac{3Ne^2}{\sqrt{2\pi}(9R^4\sigma_s^5)^{1/3}}I_1(s/\sigma_s), \tag{17}$$

$$F_y(s) = -y\frac{Ne^2}{\sqrt{2\pi}(9R^4\sigma_s^5)^{1/3}}I_1(s/\sigma_s), \tag{18}$$

where $I_1(s)$ is shown in Fig.1 by solid line.

One can see that particles at the head of the bunch are defocused by overtaking radiation fields while other particles are focused.

Let us note that effective radial force $-\partial V_0/\partial x$ is essentially defferent from the initial force F_x in (2) – the difference is the term $\propto KA_z$. In fact, the later term dominates in F_x and it is centrifugal "Talman force" [3] (it looks like $Ne^2/R \cdot \lambda(s) ln \frac{(R\sigma_s^2)^{2/3}}{\sigma_\perp^2}$).

The effect of this later force on the bunch particles is cancelled by effect of the particle energies deviation under the influence of the transverse electric field, and therefore does not lead to the emittance growth. Similar cancellation effect in the particular case of a coasting beam was found in [4]. Now we can conclude that it is valid for any relativistic bunch.

4 EXAMPLE: TTF FEL

Let us apply the obtained results to the TESLA Test Facility Free Electron Laser [5], which intends to decrease initial rms bunch length from 0.8 mm to 0.25 mm in bunch compressor C#2 at 144 MeV and from 0.25mm to 0.05 mm in compressor C#3 at 516 MeV. The FEL bunch contains N= 6.2·10^9 electrons with design normalized transverse emittance of ϵ = 2 μm. Each 5-m long compressor consists of four 50-cm-long magnets. Curvature radius for electrons in the magnets is R = 1.3m. Mean horizontal beta-function is about $\beta_x \approx$ 11m.

The overtaking length $L_o = 2(3R^2\sigma_s)^{1/3}$ is about 0.3-0.1 m, thus, it is less than the magnet length L_d and, therefore, the cooperative effects should take place. In Ref. [1] cumulated energy spread induced by longitudinal tail-head effects in the bunch after two compressors was estimated to be about 0.7 MeV (rms value) and corresponding rms emittance increases by 7μm in C#2 and 25μm in C#3 due to the dispersion in compressors.

The maximum centripetal field (at the center of the Gaussian bunch) $B_c^{max} = -\frac{2Ne}{\sqrt{2\pi}R\sigma_s}$ is about 3.7 G at the exit of C#3. As the centripetal force varies with position along the bunch, then it will induce the normalized emittance growth of the order of $\Delta\epsilon_c = \gamma\beta_x\theta_d^2\left(\frac{r.m.s.\,B_c}{B}\right)^2$, where $\theta_d \simeq 0.38$ rad is the bending angle in each magnet, and $r.m.s.\,B_c \approx 0.28 B_c^{max}$ for the Gaussian bunch. Total emittance growth in C#2 and C#3 due to centripetal forces is about 2μm and 12μm correspondingly.

Transverse focusing forces due to cooperative radiation can be characterized by minimum focusing length $f_x^{rad} = \frac{\gamma R^{4/3}\sigma_s^{5/3}}{I_1^{max} 3^{1/3} N r_0 L_d}$, ($I_1^{max} \approx$ 1.3 – see Fig.1) which falls from 168 m at the entrance of C#2 down to 6 m at the exit of C#3. The focusing radiative forces are some 7 times of the Coulomb expansion force at the exit of the bunch compressor #3 and comparable with strength of the external focusing.

5 CONCLUSIONS

We have analyzed the microbunch cooperative synchrotron radiation in bend and found that it essentially influences the microbunch dynamics. First of all, the longitudinal force redistributes radiative energy losses along the bunch, so, that head particles are somewhat accelerated by the field radiated by tail particles. The energy losses originate from derivative of the linear charge density, that is characteristic feature of the effect. Aside of the energy spread along the bunch, the effect leads to radial emittance increase due to dispersion.

The transverse radiative force consists of two components. The smaller term represents focusing forces which are produced by the back part of the bunch. Note, that the radial focusing gradient is three times the vertical one. Finally, there is radial centripetal force $F_x^c(s) = -2Ne^2\lambda(s)/R$ which is much bigger than the focusing forces. The transverse forces also cause the transverse emittance growth. We have found that the combined effects in the TTF FEL bunch compressors can lead to many-fold increase of the initial beam emittance.

Our results are applicable if the characteristic overtaking length $L_o = 2(3\sigma_s R^2)^{1/3}$ is less than the bend length $L_d = R\theta_d$ and if there is no shielding due to metallic vacuum pipe $(\sigma_s^2 R)^{1/3} \ll b$ (for example, the last condition yields $\sigma_s \ll$ 2 mm for $R \simeq$ 2m and $b \simeq$ 2cm). Thorough studies of the beam pipe shielding are underway.

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