Another Method to Measure the Low-Frequency Machine Impedance

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Abstract

The spectrum of long bunches samples the low-frequency part of the machine impedance which is mostly reactive. The voltage induced by the bunch produces the well known ”potential well distortion” when RF is on, but also affects the debunching when RF is off. In this paper we present a method of estimating the reactive impedance by measuring the evolution of bunch parameters, such as the peak line density, during debunching. This method was used to find the inductive impedance of the CERN SPS with a single proton bunch injected above transition at 26GeV.

1 INTRODUCTION

In proton accelerators bunch lengths are usually long compared to the transverse dimensions of the vacuum chamber. The spectrum of such bunches peaks at frequencies much lower than the cut-off frequency and the impedance seen by the bunches is dominantly reactive with ImZ/n usually assumed constant.

When injected into a machine with RF on, the voltage induced by the bunch current adds to the external RF voltage and the deformation of the potential well leads to bunch lengthening, (or shortening) [1]. Measurements of the changing bunch length with intensity have often been used to find the impedance [2]. This method is based on analysis of the stationary distribution that is finally reached, with an exact solution known for parabolic bunches.

In our case an intense bunch is injected into a machine with RF off where, without the focusing effect of the external RF system, the bunch starts to spread out or debunch. The fact that debunching of intense bunches is strongly affected by the induced voltage was already noticed in [3]. However evaluation of the low-frequency impedance from this effect implies knowing the time dependent solution for the variation of the bunch parameters during debunching. This time dependent solution has been found for parabolic bunches in [4], and has been used to estimate the low-frequency impedance of the CERN SPS by measuring the debunching rate, as a function of intensity, of single bunches injected at 26GeV.

2 THEORETICAL MODEL

In this section we give a brief resume of the full results presented in [4]. If the injected bunch has a parabolic line density, closed form analytic solutions can describe the behaviour of the bunch after injection into the machine with RF off in the presence of a reactive impedance. In particular the variation with time of bunch length τ or of peak line density λ_p

$$\frac{\tau(t)}{\tau(0)} = r(t), \qquad \frac{\lambda_p(t)}{\lambda_p(0)} = \frac{1}{r(t)} \tag{1}$$

can be obtained from a solution of the following equation for a positive defined function $r(t)$

$$\frac{\dot{r}^2}{2} + U(r) = 0, \tag{2}$$

with the initial condition $r(0) = 1$. This equation can be formally interpreted as the equation of motion of some particle with the coordinate r in the potential $U(r)$:

$$U(r) = -\frac{\Omega^2(r-1)(1+ar)}{2r^2}, \tag{3}$$

where $a = 1 + 2\mathrm{sgn}(\eta \mathrm{ImZ})\Omega_\epsilon^2/\Omega^2$. Here the frequency Ω depends on the shape of the injected bunch:

$$\Omega = \frac{2\eta}{\tau(0)} \frac{\Delta p_{max}}{p}, \tag{4}$$

where $\eta = 1/\gamma_t^2 - 1/\gamma^2$ and $\pm\Delta p_{max}/p$ is the initial maximum relative momentum spread in the bunch. Note that Ω would coincide with the frequency of linear synchrotron oscillations in a matched RF voltage at low intensity.

Intensity effects are presented by the parameter Ω_ϵ:

$$\Omega_\epsilon = \left(\frac{6Ne^2|\eta|}{\pi E_s \tau^3(0)} \frac{|\mathrm{ImZ}|}{n} \right)^{1/2}, \tag{5}$$

where N is the bunch intensity, $\mathrm{ImZ/n}$ is the low frequency reactive part of broad-band impedance and E_s is the synchronous energy. (In the case with RF on, Ω_ϵ would represent the incoherent frequency shift). For zero intensity, $\Omega_\epsilon = 0$ and $a = 1$. In this case the solution is independent of the bunch distribution with

$$r(t) = [1 + \Omega^2 t^2]^{1/2}. \tag{6}$$

It is only necessary that the initial distribution be a function of the Hamiltonian of linear synchrotron motion in the injector for this formula to be applicable.

For the case $a > 0$, we can find from (3) an explicit solution for the function $r(t)$,

$$\Omega t = \frac{\sqrt{\rho(r)}}{a} + \frac{a-1}{2a^{3/2}} \ln \frac{|2\sqrt{a\rho(r)} + 2ar + 1 - a|}{1+a}, \tag{7}$$

where $\rho(r) = (r-1)(ar+1)$.

For $a > 0$, motion in the potential defined by (3) can only be infinite which means continuous debunching, $r \rightarrow \infty$ as $t \rightarrow \infty$. (Finite, oscillating solutions are possible for $a < 0$). Nevertheless, the character of the debunching is different depending on the value of a. If $a > 1$, (inductive impedance above transition or space charge below), the induced voltage has a defocusing effect and debunching is faster compared to the low intensity case. For $a < 1$, (inductive impedance below transition or space charge above), debunching is slowed down by the focusing effect of the induced voltage. These possibilities are shown in Figs.1,2.

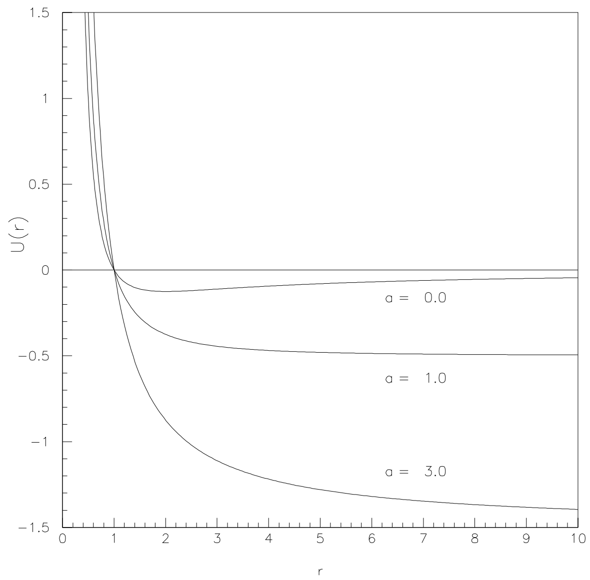

Figure 1: Effective potential for different types of induced voltage, focusing ($a = 0$), zero intensity ($a = 1$), and defocusing ($a = 3$).

A change in debunching rate due to intensity effects can therefore be used to estimate the reactive part of the broad band impedance if the parameters of the injected bunch are known. By changing the parameters Ω and Ω_ϵ one can fit the curve defined by formula (7) to the measured data.

A quite good approximation to this formula, valid at the beginning of debunching, for $t < 1/\Omega$, $(r \sim 1)$, is

$$r(t) \simeq [1 + (\Omega^2 \pm \Omega_\epsilon^2)t^2]^{1/2}. \quad (8)$$

For $t >> 1/\Omega$, $(r >> 1)$, i.e. when the initial distribution is already strongly debunched, the asymptotic solution can again be obtained from (3) and is

$$r(t) \simeq [1 + (\Omega^2 \pm\ 2\Omega_\epsilon^2)t^2]^{1/2}. \quad (9)$$

The positive sign in (8) and (9) corresponds to the defocusing and the negative to the focusing case.

Both these approximations are shown in Fig.2 together with the exact solution for $a = 3$. As can be seen the exact solution lies between these two limits.

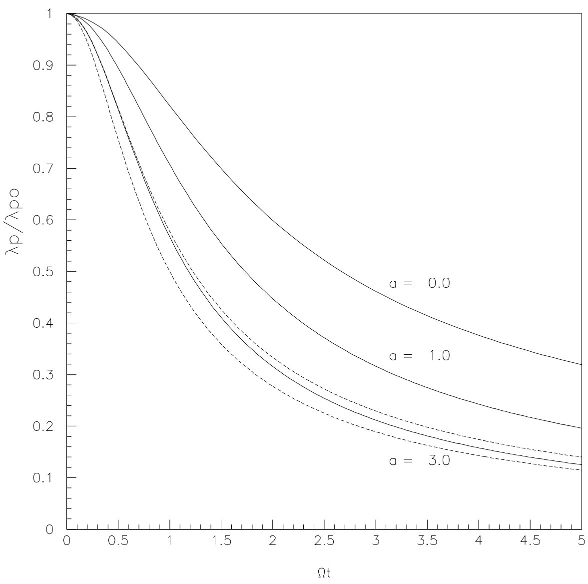

Figure 2: Peak line density variation during debunching for different types of induced voltage, focusing ($a = 0$), zero intensity ($a = 1$), and defocusing ($a = 3$) together with approximate solutions (dashed lines) for $a = 3$.

Using the approximate formula (8) simplifies the curve fitting since only one parameter, $\Omega_d^2 = \Omega^2 + \Omega_\epsilon^2$, need be varied. Frequency Ω_d is a parameter with two components, one dependent only on the injected bunch characteristics and independent of intensity and a second which varies linearly with intensity. Consequently if we plot Ω_d^2 as a function of intensity we can hope to separate the intensity and non-intensity dependent effects.

It is necessary to note that if the initial bunch was both created and later allowed to debunch in the same machine, then the measured Ω_d is defined to first approximation only by the external voltage and doesn't depend on intensity. Indeed due to the potential well distortion the matched intense bunch has dimensions defined by $\Omega \sim \sqrt{\omega_{s0}^2 \mp \Omega_\epsilon^2}$ where ω_{s0} is the zero intensity synchrotron frequency with RF on. In this situation the measured debunching frequency will always be $\Omega_d \simeq \omega_{s0}$.

3 MEASUREMENTS

One can see two possibilities for measuring the function r(t), via changes in bunch length τ or peak line density λ_p. Bunch length can be measured from the variation of bunch spectrum at low frequency kf_0, f_0 being the revolution frequency. The amplitude $I_k(t)$ changes with time as $I_k(t) = I_{kr(t)}(t = 0)$. Note that the influence of induced voltage should also be taken into account when this technique is used to estimate the momentum spread of intense bunches.

The method we used is based on the measurement of the decay of the peak line density with time for bunches having different total intensities.

Single bunches with various intensities were injected onto the 26GeV injection plateau. At this energy, above transition ($\gamma_{tr} = 23.4$), the inductive impedance produces a defocussing effect and we expect the decay rate of the peak intensity to increase with intensity. The peak intensity of the injected bunch during the debunching process was measured by peak detection of the signal from a longitudinal wideband monitor (wall-current type). The bunch profile acquired at injection was used to estimate the bunch length with a fit for a parabolic line density.

The total bunch intensity and microwave signals were monitored to be sure that no losses occurred and that the bunch remained stable during the measurement.

In Fig.3 we give, as an example, the peak line density from one measurement and the curve calculated from the approximate formula (8) having adjusted the parameter Ω_d for the best fit. For the nominal injected bunch we expect $\Omega \sim 0.2 \times 10^3 \mathrm{s}^{-1}$ and hence the approximate formula should be valid to $\sim 5\mathrm{ms}$. The use of the initial part of the curve for the fit reduces the dynamic range requirement on the measurement apparatus. Calibration becomes critical for low values of peak line density.

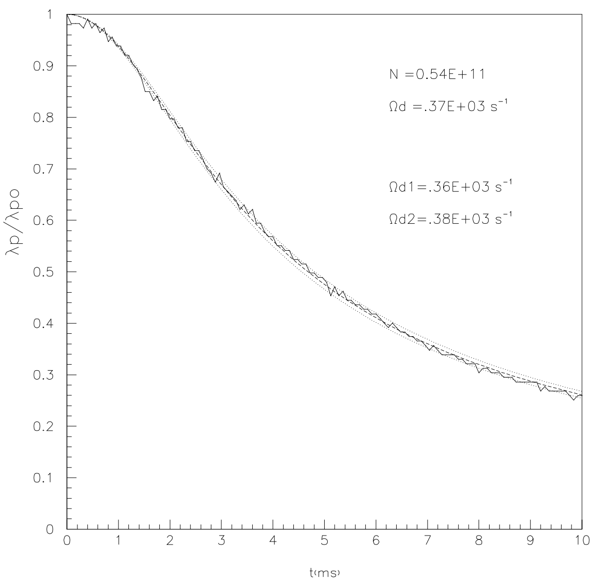

Figure 3: Measured ($N = 5.36 \times 10^{10}$) and calculated ($\Omega_d = 0.37 \times 10^3 \mathrm{s}^{-1}$, dashed line) decay of peak line density. Dotted lines correspond to decay calculated with $\Omega_d = 0.36 \times 10^3 \mathrm{s}^{-1}$ and $\Omega_d = 0.38 \times 10^3 \mathrm{s}^{-1}$.

Having found the parameter Ω_d^2 for each measurement, they are plotted as a function of intensity in Fig.4. Although Ω_d can be determined accurately from the decay curve, the value obtained is affected by the large variation in injected bunch parameters, (the bunch length was varying from 3.8 - 4.8ns). This produces the scatter on the graph. The linear fit to the data in Fig.4 gives us the following information.

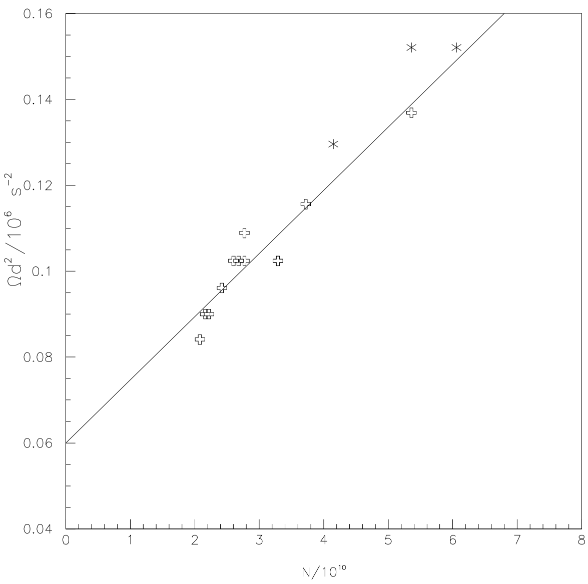

Figure 4: Measured Ω_d^2 as a function of intensity.

The intersection of this line with the vertical axis defines Ω^2 for the average zero intensity injected bunch. The slope of the line gives Ω_ϵ^2 as a function of intensity and hence allows the impedance to be estimated from (5) using the known average length of the injected bunch. For an average bunch length of 4.3ns this gives $\mathrm{Im}Z/\mathrm{n} = 18.7\mathrm{Ohms}$. In Fig.4 measurements with unstable bunches are shown as asterisks but are not used for the fit.

4 CONCLUSIONS

We suggested an alternative method to measure the low-frequency machine impedance based on an analysis of the change with intensity of the debunching rate. This method was applied to estimate the impedance of the SPS. The value obtained, 18.7 Ohms, lies within the relatively wide range of values found by previous measurements, 10 - 20 Ohms.

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6 REFERENCES

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