SORTING OF MAGNETS FOR THE BESSY II BOOSTER AND STORAGE RING *

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1 INTRODUCTION

BESSY II is a low emittance 1.7 GeV electron storage ring presently under construction in Berlin (Germany) [1]. The injection into the storage ring is done by a full energy booster [2]. Both rings have tight tolerances for magnetic field and alignment errors to achive good performance. A discussion of the effects of alignment errors for the storage ring was given in [3]. Disturbing effects of magnetic field imperfections on the machine optics are unavoidable, however, they can be lowered if these errors are known and the magnets are placed into the ring dependent on their field defects.

A scheme of constructing cost functions is presented here, for a quantitative description of these disturbing effects due to dipole, quadrupole and sextupole field errors [4]. A sorting method, based on simulated annealing [5], which minimizes the beam distortion is then applied. In case of sextupole errors, a new scheme of constructing a cost function is suggested, based on Collins distortion functions [6], which leads to an improved dynamical aperture.

There are many publications on this subject, however, most of the work discussed here goes back to the paper by E.D. Courant and H.S. Snyder [7], where a derivation of the basic formulae is given.

2 THE COST FUNCTIONS

The magnet errors discussed here are expressed as deviations of the integrated field strength from the design value, within the frame of the hard edge model. Each magnet is characterized by its design value and the integrated field strength deviation located as a kick k in the center of the magnet. A vector $\vec{\delta}$ is built up, which contains the magnet errors in a specified sequence $\vec{\delta} = (k_1, ..., k_i, ..., k_N)$ dependent on their position in the ring. For each magnet type (dipole, quadrupole, sextupole) the sorting is done independently, all definitions which are given belong only to a specific group.

A cost function $\mathcal{W}$ was developed and used to compare different permutations of the given error set. This function was used in the optimization by the method of simulated annealing.

The cost function is built up, described by a vector $\vec{W}$ with two components (A, B), which are constructed for each type of magnet differently. This vector is transformed around the ring. A kick k acts only on the A-component. Between the kicks $\vec{W}$ is simply rotated with an appropriate phase advance, changing A and B but preserving its length. Depending on the phase value a kick increases or decreases the length W of the vector. At a given observation point, W is given by $\sqrt{A^2 + B^2}$, dependent on the distribution of all other errors. For a fixed error distribution, W is changing around the ring at each kick. One observation point per kick is sufficient because of the invariant character of W. The individual W values are sumed up by an rms averaging to a single number $\mathcal{W}$; this is the value of the cost function of a specific error distribution. By permutation of the error distribution a new cost function is obtained. The goal is, to permute the errors and to minimize the cost function.

The vector components (A_i, B_i) at different observation points i are linearly related to the error vector by a matrix $\mathbf{T}$. Combining them in a common vector yields:

$$((A_1, B_1), ..., (A_i, B_i), ..., (A_N, B_N)) = \mathbf{T}\ \vec{\delta}$$

In a fast computer simulation only the error distribution of $\vec{\delta}$ is permuted to obtain the new (A_i, B_i) vectors, while the matrix stays unchanged. The matrix is dependent on the linear optics and the magnet type. From these vectors, the new cost function $\mathcal{W}$ is extracted.

2.1 The Dipole Cost Function

Dipole errors cause closed orbit deformations. The kick strength k_i is defined as the difference between the measured and designed dipole bending angle $\theta_i - \theta_0$ of the i^{th} dipole. The closed orbit is kicked by this error and oscillates in a closed loop around the ring.

A single kick k and the disturbed closed orbit vector $\vec{x}_{co} = (x, x')$ are related by the one turn transfer matrix of the ring. The amplitude of the closed orbit is given by

$$x = k/2\sqrt{\beta\beta_i}\cos(\phi/2 - |\Delta\psi|)/\sin(\phi/2),$$

where β and β_i are the beta functions at the observation point and the kick position, ϕ is the phase advance around the ring and $\Delta\psi$ is the distance between the kick position and the observation point.

Using the linear transfer properties of the ring, the vector $\vec{x}_{co}$ can be calculated at any other position. Several kicks at different locations are added by superposition to get the resulting closed orbit (x_i, x_i') at any position. The vector (A_i, B_i) is then constructed by using the local Twiss pa-

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rameters (β_i, α_i):

$$A_i = (\alpha_i x_i + \beta_i x_i')/\sqrt{\beta_i}\,, \quad B_i = x_i/\sqrt{\beta_i}\,,$$

and for the dipole case W is the Courant-Snyder invariant of the closed orbit oscillation. Minimizing this type of cost function leads to small amplitudes and slopes of the closed orbit oscillation.

2.2 The Quadrupole Cost Function

Focussing errors of quadrupoles are responsible for a beating of the beta function. The difference $k_i = g_i - g_0$ between the measured integrated focussing strength and its designed value is located as a kick in the quadrupole center, which generates the distortion $\Delta\beta = \beta - \beta_0$ and $\Delta\alpha = \alpha - \alpha_0$ of the Twiss parameters. The resulting beta function distortion is oscillating with twice the betatron phase around the ring.

Applying a matrix approach, an additional focussing kick in the quadrupole yields a distorted matrix, which can be expressed in terms of the undistorted one. The distorted beta function is given by (and similar for the α-function):

$$\Delta\beta/\beta_0 = -\beta k \cos(2\,|\Delta\psi| - \phi)/(2\sin\phi).$$

Superposing the effects of focussing errors in different quadrupoles, the distortions of the Twiss functions can be calculated. The vector (A, B) is now constructed by:

$$A = (\alpha\beta_0 - \alpha_0\beta)/\sqrt{\beta\beta_0},\ B = (\beta - \beta_0)/\sqrt{\beta\beta_0},$$

as given in [8] for chromatic focussing errors of quadrupoles. The cost function is composed as the rms-value obtained from the vertical and horizontal plane.

2.3 The Sextupole Cost Function

In the case of the storage ring, the effect of sextupole field errors is minimized. As a new approach, a cost function based on the distortion functions introduced by T. Collins is suggested. There are 5 distortion functions, describing nonlinear effects of sextupoles, given in his notation as B_1, B_3, B_s, B_d, $\bar{B}$ and A_1, A_3, A_s, A_d, $\bar{A}$, where the A_m functions are derivatives of the B_m functions. For a single sextupole, a distance $\Delta\psi_m$ away from the observation point, these functions are of the type

$$B_m = s_m \cos(|\Delta\psi_m| - \phi_m/2)/(8\sin(\phi_m/2)),$$

where the index m indicates one of the five functions. The sextupole strength s_m and the phase advance ϕ_m around the ring are scaled in an appropriate way.

The definition of the distortion functions is not fully repeated here, only the steps how to use them to construct cost functions. The functions has to be calculated twice, using sextupoles at their design values and including the errors. The difference $(A_m - A_{m,0}\,, B_m - B_{m,0})$ is now the vector (A, B) which is used to calculate the cost function, its length is conserved between adjacent kicks. The rms value of the length calculated around the ring and the rms value of all five functions yields the value of the cost function. Minimizing the cost function by sextupole error permutation reduces the nonlinear beam mismatch, on the level of the approximation valid for the distortion functions.

3 SIMULATED ANNEALING

The cost function $\mathcal{W}$ defined as discussed above, does not show a minimum, which can be reached by a gradient like approach. For N magnets all possible permutations are growing with $(N-1)!/2$. A computer simulation checking all possible cases would be too time consuming.

The method of simulated annealing is typically applied for these kinds of problems to find a solution close to the minimum in acceptable time.

For a given error vector $\vec{\delta}_0$ the value of the cost function $\mathcal{W}_0$ is calculated and compared with the cost function of a permutated error set. The new permutation is obtained from the previous one by some few, well defined steps. First, one indicates in the given error vector, a sequence of elements by randomly choosen start and end elements. Second, this string is moved to a randomly choosen position inside the error vector, or alternatly, is simply reversed. With this new error vector, a new value of the cost function $\mathcal{W}$ is calculated.

If the new distribution results in a smaller cost function, it is taken as a new reference $\vec{\delta}_0$. Also, in case the new cost function is larger and differs by an amount $\mathcal{W} - \mathcal{W}_0 = \Delta\mathcal{W} > 0$, this distribution will be taken as the new reference, if $e^{-\Delta\mathcal{W}/T}$ is above a given (randomly choosen) threshold. Here T serves as a temperature like parameter, which is lowered with increasing number of permutations.

In this way one scans the values of the cost functions in the vicinity of the present reference point but avoids being trapped in a local minimum. The resulting solution of the cost function is not necessary the smallest one, but it is close to the minimum. As a comparison, the same algorithm is used to find the worst case yielding a large cost function. The differences between the worst and best solution is a measure of the gain which is achivable with the sorting.

4 RESULTS

Measured magnetic field defects [9] of the booster dipoles and quadrupoles are used for the simulation. For the storage ring magnets, the errors are not yet available, but they are simulated by a Gaussian error distribution.

Results of sorting the dipoles of the BESSY II booster synchrotron are presented in fig. 1. The best and worst case scenarios are displayed. The rms-error of the integrated dipole strength was measured as $(\Delta\theta/\theta)_{rms} = 1.7/10^4$. For the storage ring, a calculation with simulated errors of $(\Delta\theta/\theta)_{rms} = 4.5/10^4$ yieds a closed orbit oscillation

amplitude of up to 0.1 mm for the best case, growing to 4 mm for the worst case.

For the quadrupole sorting, the results of the booster is shown in fig.2, based on measured values $(\Delta g/g)_{rms} = 2.5/10^4$. The simulation for the storage ring scenario used focussing errors of $(\Delta g/g)_{rms} = 1/10^3$. A beta beating $\Delta\beta/\beta$ for the minimized case of 5 % was achived, whereas the worst case was 6 times larger.

Optimizing the distribution of integrated sextupole errors of the storage ring resulted in a clear enlargement of the dynamical aperture, as shown in fig.3. The tracking is based on 1000 turns, nominal particle momentum, no additional errors are activated and no insertion devices are included. This simulation is based on expected integrated sextupole errors of $(\Delta m/m)_{rms} = 1/10^2$.

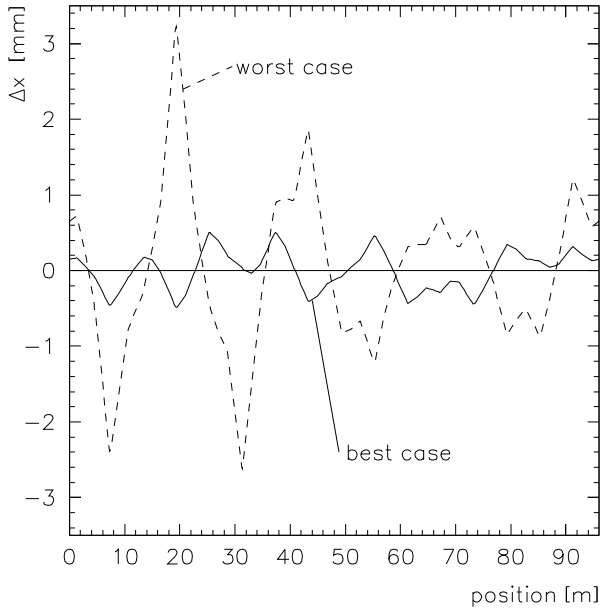

Figure 1: Best and worst case of the closed orbit amplitude along the the booster synchrotron circumference. The vertical axis is the oscillation amplitude in units of [mm].

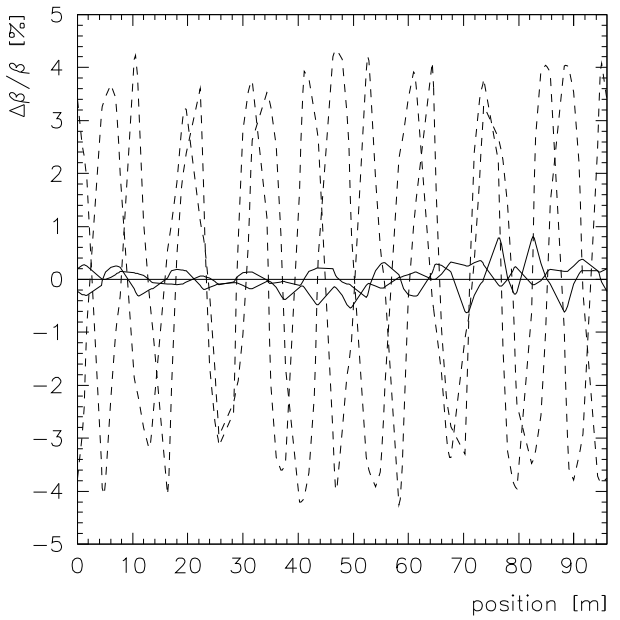

Figure 2: Best and worst case of the beating of the beta function along the booster synchrotron circumference. The vertical axis is the oscillation of $\Delta\beta/\beta$ in %.

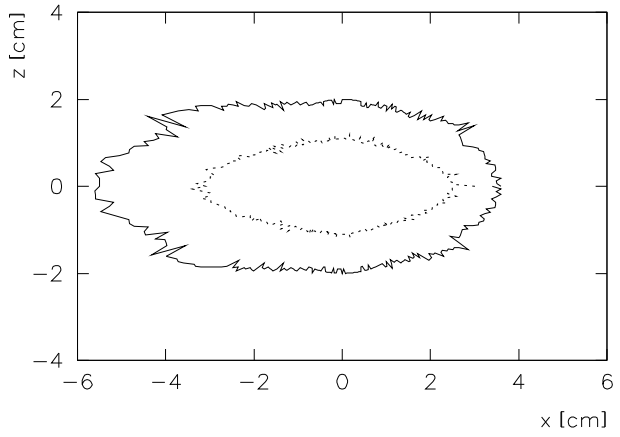

Figure 3: Best and worst case of the BESSY II storage ring dynamical aperture based on simulated sextupole errors. The vertical (horizontal) axis is the vertical (horizontal) size of the dynamical aperture in [cm].

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