

# Correction of Field Errors in ISAC RFQ

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## Abstract

This paper is an abridged version of a TRIUMF design note[1]. The most important effect of using a non-ideal vane geometry is that the longitudinal field can depart substantially from the value calculated from the usual analytic formulae; and this may lead to beam loss due to inadequate acceleration. The effect is enhanced at low particle speed and low radio-frequency, as occurs at injection into the TRIUMF ISAC heavy ion RFQ. An exact and self-consistent procedure has been invented that may be applied to any practical vane geometry, to recalculate the vane profile parameters, modulation index ( $m$ ) and minimum bore radius ( $a$ ), so as to give the ideal acceleration and focusing field-coefficients at every RFQ cell. The procedure has two steps. First, make a numerical map, for each field coefficient, of the dependence on cell parameters  $m$ ,  $a$  and cell length. Second, for given, desired fields at the synchronous particle, to invert the map numerically to find the required values of  $m$  and  $a$ . We have applied this technique to the TRIUMF ISAC RFQ accelerator design.

## 1 INTRODUCTION

We shall use the notation of [1] and take cylindrical polar coordinates  $r, \theta, z$ . Let  $k = 2\pi/(\beta\lambda)$  where  $\beta = v/c$  and  $\lambda$  is wavelength. Typically, an RFQ is designed assuming that the dynamics in each cell is based on the normalized two term potential function,

$$\Phi_2 = A_{0,1}r^2 \cos(2\theta) + A_{1,0}I_0(kr) \cos(kz), \quad (1)$$

where  $k = 2\pi/(\beta\lambda)$ . Further, the design of vane profiles is based on the assumption that for each cell the focusing and acceleration coefficients ( $A_{0,1}$  and  $A_{1,0}$  respectively) are solutions of

$$\begin{bmatrix} 1 \\ 1 \end{bmatrix} = \begin{bmatrix} a^2 & +I_0(ka) \\ (ma)^2 & -I_0(mka) \end{bmatrix} \begin{bmatrix} A_{0,1} \\ A_{1,0} \end{bmatrix}. \quad (2)$$

It is usual to impose a further constraint so that  $m$  and  $a$  cannot be chosen independently, and this is the condition of constant characteristic radius,  $r_0$ , to pole tip:  $A_{0,1}(m, a) \times r_0^2 = 1$ . Typically, one employs a design strategy to specify the variation of  $A_{1,0}(z)$  along the RFQ and then numerically inverts (2) for  $m, a$ .

That the functional dependence of the coefficients on parameters  $m$  and  $a$  is given by (2) presupposes that the two sets of RFQ electrode surfaces are exactly equipotential surfaces of  $\Phi_2$ . A practical vane geometry, e.g. that shown in figure 1, must depart significantly from the ideal electrode surfaces; with the consequences that (i)  $A_{0,1}$ ,  $A_{1,0}$  differ from the form given in (2), (ii) higher order terms are introduced into the potential. Often, the longitudinal fields are weaker than predicted by the  $\Phi_2$  function.

## 2 RFQCOEF AND PARMTEQ

In principle, the computer program PARMTEQ[2] can track particles through an eight term potential of the form

$$\Phi_8 = \sum_{n=0}^3 [A_{0,n}r^{2n} \cos 2n\theta + \sum_{l=0}^3 A_{l,n}I_{2n}(lkr) \cos 2n\theta \cos l kz] \quad (3)$$

provided that one can supply the coefficients  $A_{l,n}$  for each cell of the RFQ. Here in (3) the  $I_{2n}$  are hyperbolic Bessel functions. The program RFQCOEF[3] can calculate the coefficients for several vane geometries including that employing constant transverse pole tip radius (CTPTR), but with the radius from bore centre-line to pole tip varying from  $a$  to  $ma$ .

RFQCOEF reads a description of the RFQ that is almost identical to that used by PARMTEQ, and sets up a finite element mesh (for each cell individually) upon which to solve for the electrostatic potential; RFQCOEF then best-fits the coefficients  $A_{l,n}$  to the potential at all mesh-nodes between the pole tips, and outputs the harmonic coefficient data set (HCDS). PARMTEQ reads in a description of the RFQ (which is essentially the list of  $m, a$  and  $\phi_s$  for each cell) and the HCDS, and then calculates the electric fields and tracks ions through these fields.

### 2.1 ISAC trial case

For the ISAC RFQ, for simplicity, the CTPTR vane geometry was adopted as a trial case. The on-axis longitudinal electric field,  $E_z(z, r=0)$ , calculated from the eight term potential was found to be on average 8% weaker than anticipated, and deviations of up to 33% were seen in the low energy parts of the RFQ. The relative field error is sketched in figure 2. Particle tracking through these fields gave 100% losses, due to inadequate acceleration; this was a calamitous result in need of explanation and rectification.

### 2.2 Explanation of weak $E_z$ -field

The inadequate longitudinal electric field was due to the use of a non-ideal vane geometry. The question was: "why for the ISAC RFQ is vane geometry so crucial, when there are many proton RFQs built and working that use constant radius poles?" The answer hinges upon the Bessel function radial dependence of the acceleration term in  $\Phi_2$ . If the argument is less than unity then the field depends almost linearly on displacement; whereas for larger values the exponential dependence on radius becomes more evident. The argument is  $\simeq mr_0 2\pi/(\beta\lambda)$  and in the low energy part of the ISAC accelerator the value is  $\approx 3$  because  $r_0 = 8.6$  mm is large, and  $\beta = 0.002$  is small, and  $\lambda = 8.56$  m. A proton accelerator would have larger  $\beta$ , smaller  $r_0$ , but smaller  $\lambda$ .

### 3 FIELD CORRECTIONS

Fundamentally, the difficulty is that ideally we should like to specify the fields (for small amplitude motions about the on-axis synchronous particle) and find the geometry that gives rise to them; whereas we only have the tools for solving the inverse problem of “given a geometry, find the fields.” There are two strategies to overcome this difficulty:

- By trial and error find a vane geometry that produces fields within a few percent of values for the ideal geometry, and then rely upon the RESTORE option of PARMTEQ to correct the  $m(z)$  and  $a(z)$  profiles.
- For a given simple, cheap to machine geometry find the functional dependence of  $A_{0,1}$  and  $A_{1,0}$  on the cell description parameters  $m$  and  $a$ ; and directly chose the  $m$  and  $a$  combination to give the desired fields.

Both strategies were considered at length, but only the second was adopted.

We realized that if, for a given vane geometry, one could find the functional dependence of coefficients  $A_{0,1}$  and  $A_{1,0}$  on the cell description parameters,  $m$ ,  $a$  and length  $l$ , then profiles  $m(z)$  and  $a(z)$  could be calculated to give exactly the synchronous and small amplitude motions of the ideal geometry. Further, that if the functional dependence of all eight coefficients of  $\Phi_8$  upon the parameters  $m$ ,  $a$ ,  $l$  could be determined, then a complete, consistent and almost exact description of the fields and beam dynamics in the RFQ could be obtained. We also realized that a numerical ‘map’ of the functional form would be adequate, and that sufficiently accurate maps for each of the eight coefficients ( $A_{l,n}$ ) could be constructed by running RFQCOEF some few thousand times; which computation takes only a day or so of c.p.u. time.

#### 3.1 The ‘MAP’ and ‘RESMAP’ options

RFQCOEF was modified so as to systematically scan the parameters  $m$ ,  $a$ ,  $l$  and thus generate the basic map data at discrete points on a three-dimensional (3-D) grid. New subroutines MAP and RESMAP were written to supersede the routines RFQPOT and RESTORE already in PARMTEQ.

**MAP routine** It may be that different vane geometries are used for different parts of the RFQ, and MAP allows to sequentially utilize up to five map data sets to allow for this.

Firstly, MAP calls RFQPOT\_MAP\_INIT to read the numerical map and initialize the interpolation subroutine. Then for every RFQ cell in a specified range, subroutine RFQPOT\_MAP is called to calculate the eight harmonic coefficients. In turn, RFQPOT\_MAP calls RATINT or POLINT which perform 3-D rational or polynomial interpolation (and extrapolation) to evaluate the map at points off the grid, and so allow evaluation of partial derivatives. Finally, subroutine RFQPOT\_INTERPOL fills in any missing data by interpolating values from neighbour cells.

**RESMAP routine** will adjust  $m$  and  $a$  of each cell so that actual values of  $A_{0,1}$  and  $A_{1,0}$  are equal to the ideal desired values. The code will then recompute the remaining six coefficients of  $\Phi_8$  according to the new values of  $m$  and  $a$ . RESMAP uses, sequentially, the same map data as was used by MAP. RESMAP makes multiple calls to RFQPOT\_MAP to evaluate  $A_{0,1}$ ,  $A_{1,0}$  and their partial derivatives with respect to  $m$  and  $a$ .

**RESMAP algorithms** Given that the on-axis longitudinal electric field after restoration is equal to the ideal value, so it follows that the cell lengths are unchanged as is the synchronous phase profile; and this property of constancy makes the procedure of fixing up the cells straight forward. For each cell the basic procedure is (for fixed  $l$ ) to perform a two-dimensional Newton-Raphson iteration that finds doublets  $m_{\text{new}}$  and  $a_{\text{new}}$  for which focusing and acceleration coefficients of the actual geometry are equal to their values for the ideal geometry with original cell parameters  $m_{\text{orig}}$  and  $a_{\text{orig}}$ . We desire the conditions (4):

$$A_{0,1}^{\text{actual}}(m,a)_{\text{new}} = A_{0,1}^{\text{ideal}}(m,a)_{\text{orig}} ; A_{1,0}^{\text{actual}}(m,a)_{\text{new}} = A_{1,0}^{\text{ideal}}(m,a)_{\text{orig}} .$$

The algorithm to achieve this is iterative and consists of two parts. Firstly, find the direction in which to increment the vector  $\mathbf{v} = (m, a)$ , thusly:

$$\mathbf{v}_{\text{new}} = \mathbf{v}_{\text{old}} + \mathbf{A}^{-1} \begin{bmatrix} A_{0,1}^{\text{ideal}}(m,a)_{\text{orig}} - A_{0,1}^{\text{actual}}(m,a)_{\text{old}} \\ A_{1,0}^{\text{ideal}}(m,a)_{\text{orig}} - A_{1,0}^{\text{actual}}(m,a)_{\text{old}} \end{bmatrix} \quad (4)$$

$$\text{where } \mathbf{A} = \begin{bmatrix} \frac{\partial A_{0,1}^{\text{actual}}}{\partial m} & \frac{\partial A_{0,1}^{\text{actual}}}{\partial a} \\ \frac{\partial A_{1,0}^{\text{actual}}}{\partial m} & \frac{\partial A_{1,0}^{\text{actual}}}{\partial a} \end{bmatrix}_{m=m_{\text{old}}, a=a_{\text{old}}} \quad (5)$$

and  $\mathbf{A}^{-1}$  indicates the matrix inverse. Secondly to make the substitutions

$$A_{0,1}^{\text{actual}}(m,a)_{\text{old}} \Rightarrow A_{0,1}^{\text{actual}}(m,a)_{\text{new}} ; A_{1,0}^{\text{actual}}(m,a)_{\text{old}} \Rightarrow A_{1,0}^{\text{actual}}(m,a)_{\text{new}} .$$

The iteration commences by setting  $m_{\text{old}} = m_{\text{orig}}$  and  $a_{\text{old}} = a_{\text{orig}}$ ; and terminates when (4) are satisfied.

## 4 APPLICATION TO ISAC RFQ

To use the new options in PARMTEQ, one must first decide upon a vane geometry. In general, for the same given  $A_{0,1}$  and  $A_{1,0}$ , different geometries will have different higher harmonic contributions  $A_{l,n}$ . Ideally, one should pick a geometry for which the high order terms make no perceptible change to the beam dynamics, but this is largely a matter of ‘educated guessing’.

### 4.1 Effect of perturbations

In the ISAC RFQ accelerator, the betatron and synchrotron tunes are small, and so velocity and transverse displacement are almost constant across a structure period ( $\beta\lambda$ ). Consequently, if we integrate the perturbing forces along one period of the unperturbed particle motion, then (provided coefficients do not change sign from cell to cell) any  $A_{l,n}$  with index  $l > 1$  will average to zero. Hence, the parts of  $\Phi_8$  which will affect the transverse motion are  $A_{0,3}$  and  $A_{1,2}$ ; and the only term to affect the longitudinal motion is  $A_{1,2}$ .

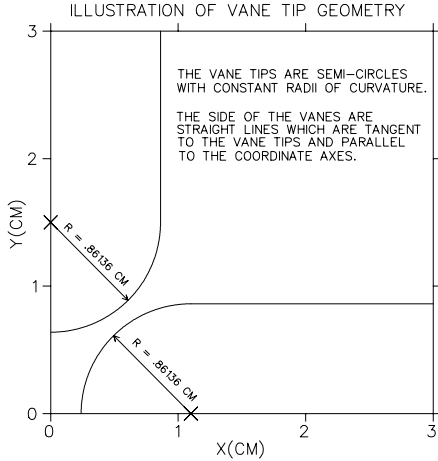


Figure 1: Vane geometry for RFQCOEF

These arguments (and careful analytical work on the effect of gradient errors[4]) gave enough confidence to apply the new MAP and RESMAP options to an RFQ with a CTPTR vane geometry, i.e. the simple and cheap geometry which had originally produced field errors of up to 40%. As shown in figure 1, the vanes are cut from parallel faced blanks and the transverse pole tip radius is equal to the characteristic bore radius; only a single quadrant is shown.

#### 4.2 Effect of MAP and RESMAP

After the MAP option, the two term potential function values of coefficients  $A_{0,1}$  and  $A_{1,0}$  are replaced by the eight term potential function values for the actual vane geometry. After the RESMAP option, the  $A_{0,1}$  and  $A_{1,0}$  coefficients of  $\Phi_8$  are brought into coincidence with the  $\Phi_2$  potential values. To effect this restoration, the vane profile parameters  $m$  and  $a$  are changed. Figure 2 shows the relative fractional changes between longitudinal electric field, and  $m$ ,  $a$  before and after the RESMAP option.

#### 4.3 Emittances and phase space

The transmission of the RFQ after RESMAP is 90.7%, which is identical with calculations using  $\Phi_2$ ; statistics based on tracking  $2 \times 10^3$  non-interacting particles. Due to radial emittance increases, however, not all of these ions are useful; and the RFQ should be followed by collimators.

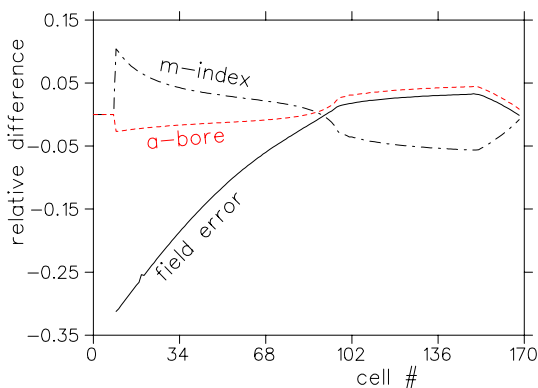


Figure 2: Comparison

**Transverse beam quality** Using the ideal  $\Phi_2$ , the transverse emittances (equal to 0.10 mm.mradian) are preserved in the RFQ. New calculations with  $\Phi_8$  were made after adjusting the vane profiles with RESMAP. The table below compares the ratio of normalized X-X' and Y-Y' emittances (as a function of % of ions) at entrance and exit. Evidently, the effect of the higher order terms in the potential is to produce a wide halo containing some 2% of the ions. However, the central core of particles is unperturbed.

	horizontal			vertical		
fraction	100%	95%	90%	100%	95%	90%
$\varepsilon^{\text{out}}/\varepsilon^{\text{in}}$	1.59	1.00	0.99	1.91	0.97	1.00

**Longitudinal beam quality** There is no difference between the longitudinal emittance when tracking with  $\Phi_2$  or  $\Phi_8$ . This is not surprising as all the emittance increase is due to the fundamental, sinusoidal non-linearity of longitudinal focusing. The table below gives the occupied areas in phase space (units of MeV.deg at 35 MHz). Note, the values given are 'per nucleus'; to get the values per nucleon, one must divide by the mass number, equal to 30.

Emittance fraction (longitudinal)			
r.m.s	100%	95%	90%
0.0063	0.254	0.0497	0.0360

## 5 CONCLUSION

The longitudinal field coefficient  $A_{1,0}$  can depart substantially from the values calculated from the usual formulae (2). We have presented a procedure, that may be applied to any realistic vane geometry, to recalculate the vane profile parameters  $m$  and  $a$ , so as to give the ideal field coefficients  $A_{0,1}$  and  $A_{1,0}$  at every RFQ cell. We have applied this technique to the ISAC RFQ design, assuming a constant transverse radius of curvature vane geometry, and performed particle tracking with  $\Phi_8$ . The losses, 99% longitudinal emittance and 96% transverse emittance contours are unchanged compared with the  $\Phi_2$  tracking.

To summarize, the most important effect of using a non-ideal geometry is the reduction of longitudinal electric field; the disturbance caused by non-linear high order terms in the potential function (associated with the non-ideal geometry) is negligible except for some halo formation.

## 6 REFERENCES

- [1] S. Koscielniak: *History of RFQ beam dynamics investigation with higher order potential function*; TRI-DN-95-31.
- [2] K.R. Crandall: RFQ beam dynamics design studies; Proc. of 1979 Linac Conf. Montauk N.Y. USA, BNL-51134.
- [3] N.J. Diserens: *RFQCOEF: a package for extracting the harmonic coefficients for the potential function in RFQ cell*; Proc. of 1984 Linac Conf. Darmstadt, Germany, GSI-84-11.
- [4] D. Kaltchev and S. Koscielniak: *Effect of gradient perturbations up on tune and beta-function of the TRIUMF ISAC RFQ accelerator*; TRI-DN-95-30.