IMPEDANCE OF RECTANGULAR SLOTS IN A ROUND COAXIAL TUBE

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Abstract

For the LHC liner the impedance of small slots has to be known with good precision. A computercode based on the mode matching technique is applied to get results valid over the whole frequency domain for the real and imaginary part of the impedance. For low frequencies the values calculated with this code are compared with the results from Bethe small hole approximation. Finally theoretical numbers and measurements are discussed.

1 CALCULATIONS

1.1 MODE MATCHING

Figure 1 shows a scetch of the treated liner structure. For an infinite thin inner wall containing the holes the impedance can be calculated by matching the fields of the spaces I (inner wave guide) and II (hole) as well as II and III (outer waveguide) in the area of the holes [1]. The coefficients of the fields are evaluated by integration in the complex plane using the residuum theorem.

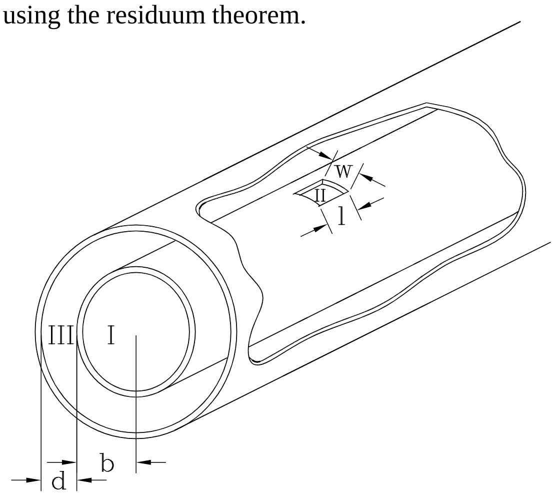

Figure 1: scetch of the liner structure

1.2 BETHE HOLE COUPLING

Using the well known Bethe small hole approximation the imaginary part of the impedance below cutoff is given as [2]:

$$Z(\omega) = \mathrm{j}Z_0 \frac{\omega}{c_0} \frac{\alpha_m + \alpha_e}{4\pi^2 b^2} \tag{1}$$

with $Z_0 = 377\Omega$, b the tube radius, c_0 the velocity of light, α_m und α_e the magnetic and electric polarizabilities and ω the angular frequency. For rectangular slots the sum of these are given in [2] as:

$$\alpha_m + \alpha_e = w^3(0.1814 - 0.0344\frac{w}{l}) \tag{2}$$

with w the width and l the length of the slot.

The real part of the impedance for a narrow slot can be calculated as [3]:

$$\Re\{Z(\omega)\} = Z_0 \frac{1}{(2\pi b)^2} \frac{(\alpha_e^2 + \alpha_m^2)\omega^4}{6\pi c_0^4} \quad . \tag{3}$$

with the polarizabilties valid for rectangular slots [2]:

$$\alpha_e = -\frac{\pi}{16} w^2 l \left(1 - 0.5663\frac{w}{l} + 0.1398\frac{w^2}{l^2}\right) \tag{4}$$

$$\alpha_m = \frac{\pi}{16} w^2 l \left(1 + 0.3577\frac{w}{l} - 0.0356\frac{w^2}{l^2}\right) \tag{5}$$

2 COMPARISON OF THEORETICAL RESULTS

Figure 2 shows the imaginary part of impedances for a slot with several widths w and lengths l at 1 GHz.

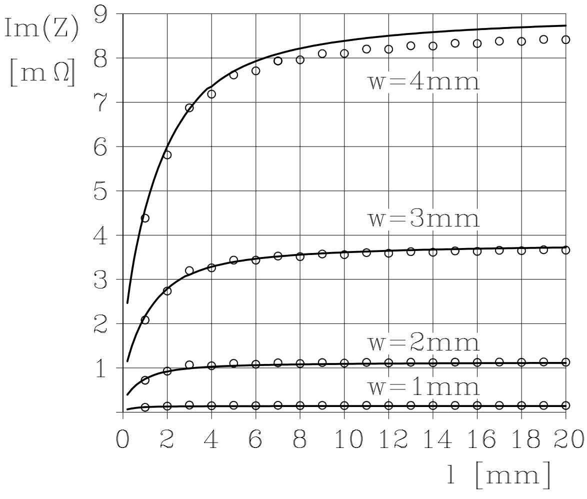

Figure 2: comparison of imaginary part of impedance for slots with the width w and the length l at 1 GHz calculated by Bethe hole coupling (solid line) and by mode matching (dots); b=16mm, d=5mm

One sees a good correspondence between the two methods for narrow as well as short slots. For increasing width and length the approximate formulas of Bethe are not longer valid.

Up to 16 GHz the imaginary parts of the impedances for a square hole with 4 mm edge length are compared. Bethe predicts a linear behaviour. Due to higher order modes the exact value increases for higher frequency as can be seen in figure 3.

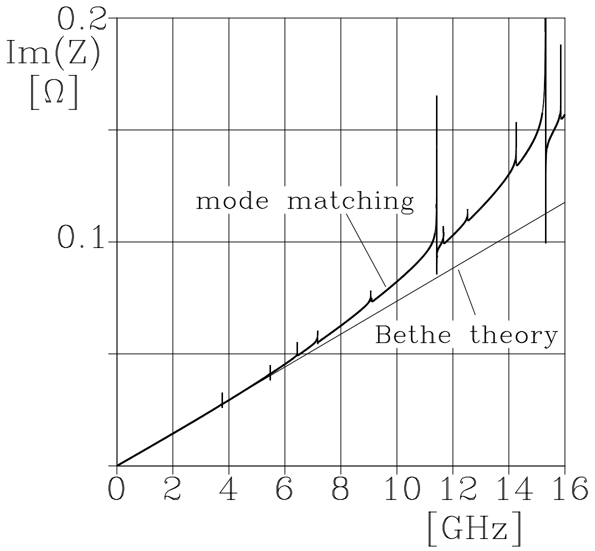

Figure 3: comparison of imaginary part of impedance for a square hole with 4 mm edge length versus frequency calculated by mode matching (thick line) and by Bethe hole coupling (thin line); b=16mm, d=5mm

In table 1 imaginary parts of impedance for quadratic holes are given for several edge lengths.

edge length	$\Im(Z)$		relative
	mode matching	Bethe	deviation
[mm]	[mΩ]	[mΩ]	[percent]
1.0	0.111	0.115	3.6
2.0	0.928	0.919	-1.0
3.0	3.198	3.103	-2.9
4.0	7.185	7.355	2.4
5.0	13.921	14.366	3.2
6.0	23.150	24.966	7.8
7.0	35.960	39.645	10.2
8.0	51.520	59.179	14.9
9.0	70.273	84.261	20.0
10.0	94.023	115.584	22.9
11.0	120.410	153.843	27.8
12.0	150.967	199.730	32.3

Table 1: imaginary part of impedance for quadratic holes with different edge length; b=16mm, d=5mm

For large holes Bethe theory looses validity.

In figure 4 the real part of the impedance at 1 GHz for the same slots as in figure 2 calculated with mode matching is depicted. The numbers calculated with Bethe are two orders smaller for these parameters.

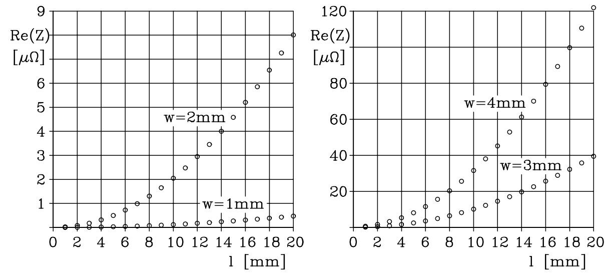

Figure 4: real part of impedance for slots with the width w and the length l at 1 GHz calculated by mode matching; b=16mm, d=5mm

For slots with greater length than width a nearly quadratic behaviour has been found as exspected from equation (3) to (5).

Figure 5 shows the real part of the impedance for the configuration of Figure 3. The calculation by Bethe theory is done without an outer wall.

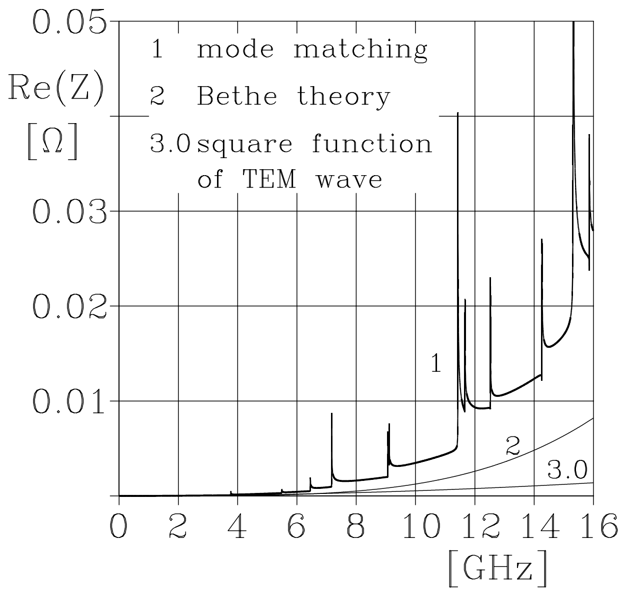

Figure 5: comparison of real part of impedance for a square hole with 4 mm edge length versus frequency calculated by mode matching (1); compared with the square behaviour for TEM wave (3.0); and by Bethe hole coupling (2); b=16mm, d=5mm

The same is done in Figure 6 for a smaller frequency range. As a result the real part of impedance versus the frequency in a coaxial line for each propagated mode is quadratic in ω in contrary to the ω^4 behaviour for the radiation of a hole into free space. This can easily be proved by using the reciprocity theorem [4] for TEM-mode as well as for waveguide modes. The influence of the factor $\omega/\sqrt{\omega^2-\omega_c^2}$ which is important close to cutoff ω_c of waveguide modes is neglected here. Curve (3.0) is the quadratic function for the TEM mode, (3.1) is the sum of the TEM and the first waveguide mode, in (3.2) also the second waveguide mode is included, in (3.3) the first 3 waveguide modes.

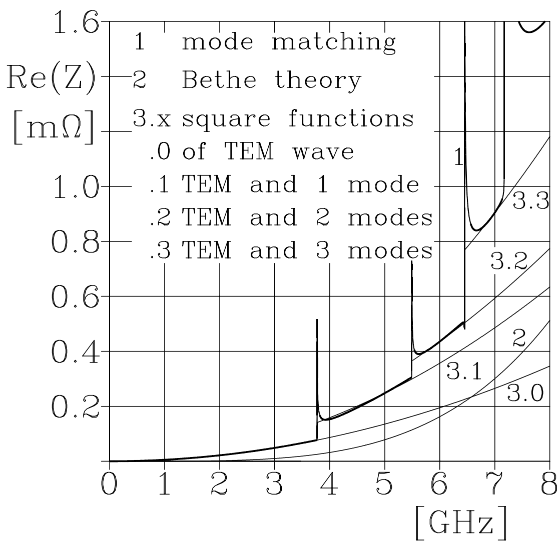

Figure 6: comparison of real part of impedance for a square hole with 4 mm edge length versus frequency calculated by mode matching (1); compared with the square behaviour (3.x), (3.0) including TEM wave, (3.1) including also 1 waveguide mode, (3.2) 2 modes, (3.3) 3 modes; and by Bethe hole coupling (3); b=16mm, d=5mm

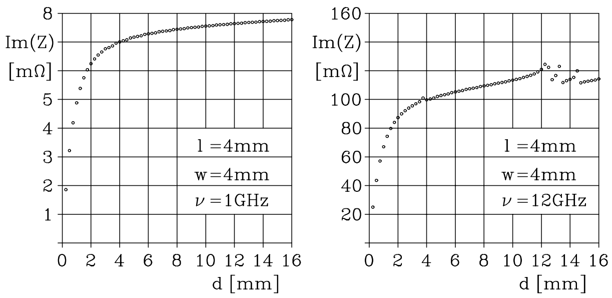

Figure 7: imaginary part of impedance versus the distance d between inner and outer conductor at 1 GHz (left) and 12 GHz (right); b=16mm

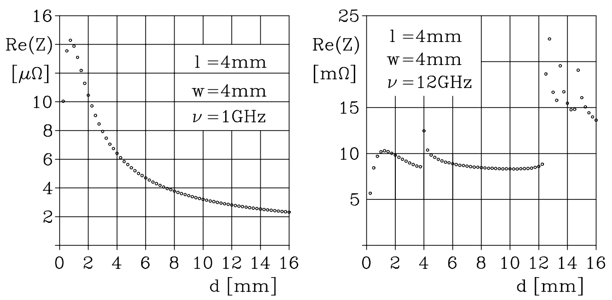

Figure 8: real part of impedance versus the distance d between inner and outer conductor at 1 GHz (left) and 12 GHz (right); b=16mm

Figure 7 and 8 shows the influence of the distance d between inner and outer conductor on the imaginary and real part of impedance at 1 GHz and 12 GHz. The imaginary impedance is asymptotic for large d. The variations at the higher frequencies mark the cutoff frequencies of higher modes. For a frequency below cutoff the real impedance decreases after a maximum for 1 mm. Above cutoff the influence of the higher modes is dominant.

3 MEASUREMENTS, RESONATOR METHOD

The most accurate method for measuring impedances below cutoff caused by small holes ist the resonator method [5] using an inner conductor centered in the liner tube. The frequencies of the resonances of this TEM-line are shifted due to the holes acting as small inductances an thus increasing the electrical length. This frequency shift for 200 holes, calculated from the imaginary impedances from Bethe theory and measured, are presented in figure 9.

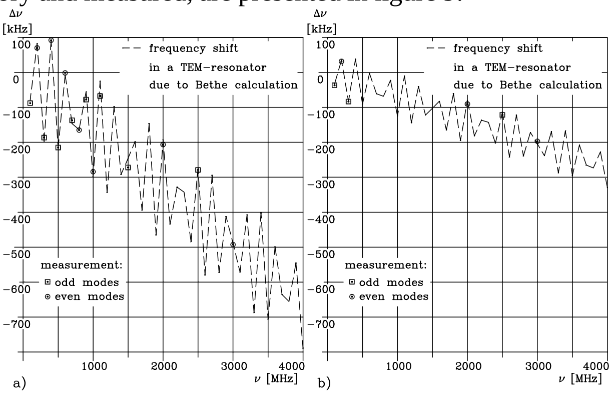

Figure 9: frequency shift versus resonance frequency of the TEM-line for 200 holes with a) 4mm diameter and b) 3mm diameter, calculation (dashed line) and measurement (points)

From the change of the Q-value without and with holes due to the radiation through the holes the real part of impedance could be calculated. But the measured effect here is to small for an exact calculation and only a tendency is seen.

4 ACKNOWLEDGEMENT

I would like to thank M.Filtz for helpful theoretical discussion, F.Caspers for valuable experimental hints and H.Henke for general support.

5 REFERENCES

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