# A 3-Dimensional Simulation of Collective Effects in Particle Accelerators

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## Abstract

Collective effects due to the impedance of structures along the vacuum chamber can limit the performance of accelerators and storage rings. In particular in LEP, transverse impedances are limiting single bunch currents in the vertical, but - to a somewhat lesser degree - also in the horizontal direction. The head-tail motion generally appears in one or the other direction. Previous simulations have taken into account only a single transverse dimension. This neglects possible effects in both transverse directions which could lead to coupling. The 3-D program is based on the simulation program TRISIM, which uses triangular basis functions to represent the particle distributions in 2 dimensions and has shown good agreement with observations in a single transverse plane. The results of the 3-D simulations will be compared with measurements on LEP.

#### **1 INTRODUCTION**

In the operation of the large electron-positron storage ring LEP, head-tail oscillations of the bunches are often visible on the streak camera. By splitting the synchrotron light coming from the beam, the display shows both the side view - i.e. motion in the vertical plane - and the top view, i.e. horizontal motion - at the same time. In addition to transverse dipole oscillations of the bunch as a whole, "head-tail" oscillations of the bunches can appear in the vertical or in the horizontal plane, mainly depending on current and chromaticity settings. With increasing current per bunch, the range of chromaticities where the bunches remain stable shrinks to a very small region, and it is sometimes even necessary to work with negative chromaticities and apply transverse feedback to stabilise the dipole mode.

# 2 TRANSVERSE MODE COUPLING

As the bunch currents increase during injection, the strongly detuned m = 0 mode approaches the frequency of the m = -1 mode and the "transverse mode coupling instability" (TMCI) sets in. It usually leads to loss of only a small part of the bunch current, and further injection can continue up to the same level. Nevertheless, it constitutes a major limitation of current and hence of luminosity in LEP. For a given machine impedance and optics, the threshold current is proportional to energy, which has been raised from 20 to 22 GeV last year, increases with longer bunch length, which is achieved by powering all installed wigglers during injection, and with the synchrotron tune which will be raised to 0.15 by using the additional RF voltage available with the installation of super-conducting cavities installed for higher energy operation of LEP2.

At higher currents also the head-tail oscillations have a finite dipole moment, and the m = 1 synchrotron sideband frequencies  $f_{\beta} \pm f_s$  are clearly visible on a spectrum analyser even in the absence of external excitation. Therefore also these modes are detuned by a reactive feedback system, and the TMCI threshold remains essentially unchanged by it. More complicated feedback systems using additional oscillator circuits have been proposed to circumvent this problem, but have not been very successful so far.



Figure 1: Average and rms values of particle coordinates in longitudinal, horizontal, and vertical directions as function of number of turns in LEP

With LEP operating at 45 GeV, TMCI appeared at about the same bunch currents as the beam-beam effect, and there was no strong incentive to increase them. However, for LEP2 operating at higher energies, the beam-beam effect will occur at correspondingly higher bunch currents, and hence it would thus be desirable to overcome the TMCI limitations.

In LEP, TMC occurs first in the vertical direction, where detuning of the betatron frequency with current is almost twice as large as in the horizontal one. This large ratio is somewhat surprising, as the biggest part of the transverse impedance in LEP comes from the RF cavities, which have circular symmetry and - on average - equal beta function, hence their contribution to the tune shifts in both transverse planes should be the same. The contributions from numerous small, not-rotationally symmetric obstacles in the oval vacuum chamber - such as shielded bellows, flange gaps, wall resistance etc. - are considerably weaker, in particular for bunches which are lengthened by wigglers needed to reach higher currents. However, asymmetric objects often cause quadrupolar wake fields, which actually reduce the detuning in the horizontal plane, while adding to the vertical one, which can explain their large ratio.

### **3 CROSS-COUPLING IMPEDANCES**

The compensation of betatron coupling by the experimental solenoids in the interaction regions of LEP is done with sets of skew quadrupoles which are located in the RF sections. The current dependent wake field kicks of the RF cavities will change the betatron phase at these quads, and thereby influence the compensation.

In addition to this effect, non-symmetric objects may also cause cross terms which couple horizontal displacement to vertical impedance and vice versa. These "cross coupling terms" of most impedances are usually quite small, but could have increasing importance at the high bunch currents desired for LEP2. Therefore it was thus considered desirable to extend our existing simulation programs to all 3 spatial dimensions, in order to be able to include coupling of the motion in both transverse planes.

# **4 SIMULATION PROGRAM TRISIM**

The computer program TRISIM[1] simulates particle motion in the longitudinal and in one of the transverse planes, which is usually assumed to be the more sensitive vertical one. Collective single bunch effects in LEP have been investigated in detail, and the results are generally in quite good agreement with experimental observations. The code works with a sufficiently large number of super-particles, typically a few thousand, whose distribution is approximated by a superposition of a number of shifted, equilateral triangular base functions, typically 20 - 50. The wake potentials of all significant impedances in LEP have been pre-computed for such triangular distributions with the 2-D wake field code ABCI[2]. The values are stored in tables which can be used to obtain the wake potential of an arbitrary distribution in a fast and efficient way. For transverse effects, the super-particle charges are multiplied by their offsets from the axis, and the resulting dipole distribution is ex-



Figure 2: Horizontal and vertical loss factors (V/nC) in shielded LEP bellows as function of offset.

panded in a similar fashion into triangular base functions.

The evolution of the particle motion is followed for a few radiation damping times, which is about 6000 turns for LEP at injection energy. The average bunch positions and rms values are displayed as function of time, as well as their Fourier transforms, in graphical form similar to that available in the LEP control room. One can thus observe bunch deformations, displacements, tune shifts, or TMC thresholds, generally in excellent agreement with measurements in LEP Furthermore, it was possible to study various methods to increase the threshold currents such as bunch lengthening, operation with high synchrotron tunes, or reactive feedback systems, and to optimise parameters, thereby saving precious machine time[3].

#### 5 PROGRAM TRISIM-3D

In order to extend this computer program to handle particle motion in all three spatial dimensions (TRISIM-3D), the equations of motion were re-written in matrix formulation, thereby allowing coupling between horizontal and vertical planes. An expansion of the distribution of first moments in the second plane was added. Also the graphical output was changed to show both transverse planes side by side (see Figure 1).

# 5.1 Equations of motion

The longitudinal motion is described by the energy deviation  $\epsilon$  and time delay t with respect to a synchronous particle. For the transverse motion the horizontal and vertical positions and slopes x', y' are represented by the 4dimensional vector  $\mathbf{z} = \{x, x', y, y'\}$ . The interaction of the particles with the i-th impedance element can be written:

$$\begin{aligned} \mathbf{z}_{i+1} &= \mathbf{M}_{w,i} \cdot \mathbf{z}_i \\ \epsilon_{i+1} &= \epsilon_i + \delta \epsilon_i + e V_i^{RF} \sin(\Phi_i^s + \omega_i^{RF} t_i) \end{aligned}$$

where  $\Phi_i^s$  is the phase of the synchronous particle with respect to the RF wave, and the wake-field matrix  $\mathbf{M}_w$  is given by:

$$\mathbf{M}_w = egin{pmatrix} 1 & 0 & 0 & 0 \ w_{x\,x} & 1 & w_{x\,y} & 0 \ 0 & 0 & 1 & 0 \ w_{y\,x} & 0 & w_{y\,y} & 1 \end{pmatrix}$$

where  $w_{xx}$  and  $w_{yy}$  are the usual horizontal and vertical wake potentials, and  $w_{xy}$  and  $w_{yx}$  linear coupling terms which vanish for cylindrically symmetric structures. For a rectangular model of the LEP shielded bellows the horizontal and vertical wake potentials were calculated with MAFIA for different offsets in both transverse directions.

A major part of the work was to identify impedances in LEP which contribute significantly to the off-diagonal elements of the impedance matrix, and to compute their values with 3-D wake field codes such as MAFIA[4]. The most likely sources of such coupling elements are asymmetric structures such as the shielded bellows or slotted inserts for the pumping-T's, which have obstacles in or near the 45 degree plane. The cross-section changes are usually quite small and reasonably exact computation of their wake potentials is rather difficult. However, since there is often a large number of such elements, their effects can not be neglected. First estimates of non-zero, off-diagonal coupling terms of shielded bellows and pumping slots were obtained and have been used in the program TRISIM-3D, together with fixed coupling by solenoids and skew quadrupoles. In Fig.2 we show the corresponding loss factors for a Gaussian bunch of rms length  $\sigma = 10$  mm.

## 5.2 Results

Some preliminary results obtained with this program are shown, but a more systematic study of such wake-field coupling effects is still in progress.

The bunch positions and rms dimensions in all 3 spatial directions are shown in Fig.(1) as function of number of turns. The influence of this coupling can be seen more clearly in the spectra of the horizontal and vertical bunch oscillations which are shown in Figs.(3) and (3, without and with wake field coupling.

Furthermore, we are studying the influence of wake potentials of the (axially symmetric) RF cavities on the coupling compensation of the solenoids with skew quadrupoles which is induced mainly by the current dependent tune change.

#### **6** CONCLUSIONS

Coupling between horizontal and vertical motion is influenced by off-axis terms in the wake potential matrix, caused



Figure 3: Spectra of the horizontal and vertical bunch oscillations without and with wake field coupling.

by axially non-symmetric elements such as shielded bellows or screens for the pumping-T's with many slots in LEP. Such elements can cause a negative loss factor in the horizontal direction when they are wider than high, which subtract from the always positive contributions of the axially symmetric RF cavities. This may also explain why horizontal betatron tune shifts are much smaller than vertical ones in LEP, although the main contribution comes from the axially symmetric RF cavities. By introducing properly designed elements one could thus actually reduce the loss factors in the vertical direction, and thereby increase the threshold current for the vertical transverse mode coupling instability, which is the main limitation of single bunch current in LEP2.

# 7 REFERENCES

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