# Numerical Simulation of a Gridded Inductive Output Amplifier

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*Abstract*

A set of codes for the numerical simulation of a gridded Inductive Output Amplifier has been developed in close collaboration between BINP and Philips RHW. The following problems are investigated: nonstationary gridded gun simulation, self-consistent solution for beam-cavity interaction and passband calculations. The Finite Element Method is used for the field calculation in the electron gun. Space charge forces are taken into account by the long-wave approach. A special method is used to obtain a self-consistent solution for the beam-cavity interaction with a reduced number of iterations.

## 1 IOA MODEL

The Inductive Output Amplifier ( IOA ) is a gridded RF device in which electron beam modulation is achieved by applying a RF voltage to the grid of the electron gun. The electron bunches are accelerated by the anode voltage of the gun and pass through the RF cavity gap interacting with the cavity fields. The output cavity of IOA consists of two coupled cavities to extend the passband. The electron gun of this device is axisymmetric. The design of the gun grid is such that it may be described by set of axisymmetric rings. The region of the cavity, in which the electron beam interacts with the RF fields is axisymmetric too. So, for the simulation of the IOA a set of 2D, axisymmetric codes was developed and used.

There are several steps in the device simulation:

1. Calculation of non-stationary electron emission in the gridded gun.
2. Calculation of the beam dynamics in the gun and of the beam-cavity interaction in the output cavity.
3. Obtaining a self-consistent solution together with the frequency characteristic of the divice.

## 2 GRIDDED GUN MODEL

For the numerical simulation of the electron emission in the gridded gun with applied RF voltage the PIC method is used. The macroparticles are described by the infinite thin rings with uniform charge distribution. The length of the rings is defined by the time step. We suppose that the wave length of the RF oscillation is much greater than the device and gun size. So, we can neglect wave effects on the beam dynamics and calculate the influence of space charge by using Poisson's equation, because the beam is not relativistic:

$$\Delta U\ (t) = -\frac{\rho(t)}{\varepsilon_0},$$

where $\rho\ (\ t\ )$ and $U\ (\ t\ )$ are the time depending space charge distribution and potential, respectively.

The Poisson equation is solved by FEM as in SSAM[1] for each time step. Because the system matrix does not depend on time, it is calculated and transformed to LU form only once. So, Poisson's equation is solved quite fast. For bunch simulation the RF periode is devided into N time intervals and in the middle of each interval one layer of macroparticles is started. The Poisson equation is solved once for each time interval. The charge of each macroparticle is defined by the current density on the cathode and the length of the time interval. The current density on the cathode is calculated by Child’s law. In order to use this law a small gap has to be introduced between emitting surface and starting surface of the particles. To avoid transit time effects this gap has to be small enough.

On the electrodes the dc and RF voltages are set:

$$U\ (t) = U_d + U_a \cos(\omega t)\ .$$

Furthermore, each time interval is divided into M steps, for a better description of the macroparticle movement. For the integration of the equations of motion the second order Runge-Kutta method is used. Macroparticles move in the fields from previous time step. Then the charge distribution is calculated and the Poisson equation is solved again to obtain the fields for the following time step.

To describe the particle dynamics within the cathode-grid region correctly a large number of macroparticles is needed. The data describing the motion of the particles has to be passed to a seperate code module which calculates the beam-cavity interaction. Because of the large number of particles this procedure is inexpedient, as this program must be started several times in order to obtaining of self-consistent solution and the time of calculation is an important topic. Therefore the following procedure is used to pass the relevant information from one program unit to the other. On a defined surface, usually the grid surface towards the anode is taken, the distribution of current density and speed of particles in dependence on time and longitudinal coordinate along this surface is calculated. Then these distributions are smoothed in time and coordinate. The noise in time is connected with numerical noise because of the approximation of continuous flow by macroparticles.

The results of the program SMASE for a plane diode with applied RF voltage was compared with a semi-analytical model [2]. Also the calculations of the gridded gun in static regime were made. The calculation results were compared with results obtained by SSAM and the difference in calculated current were less than 1%.

In Figure 1 the finite element mesh near the cathode is shown. In Figure 2 the calculation of a gridded gun is represented .

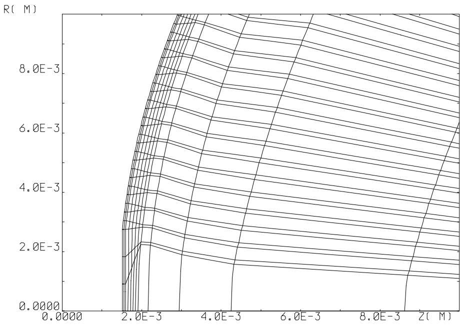

Figure: 1 Finite element mesh near cathode-grid region.

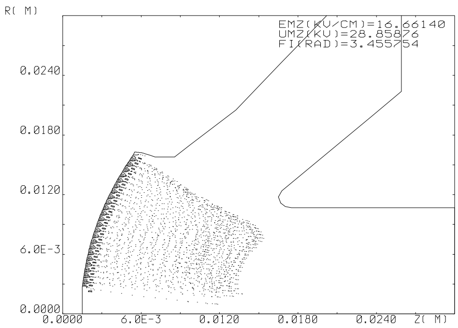

Figure:2 Example of calculation by SMASE code.

## 3 SIMULATION OF THE BEAM-CAVITY INTERACTION

For the calculation of the beam dynamics in the gun and cavity region the same assumption as in the case of emission calculation for cathode-grid region is made: the wave length of RF oscillation is much more than device size. Therefore we take into account the eigen field of bunch by solving the Poisson equation by finite element method, as was described above. The bunch is simulated by macroparticles, as well as in case of emission calculation.

The particles are emitted from the starting surface on which the distribution of current density and the initial energy of particles have been calculated before. These distributions are passed from the programs which calculates the emission (SMASE).

The focusing magnetic field can be determined on axis and then being expanded to the points of the finite element mesh in paraxial approximation, or it can be calculated directly in the points of the finite element mesh by program SAM [3]. The distribution of RF fields in the area of the cavity is calculated by program SUPERLANS [4]. The amplitude of the RF field is considered to be constant and not varying with time. The shift of phase between RF oscillation and the beginning of the bunch emission is also considered to be constant. The amplitude of the RF field is determined by the cavity voltage. As the time of emission takes only a part of the RF period, only this part is divided into N intervals on each of which a layer of particles is started from the starting surface.

If the device works in regime C, it is usually enough to start one bunch, because bunches do not interact with each other. For regime A or B a minimum of 3 bunches is started, and the relevant data is calculated for the middle one which feels the forces of the preceding and the following bunch.

While the bunch passes through the cavity the real and imaginary parts of the integral of beam-cavity interaction are calculated:

$$R^* = \mathrm{Re}\left(\int \vec{j}\vec{E}_{rf}\,dV\right) = \sum_i \int Q_i \vec{E}_{rf}(t)\cos(\omega t - \varphi)\,dl_i$$

$$I^* = \mathrm{Im}\left(\int \vec{j}\vec{E}_{rf}\,dV\right) = \sum_i \int Q_i \vec{E}_{rf}(t)\sin(\omega t - \varphi)\,dl_i$$

Where the summation is made over all macroparticles. To obtain optimal electron efficiency one should define the cavity voltage and shift of phase between bunch and RF field. Knowing the integrals of interaction, cavities voltage and characteristic impedance, it is possible to obtain the necessary loaded Q-factor and detuning of output cavity in case of single cavity .

$$Q = \frac{U^2 T}{2\rho R^*}, \quad \frac{\delta\omega}{\omega} = \frac{\rho I^*}{2U^2}.$$

Figure 3 and 4 illustrate SMASON work. In these figures the bunch is shown for two different phases. The transverse and longitudinal phase space distribution is shown at the right part of pictures.

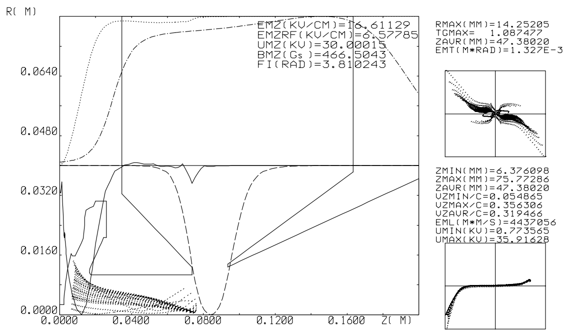

Figure: 3 SMASON calculation: bunch in front of cavity.

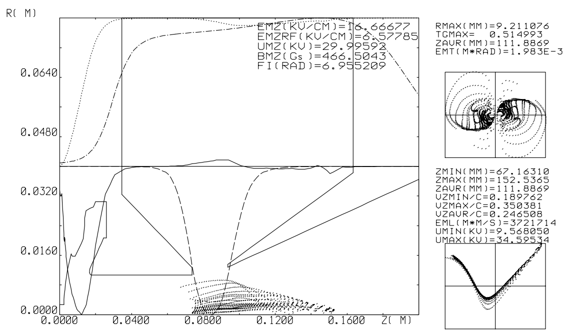

Figure: 4 SMASON calculation: bunch in the cavity gap.

## 4 CALCULATION OF IOA PASSBAND

To extend the bandwidth of the device the output resonator consists of the two coupled cavities, so it is necessary to take this into account during passband calculation.

For the description of the IOA output cavity the single mode model of two coupled cavities is used. In this model it is assumed, that the bunch interacts with the field of the first cavity, and RF power is extracted from the second.

Input parameters for this model are the following: eigen frequencies, Q-factors and coupling coefficients (which in general case are 4) of the cavities and the characteristic impedance of the first cavity.

In this model the input parameters of the cavities are selected in order to obtain wide band impedance of the first cavity, which is seen by the beam. In principle, if the problem is linear, it is possible to calculate the output power, knowing the harmonic of beam current at the given frequency. It is possible to obtain the cavity voltage from the power, which is given to the RF field by the beam, for fixed voltage value. But because the problem of output power calculation is non-linear (harmonic of current depends on cavity voltage), the following procedure is applied to calculate the dependence of output power on frequency.

The value of impedance is fixed for the center frequency of the passband and for this impedance the self-consistent problem is solved: the initial cavity voltage and phase shift are set, then SMASON is started and the impedance is calculated for this values of voltage and phase shift, then in linear approximation the new voltage and phase shift values are determined. With these new values SMASON is again started and the impedance is again calculated, then all is repeated. This method produces a self-consistent solution for the problem of beam-cavity interaction with a small number of iterations.

Furthermore, we make the assumption, that the beam characteristics do not vary within the frequency band of the device: the phase duration and starting distribution of current and speed of particles are preserved in dependence on phase. Also it is assumed, that the distribution of the RF fields in the cavity does not vary. So during SMASON calculation for other values of frequency the distribution of current density, speed and RF fields are taken such as for the center frequency. Points within the frequency band are calculated consistently to the left and to the right from the central point by the iterative procedure, described above. As the initial value, the calculated values of voltage and phase from the previous point are taken. This procedure permits to obtain the self-consistent solution for all points of the frequency band very fast.

In Figure 5 and 6 the dependence of impedance and output power on frequency obtained by code KOP are shown.

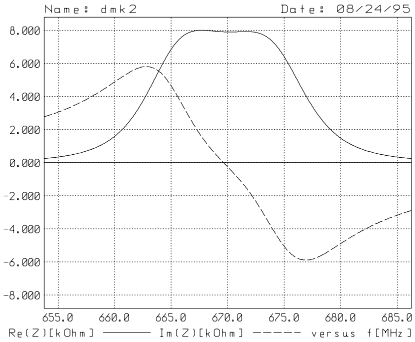


Figure: 5 Impedance of coupled cavities calculated by KOP.

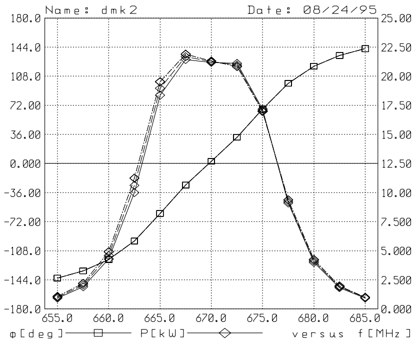


Figure: 6 Calculation of passband by KOP.

## 5 CONCLUSION

A model and set of codes has been developed to simulate the nonstationary behavior of a gridded gun and beam-cavity interaction in the IOA. This set of codes can be used for the design of RF generators, amplifiers and RF guns for accelerators.

## REFERENCES


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