HALOS AND CHAOS IN SPACE-CHARGE DOMINATED BEAMS

J-M. LAGNIEL

Commissariat à l'Energie Atomique
DSM-GECA, CEA-Saclay, LNS
91191 Gif-sur-Yvette Cedex, France

Abstract

For the high-current ion accelerators which are needed as drivers to produce high neutron flux in numerous new projects, beam losses must be extremely low in order to avoid an unacceptable radioactivity level in the machine area. The mechanisms leading to the formation of a diffuse halo around the beam core are then extensively studied. The first aim of this paper is to summarize the main results obtained from both analytical studies and numerical simulations related to emittance growth and halo formation in continuous and periodic focusing channels. The nonlinear resonances and the chaotic particle trajectories which are induced by the space-charge forces are analysed. Finally, some preliminary results concerning the longitudinal motion are presented.

I- INTRODUCTION.

Particle accelerators able to deliver high-power beams are presently proposed as drivers for numerous applications which require the production of high neutron flux. High-power proton beams (40...200 mA CW, 40...200 MW) are needed for applications such as transmutation of radioactive waste, energy production and tritium production. The new generation of pulsed spallation sources is based on ~ 5 MW H^- beams with peak current greater than 100 mA. IFMIF (International Fusion Materials Irradiation Facility) needs two 125 mA CW deuteron beams accelerated up to 35 - 40 MeV.

For these high-intensity accelerators, the most important aim is to keep beam losses along the structure below an extremely low threshold in order to limit the radioactivity in the machine area. The maximum tolerable losses for proton beams range from ~200 nA/m at 10 MeV to less than 1 nA/m at 1 GeV (table 1). Dose rates at 30 cm from copper and niobium 35 MeV deuteron accelerators have been estimated for a constant 1 nA/m loss. The dose is ~3.5 mrem/hr 8 hours after shutdown and it is still ~2.5 mrem/hr 24 hours after [3]. Then, for the new high-power machines, relative losses $\Delta I/I$ in the range ~10^{-6} /m to ~10^{-9}/m must be achieved.

Due to this very low fraction of the total beam which must not reach large amplitudes, more and more accelerator physicists have extended their works towards what is called ″halo formation″ studies. Actually, both numerical simulations and measurements of high-current beams show the formation of a diffuse ″halo″ of particles surrounding the central beam core. In low duty factor linacs used as synchrotron injector, rms-emittance growths must be limited to achieve a high injection efficiency. The evolution of the core is then the major concern. For high-power machines, particles with large amplitudes can be lost. Halo formation due to space-charge forces and imperfections along the accelerator is then the most important effect which can limit the machine performances.

Energy (MeV)	tolerable loss (nA/m)	relative level ($\Delta I/I$ /m) for I = 100mA
10	200	2 10^{-6}
20	15	1.5 10^{-7}
50	2.5	2.5 10^{-8}
100	1	1 10^{-8}
200	.2	2 10^{-9}
500	.05	5 10^{-10}
1000	.03	3 10^{-10}

Table 1 : Proton losses producing a gamma dose of 2.8 mrem/hr at 1 m from the machine one hour after shutdown (rough values from ref. [1] & [2]).

Emittance growth and halo formation induced by space charge could be due to different physics, or could be two different manifestations of the same physics. This will be analysed in the three following sections for unbunched beams evolving in continuous or periodic focusing channels. It must be pointed out that these studies concern only beams which are *unneutralysed* and *collisionless* (″Liouvillean beams″). For a recent study taking into account non-Liouvillean effects (intra-beam scattering or any other Markov process), see ref. [4].

II- EMITTANCE GROWTH.

Many theoretical and numerical studies have been done to investigate emittance growth. The brief summary presented here is based on two key papers by I. Hofmann [5] and T.P. Wangler [6]. In the latter, four distinct emittance growth mechanisms are described :

- *rms-mismatch mechanism* (see [5][6] and ref. therein). The betatron frequency being amplitude dependent for nonlinear space-charge force, a mismatched distribution evolves towards a filamented pattern in phase space. The equilibrium corresponds to an *internally matched* state for which the distribution isodensity contour coincides with the particle phase-space trajectories. An emittance growth formula has been derived considering that the free energy available in the rms-mismatched beam is transformed to ″thermal″ energy.

- The *charge-redistribution mechanism* affects beams which are rms-matched but internally mismatched (see [5][6] and ref. therein). The rms-emittance growth results from internal plasma oscillations which drive the nonuniform initial density towards an internally matched charge density with a central uniform core and a finite thickness boundary. Again, the emittance growth can be estimated from the conversion of nonlinear field energy to thermal energy. Collective interaction is considered to be the source of instabilities. Both analytic theory and computer simulation concerning these *coherent space-charge effects* are reviewed in ref. [5].

- *energy-transfer mechanism.* An observation of emittance transfer between two degrees of freedom led P. Lapostolle to the suggestion of equipartitioning. While the collisionless character of the beam precludes any thermodynanical behaviour, this effect was first demonstrated by M. Promé in computer simulation studies [7]. The space-charge force couples the longitudinal and transverse motions and drives the beam towards an equipartitioned state associated with a ″hyperemittance″ growth. This mechanism of *charge redistribution* between different degrees of freedom has been extensively studied after that by I. Hofmann, R.A. Jameson, T.P. Wangler, M. Reiser and many others (see [5] [6] and ref. therein). Hofmann found that *collective instabilities* can be excited and lead to equipartitioning. He defined the instability thresholds which can be used to avoid energy transfer by a proper choice of the accelerator parameters. Nevertheless, to keep the beam equipartitionned is still considered as a fundamental condition for an accelerator design [8].

- *structure-resonance mechanism.* Several studies have shown that periodic structures induce envelope growth and excite *coherent modes* in a nonuniform density beam. It is recommended to design the accelerator with zero-current phase advances per focusing period below $\sigma_{0t} = 90^o$ in order to avoid these instabilities.

III- NONLINEAR RESONANCES.

Two papers presented at PAC93 [9][10] pointed out that the interactions of single particles with an oscillating beam core can be resonant. To study this source of halo formation which can be called *incoherent space-charge effects* by analogy with the incoherent beam-beam effect, we must first analyse the resonances which can be excited (see [11] and ref. therein).

The envelope equations which give the beam radius evolutions in the horizontal (a) and vertical (b) planes can be written in smooth approximation :

$$\frac{d^2a}{ds^2} + \sigma_{ot}^2 a - \frac{\varepsilon^2}{a^3} - \frac{2K}{a+b} = 0$$

$$\frac{d^2b}{ds^2} + \sigma_{ot}^2 b - \frac{\varepsilon^2}{b^3} - \frac{2K}{a+b} = 0$$

where K is the generalised perveance and $\varepsilon = \varepsilon_x = \varepsilon_y$ is the total emittance. For weak mismatches, these coupled equations can be linearized and, following Hofmann [5], the two eigen modes (even and odd) of the envelope oscillations can be calculated. They are characterised by

$$\sigma_e = \sqrt{2(\sigma_{0t}{}^2 + \sigma_t{}^2)} \qquad \sigma_o = \sqrt{\sigma_{0t}{}^2 + 3\sigma_t{}^2} \quad (1)$$

where σ_t is the betatron phase advance with space charge. The even and odd modes are also called ″breathing″ and ″quadrupolar″ modes. The envelope oscillations are function of the focusing system :

- for a *continuous focusing channel*, these two modes can be excited only if the beam is mismatched.

- for a *periodic solenoidal channel*, the breathing mode is ″intrinsically excited″. The period of the intrinsic envelope oscillation for a matched beam is obviously the focusing channel period ($\sigma_i = 2\pi$). For a mismatched beam, this intrinsic oscillation is still present and, in addition, the two eigen modes σ_e and σ_o can be excited.

- for a *FODO channel*, the intrinsic quadrupolar mode ($\sigma_i = 2\pi$) is permanently excited and the two eigen modes are added when the beam is mismatched. A numerical integration of the envelope equations without smooth approximation (figure 1) shows that the amplitudes of the odd and even modes are already large compared to the one of the intrinsic mode for a weak mismatch (10%) [11]. The frequencies obtained by FFT are very close to those calculated using (1).

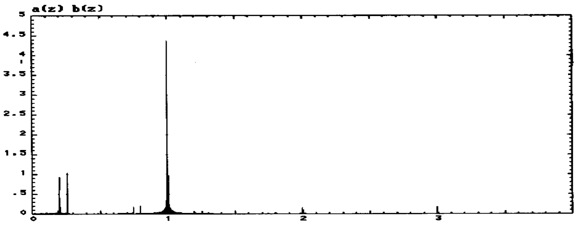

Figure 1 : Fourier spectrum of a mismatched beam envelope in a FODO channel. The intrinsic quadrupolar mode frequency is f = 1, the two peaks at low frequency are the two eigen modes.

Incoherent space-charge effects can then be ″excited″ even if the beam is matched. Beam mismatching is an additional source of excitation but is not a *sine qua non* condition for emittance growth and halo formation.

Beam core oscillations (due to the focusing channel periodicity and/or to a mismatch) induce an oscillation of the space-charge forces. Resonant interactions between the particle motion and these oscillating forces can be characterized by the space-charge tune :

$$\nu = \sigma_{particle} / \sigma_{core} \quad \text{with} \quad \sigma_{core} = \sigma_i, \sigma_e \text{ or } \sigma_o.$$

The minimum value of $\sigma_{particle}$ is σ_t when σ_t is defined as the phase advance near the axis of monotonically-decreasing or uniform distributions. Its maximum value is σ_{ot} because particles travelling at large amplitudes are weakly influenced by the space-charge force. Then, the resonances excited by core oscillations are in the range :

$$\sigma_t / \sigma_{i,e,o} \le \nu_{i,e,o} < \sigma_{0t} / \sigma_{i,e,o}$$

The range of intrinsic resonances excited by a matched beam in a periodic focusing channel is then given by :

$$\sigma_t / 2\pi \le \nu_i < \sigma_{0t} / 2\pi$$

and the ranges of the additional resonances excited by a mismatched beam are such that :

$$\eta / \sqrt{2(\eta^2+1)} \le \nu_e < 1/\sqrt{2(\eta^2+1)} \quad \text{(even mode)}$$

$$\eta / \sqrt{3\eta^2+1} \le \nu_o < 1/\sqrt{3\eta^2+1} \quad \text{(odd mode)}$$

where $\eta = \sigma_t / \sigma_{0t}$ is the tune depression [11].

The choice of $\sigma_{ot} < 90^o$ made to avoid instabilities induced by the *structure-resonance mechanism* means that the low-order intrinsic resonances ν_i = 1/2, 1/3 and 1/4 are avoided. Figure 2 shows that the strong $\nu_{e,o}$ = 1/2 resonances are always excited by a mismatch, but that $\nu_{e,o}$ = 1/4 is not present for $\eta > 0.4$. This threshold which is considered as a space-charge limit to avoid emittance growth [5][6] can then be explained by this analysis.

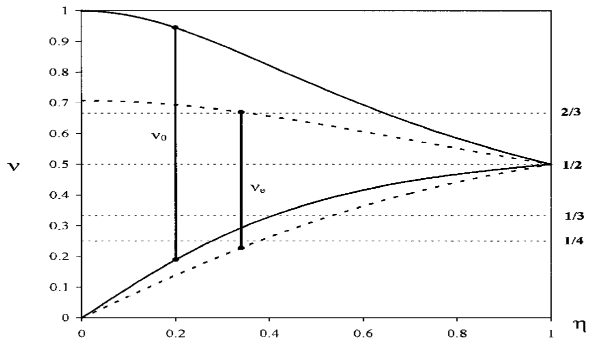

Figure 2 : Resonances excited by a mismatch versus η. Each pair of curves defines the range of resonances (ν = 2/3...) excited by the even (ν_e) and odd (ν_o) modes.

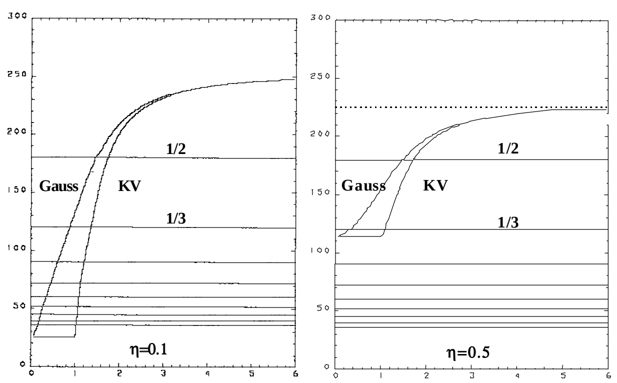

Figure 3 : Even mode resonances vs particle amplitude for η= 0.1 (left) and η= 0.5 (right) in KV and Gaussian distributions with R and R_{rms} = 1.

Figure 3 shows the positions of the even mode resonances as a function of the particle amplitudes for both uniform (KV) and Gaussian beam distributions [11]. Obviously, incoherent nonlinear resonances can be excited into the beam core of a nonuniform distribution. In this case, they can drive the nonuniform beam core towards an uniform distribution, even if the beam is not strongly ″space-charge dominated″.

For a high-intensity beam accelerated by a linac (at relatively low energies), the tune spread induced by space charge can be large. In an accelerator designed for energy production and nuclear waste treatment, η rises as {0.4, 0.62, 0.91} at {20, 200 1600 MeV} in the transverse plane and falls as {0.5, 0.24, 0.08} in the longitudinal plane for the same energies [12]. The space charge is weaker in the high-energy part of ESS linac (W > 100 MeV) where the tune depressions are greater than 0.7 [13]. Nevertheless, it seems difficult to avoid the *incoherent space-charge resonances*, at least in the low energy part of the accelerator (W < 50 MeV).

IV- HALOS AND CHAOS.

In order to get a meaningful insight of the physics underlaying any *complex system*, models must be built getting rid of details of the real system by seeking some hierarchy in the physical processes involved. Simplifying assumptions are essential to understand the system dynamic properties, an understanding which is impossible looking at computer simulations done with a large number of particles. Following this idea, the Particle-Core Model (PCM) [9] is the best tool to study the *incoherent space-charge effects*. This is a two-step method : the *beam core* envelope evolution is first computed, then, the behaviour of *test particles* injected into or around the beam core is analysed.

The very first observation of chaotic particle trajectories (see [11]) has been done using the PCM for space-charge dominated beams in *continuous focusing channels*.

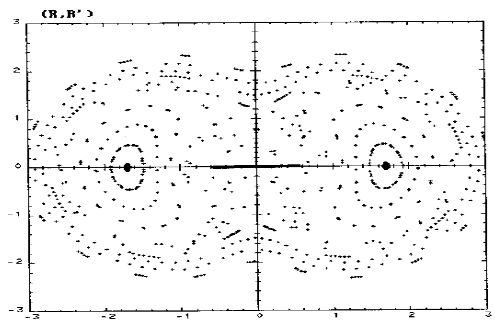

Figure 4 : Poincaré surface of section for η = 0 [11]. The two ν_e = 1/2 islands are surrounded by "KAM tori" which limit the chaotic area.

This chaotic behaviour induced by the *resonance overlap mechanism* has been clearly observed using the Poincaré surface of section technique (figure 4). These analyses (see [11]) have been confirmed for different values of the tune depression and for a nonuniform distribution [14]. Analytic models for halo formation which reproduce the main features seen in these simulations have been developped by R.L. Gluckstern [15], S.Y. Lee and A. Riabko, D.L. Bruhwiler and others. These results have been also confirmed by many self-consistent numerical simulations done using multiparticle PIC codes (R. Ryne, C. Chen, A. Piquemal...). For an overview not restricted to continuous focusing channels, see the contributions to the 8th ICFA advanced beam dynamic workshop on space charge dominated beams and applications of high brightness beams, Bloomington, USA, Oct. 1995.

The PCM has also been used to study the behaviour of matched beams in a FODO channel [11]. The Poincaré surface of section technique is used again to analyse the phase space topology. Figure 5 clearly shows the position of some intrinsic resonances in the (x,x') phase plane for uncoupled particles (y = y' = 0).

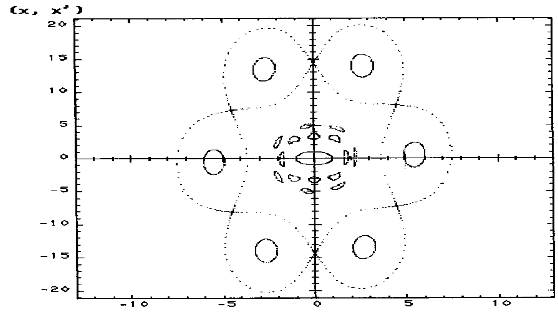

Figure 5 : Poincaré surface of section for σ_{0t} = 62° and σ_t = 20°.

Taking into account the coupling force induced by space charge leads to the analysis of a nonautonomous system with N = 2.5 degrees of freedom {(x,x')+(y,y')+z}. In this case, resonances form a dense "Arnol'd web" into which particles can diffuse (see [11]). This phenomena which is a source of energy transfer between different degrees of freedom could explain the equipartition thresholds defined by I. Hofmann.

For mismatched beams in a FODO channel, it has been shown that the additional incoherent resonances excited by the even and odd modes can induce a large chaotic zone around the beam core. This chaotic sea is induced by resonance overlap of the $\nu_{e,o}$ = 1/2 resonance with the low order intrinsic resonances which can be observed in figure 5. Nevertheless, the ν_i = 1/6 resonance which is localized at large amplitude is not very affected by a weak mismatch. For a strong mismatch, C. Chen and R.C. Davidson have shown that the envelope oscillation itself becomes chaotic [16]. In this case, each eigen mode is characterised by a large frequency spectrum indicating that the beam core becomes unstable.

It is clear that incoherent space-charge resonances can scatter particles around the beam core, an emittance growth and halo formation mechanism which is highly enhanced when chaotic areas are formed due to resonance overlap. To avoid this effect, nonlinear correctors can be used to forbid resonance overlap in the beam core vicinity. This has been demonstrated with octupoles used to cancel emittance growth and halo formation in a FODO channel tuned with σ_{0t} = 100°. (see [11]) as well as with duodecapoles by Y.K. Batygin [17].

V- ACCELERATED BEAMS.

The previous sections concern the transverse dynamics of continuous beams. These studies undertaken in a restricted framework are essential for the understanding of space-charge dominated beam dynamics (emittance growth and halo formation). Nevertheless, the results can not be extrapolated to accelerated beams. Following are some preliminary reflexions on additional effects induced when the beam is accelerated :

a- An obvious effect is that the particles being bunched, the peak current increases by almost a factor ten.

b- There is a strong coupling between the radial and longitudinal planes (synchro-betatron coupling) induced by space charge but also by the defocusing effect of the RF field which can be strong in a linac.

c- A sinusoidal RF voltage provides the acceleration, the longitudinal focusing force is then nonlinear.

d- The Hamiltonian system is no longer conservative and the motion appears to be damped [18].

Point b- should be studied carefully in the future because synchrobetatron coupling is a strong source of chaos, then a strong source of particle diffusions (see [11] for a preliminary study). Nevertheless, the two last points seem to be almost important. Point c- because the sinusoidal accelerating voltage increases the system nonlinearities and gives a possibility for particules which escape from the potential well to diffuse far from the beam core. Point d- because the deep character of the dynamics is modified when acceleration is taken into account. A damping term must be introduced in the equation of motion, the assymptotic behaviour is then characterized by a bassin of attraction and simple or strange attractors.

The equation of motion for particle trajectories in the longitudinal plane can be writen :

$$\frac{d^2\delta\varphi}{ds^2}+A(s)\frac{d\delta\varphi}{ds}+B(s)\left[\cos(\varphi)-\cos(\varphi_S)\right]+C(s,\delta\varphi)=0$$

where $\delta\varphi = \varphi - \varphi_s$ with the synchronous phase φ_s defined using the linac convention. With suitable scaling of displacement ($\delta\varphi \rightarrow x$) and ″time″ ($s \rightarrow z$), this equation can be reduced to ″standard forms″ which keep the nonlinear character of the original equation :

$$x'' + \alpha x' + \{ 1 + F\cos(\omega z) \} x - x^2 = 0 \qquad (2)$$

$$x'' + \alpha x' + x - x^2 = F\cos(\omega z) \qquad (3)$$

where the perturbing force C(s,δφ) (space charge or others) is noted F cos(ωz). These equation can be studied in order to get the feeling on the sensitivity of the longitudinal motion to perturbations.

Equation (2) is a nonlinear Mathieu's equation which can be numerically integrated with $\alpha = 0$ to keep only the nonlinear effect. Figure 6 shows that even for a weak perturbation (5% of the main accelerating force) the stable phase-space area is strongly reduced.

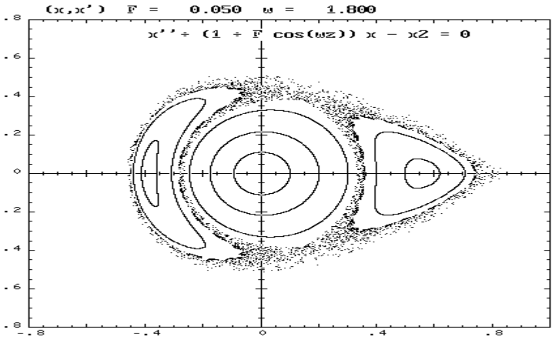

Figure 6 : Poincaré surface of section for equation (2) with $\alpha = 0$, $F = 0.05$ and $\omega = 1.8$.

Equation (3) has been extensively studied because it is a model for capsizing of ships in waves [19]. In a linac with 1 MeV/m mean accelerating field, the damping term is roughly constant (~ 0.1) above 10 MeV. Figure 7-a gives the position of the two attractors when $.047 < F < .069$, they prevent the unnormalized emittance from being damped as the energy increases. For higher perturbing forces, the bifurcation diagram shows that the inner attractor disappears and that the outer one is perturbed by several sequences of period doubling ending with a strange attractor at F~.11. The pertubing force also induces an erosion of the bassin of attraction (see [19] and figure 7-b).

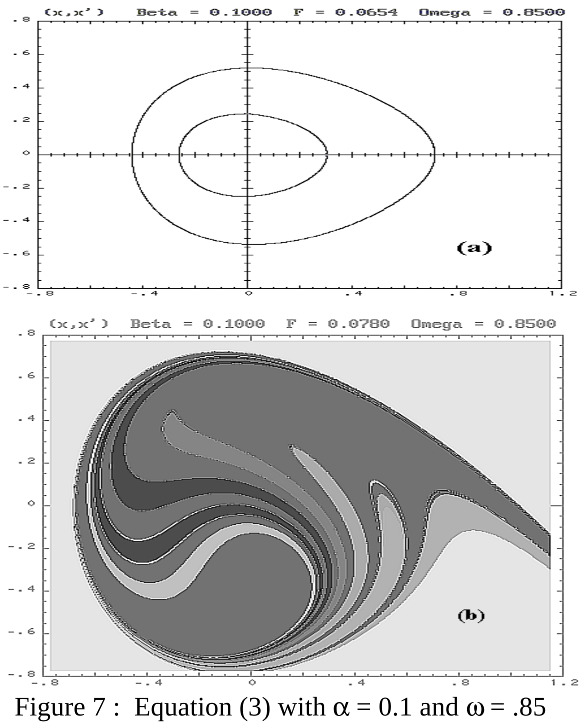

Figure 7 : Equation (3) with $\alpha = 0.1$ and $\omega = .85$
Position of the attractors for F = .0654 (a)
Bassin of attraction for F = .078 (b).

VI- CONCLUSION.

The *incoherent space-charge resonances* seem to be a major source of emittance growth and halo formation because they are excited by envelope oscillations which are unavoidable in a real accelerators (periodic focusing system and mismatching). A large number of simulations have been done for continuous beams in FODO channels. For realistic tunes and mismatches, the particles diffuse along the resonance web but the maximum radius reached by the halo has never exceded ~ 4 times the core radius. The diffusion rates from the beam core vicinity towards a resonance located far from it are very low. Nevertheless, acceptable particle losses are so low that the system chaotic behaviour precludes any definitive conclusion. The longitudinal motion seems to be more sensitive to perturbations and particles can escape from the potential well. Further work is needed to analyse these phenomena with more realistic simulations.

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