MERGING BEAM-BEAM COLLISIONS AT RIKEN RI BEAM FACTORY

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ABSTRACT

Merging ion-ion collisions is an important feature of the proposed RIKEN Radioactive Isotope Beam Factory [1]. In the merging collision case, the value of luminosity 10^{26} 1/cm^{2}sec is several orders of magnitude less than for head-on collisions even when the stored number of ions is close to the space charge limit of 10^{12} particles because both beams have almost the same vector of velocity and merging angle is rather small (1-10^o). In the present paper, the beam-beam effects are studied for the coasting merging beam collisions using particle-in-cell (PIC) model in 4D phase space. Beam-beam instability and beam disruption effect such as emittance growth have been examined from high order nonlinear resonances study. Beam luminosity is estimated as a function of main collider parameters.

1 INTRODUCTION

Proposed Radioactive Isotope Beam Factory [1] is aimed to be used for wide range of experiments with unstable nuclear beams. Among variety of planning experiments the ion-ion merging collisions are of the most importance. Merged beam technique is very useful method for the study of nuclear fusion processes. Merging two RI beams deliver low energy collisions just above the Coulomb barrier threshold that is difficult to be realized in other experimental methods. The most important collider parameter is luminosity which is limited among other reasons by physics of beam-beam interaction. In this paper we analyze the luminosity constraints originated from beam-beam interaction as a function of main parameters of storage ring.

2 LUMINOSITY OF MERGE BEAM-BEAM INTERACTION

Expected values of beam sizes at the interaction point are $\sigma_x = \sigma_y = 0.5$mm, σ_z =40 cm. Therefore, it is enough to consider interaction of two coasting merge ion beams with particle densities n_1, n_2 and beam velocities β_1, β_2 colliding with angle α (see fig. 1). Luminosity L is defined as a ratio of interaction rate to cross section for particle interaction $L=1/\sigma \, dN/dt$. Using expression for invariant cross section [2], the number of collisions dN during the time dt is

$$\frac{dN}{dt} = \int_V \sigma \sqrt{(\vec{v}_1-\vec{v}_2)^2 - \frac{[\vec{v}_1 \times \vec{v}_2]^2}{c^2}} \, n_1 n_2 \, dV \quad , \qquad (1)$$

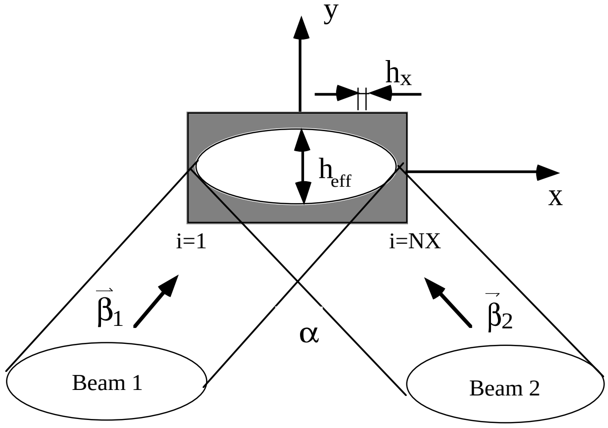

Fig.1 Merge beam - beam interaction.

where integration is performed over the volume of interaction. It is convenient to express luminosity as a function of collision angle α, number of particles per beam $N_1 \approx N_2 = N$, ring circumference $2\pi R$ and effective size of the beam h_{eff} at the interaction point:

$$L = \frac{\sqrt{\beta_1^2+\beta_2^2-2\beta_1\beta_2\cos\alpha-\beta_1^2\beta_2^2\sin^2\alpha} \; cN_1N_2}{(2\pi R)^2 \sin\alpha \; h_{eff}} \approx \frac{N^2 c \;\, tg\frac{\alpha}{2}}{(2\pi R)^2 h_{eff}} . \qquad (2)$$

Let us note that for merge coasting beams the luminosity is inversely proportional to beam height and does not depend on beam width.

3 PARTICLE-IN-CELL MODEL OF BEAM-BEAM INTERACTION

Beam-beam interaction was studied by combination of particle-in-cell treatment of space charge problem at the crossing point and transfer matrix for particle revolution in storage ring:

$$\begin{aligned}
x_{n+1} &= x_n \cos 2\pi Q_x + x_n' \, \beta_x^* \sin 2\pi Q_x \\
x_{n+1}' &= -\frac{x^n}{\beta_x^*} \sin 2\pi Q_x + x_n' \cos 2\pi Q_x + \Delta x' \\
y_{n+1} &= y_n \cos 2\pi Q_y + y_n' \, \beta_y^* \sin 2\pi Q_y \\
y_{n+1}' &= -\frac{y^n}{\beta_y^*} \sin 2\pi Q_y + y_n' \cos 2\pi Q_y + \Delta y' \quad ,
\end{aligned} \qquad (3)$$

where Q_X, Q_y are betatron tunes and β_x^*, β_y^* are beta-functions of a lattice. Self-consistent beam-beam kicks $\Delta x'$, $\Delta y'$ are calculated from numerical treatment of space charge problem:

$$\Delta x' = \frac{1}{tg\alpha\beta_s^2\gamma^3}\sum_{i=1}^{NX}\frac{qE_{x\,i}(y)h_x}{mc^2} \quad ; \qquad (4)$$

$$\Delta y' = \frac{1}{\sin\alpha\beta_s^2\gamma^3}\sum_{i=1}^{NX}\frac{qE_{y\,i}(y)h_x}{mc^2} \quad , \qquad (5)$$

where mc^2/q is a rest energy divided by charge of particle, E_{xi} (y), E_{yi} (y) are space charge field of the beam calculated at spatial grid points, NX is a number of equidistant mesh points along x - axis located with step h_x (see fig.1). Space charge field of the beam is calculated from numerical solution of Poisson's equation in Cartesian coordinates with Dirichlet boundary conditions for potential U on the surface of conducting pipe using Fast Fourier Transforms.

In coasting merge beams the interaction is not x-y symmetric. In the median plane of the beams (x-direction in fig. 1) particles experience much smaller kick than in the vertical plane (y-direction). As a result, initially round beams become prolonged in the direction perpendicular to median plane. Linear betatron tune shift due to merge beam-beam collisions is :

$$\xi_y = \frac{N\, r_o}{2\,\pi^2\sigma_y \sin\alpha\, \beta^2\, \gamma^3\, Q_y} \quad , \qquad (6)$$

where $r_o = q^2/ 4\pi\varepsilon_o\, mc^2$ is a value of classical radius of particle and σ_y is a standard deviation of beam size.

4 BEAM-BEAM INSTABILITY

For two coasting merge beams (see fig. 1) only y - direction is responsible for degradation of beam luminosity due to compensation of beam-beam kick in x-direction. Suppose, the fractional value of tune shift Q_y between two consequent collision is .168. The closest resonance value of the 6th order resonance is 0.1666. In Table 1 the results of numerical calculations for the value of beam-beam tune shift ξ_y = - 0.005 are presented. Evolution of RMS beam envelope $R = 2\sqrt{<y^2>}$ and beam emittance $\varepsilon = 4\sqrt{<y^2>\,<p_y^2> - <yp_y>^2}$ were controlled during the simulations as statistical averaged values over large number of modeling particles.

From results of simulations it follows that if beam-beam kick is a linear function of coordinate (KV beam), the beam-beam instability is not observed. Instability appears when beam-beam kick is a weak nonlinear function ("water bag" distribution). With the further increasing of nonlinearity in kick function (parabolic, Gaussian distributions), the instability increases.

To explain beam-beam instability in considered simple 1-dimensional model, extra calculation were done, where beam-beam kick was approximated by analytical Gaussian function:

$$\Delta y' = -\frac{4\pi\,\xi_y}{\beta_y^*}\;\frac{1 - \exp(-y^2/2\,\sigma_y^2)}{y^2/2\,\sigma_y^2}\, y \;. \qquad (7)$$

Table 1. Results of PIC simulation of beam-beam effects (Number of particles $3\cdot10^3$; Number of turns $4\cdot10^4$; mesh NX x NY = 64 x 64).

Beam Distribution		Envelope Growth (per 10^4 turns)	Emittance Growth
KV	$\rho(r) = \rho_o$	1.0	1.0 (no growth)
Water Bag	$\rho(r) = \rho_o\,(1 - r^2/R^2)$	1.0005	1.002
Parabolic	$\rho(r) = \rho_o(1 - r^2/R^2)^2$	1.002	1.003
Gaussian	$\rho(r)=\rho_o\exp(- 2r^2/R^2)$	1.002	1.007

It is well known , that a periodic nonlinear kick in pure linear system induces set of nonlinear resonances. Overlapping of nonlinear resonance is a universal mechanism of stochastic particle instability in nonlinear systems [3]. To observe beam-beam instability by overlapping of resonance islands in 1-dimensional model, the value of beam-beam kick should be large enough to create many-island structure (ξ = – 0.2). Another mechanism of stochastic unstable particle motion is presented at figs. 2, 3. This is a case of stable isolated nonlinear resonance for small value of ξ = – 0.005. If parameters of the kick (7) are fixed, phase space trajectories of particles are closed and not changed with time (fig. 2). Introducing a noise in parameter σ_y of beam-beam kick (7) completely destroys the stability. In the calculations presented at fig. 3, standard deviation σ_y was changed from turn to turn according to the expression

$$\sigma_y^{(n)} = \sigma_y^{(o)}\,(0.975 + 0.05u_n) \quad , \qquad (8)$$

where u_n is a random noise function within the region (0,1). As shown at fig. 3, weak noise in opposite beam size creates instability. Analysis shows, that beam-beam instability appears if two conditions are valid simultaneously: (i) beam-beam kick is a nonlinear function of coordinate; (ii) parameter of beam-beam kick (beam standard deviation σ_y) is a subject of noise.

5 LIMITATION OF LUMINOSITY

From eq. (6) the limited number of stored particles due to beam-beam interaction is:

$$N_{beam\text{-}beam} < 2\,\pi^2\;\frac{\xi_{max}\, Q_y\, \sigma_y \sin\alpha\, \beta^2\, \gamma^3}{r_o} \quad . \qquad (9)$$

On the other hand, the limited number of particles in the ring due to incoherent space charge tune shift (Laslett tune shift) $\Delta\nu_{max}$ is as follows:

$$N_{SC} < 4\pi\;\frac{\Delta\nu_{max}\, Q_y\, \beta^2\, \gamma^3\, \bar{\sigma}_y^2}{R\, r_o} \quad , \qquad (10)$$

where $\bar{\sigma}_y$ is an average value of standard deviation along the ring. Let us take typical parameters for the ring:

ξ_{max}=0.005, $\Delta\nu_{max}$= 0.25, α =10^{o}, R=40m, $\bar{\sigma}_y$ = 4mm. The ratio of limited number of particles due to beam-beam effects and due to incoherent space charge tune shift is given by:

$$\frac{N_{beam\text{-}beam}}{N_{SC}} = \frac{\pi}{2}\,\frac{\xi_{max}}{\Delta\nu_{max}}\,\frac{R\,\sigma_y \sin\alpha}{\bar{\sigma}_y^2} = 55\,\frac{\sigma_y}{\bar{\sigma}_y}\ . \qquad (11)$$

Equation (11) indicates, that limitations, caused by merging beam-beam interaction, could be the same order of magnitude as incoherent space charge effects if ratio of beam sizes is around 50. Taking the limited number of particles from eqs. (10), (11) as $N_{max} = 3 \times 10^{12}$ (A/Z^2) and assuming h_{eff} =$1.5{\cdot}10^{-3}$ m, $\beta_1 = \beta_2 \approx 1$, the limitation in luminosity is:

$$L < \frac{N_{max}^2\ c}{(2\pi R)^2\ h_{eff}}\ \mathrm{tg}\,\frac{\alpha}{2} = 2 \times 10^{26}\ \left(\frac{A}{Z^2}\right)^2\ \frac{1}{\sec\ cm^2}\ . \qquad (12)$$

6 CONCLUSIONS

Beam-beam luminosity constraints were studied from strong-strong model of particle interaction. Beam-beam instability in simple 1-dimensional case was observed for nonlinear kick with a noise parameter. Maximum number of stored particles estimated from beam-beam interactions exceeds the limit defined by the incoherent space charge tune shift for circulated beam. Maximum achieved luminosity of merge beam-beam interaction was estimated.

7 REFERENCES

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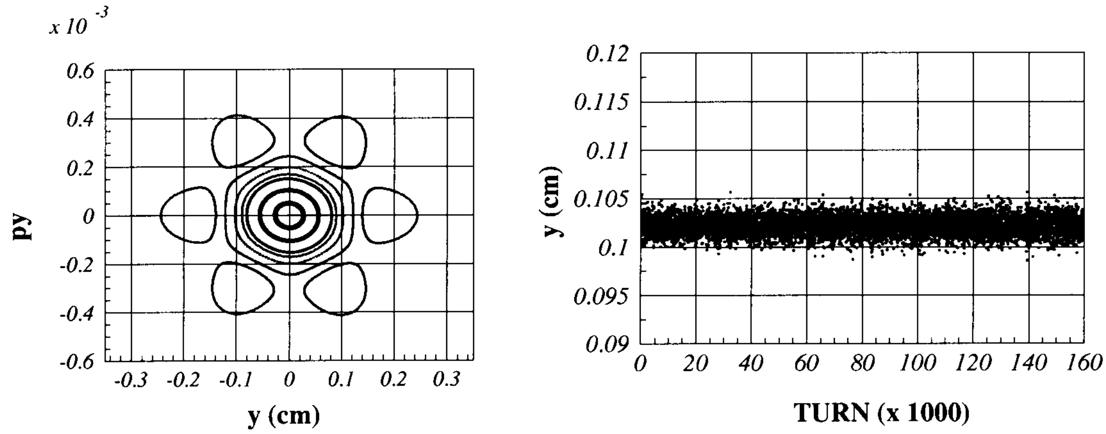

Fig.2. Phase space trajectories (left) and beam envelope (right) near nonlinear resonance of the 6th order under the value of tune shift ξ_y = - 0.005 without noise in driving beam size.

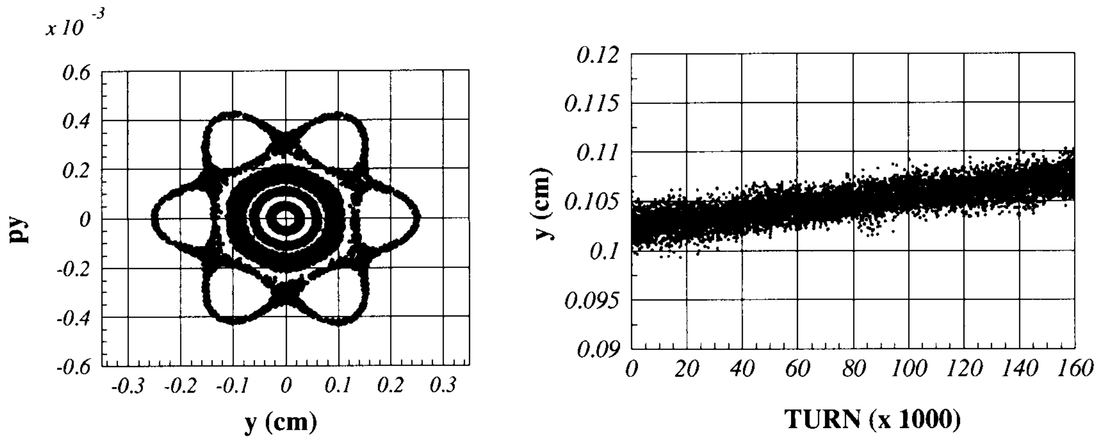

Fig.3. Phase space trajectories (left) and beam envelope (right) near nonlinear resonance of the 6th order under the value of tune shift ξ_y = - 0.005 with 5% noise in driving beam size.