BEAM-BEAM INTERACTION OF ELECTRONS AND IONS AT RIKEN RI BEAM FACTORY

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Abstract

One of the main experiment at the new RIKEN radioactive isotope beam factory [1] is a collision of 2.5 GeV electron beam with unstable ion beams. This experiment is intended to be used for determination of the charge and current distribution in the radio active nuclei. The physics of beam-beam interaction at the collision point is an important issue for determination of the value of collider luminosity. Electron - ion collisions are studied by numerical model combining particle-in-cell method for nonlinear space charge problem of interacted particles and beam mapping at every revolution.

1 INTRODUCTION

Beam-beam interaction is an essentially nonlinear phenomenon, which can results in complicated unstable beams behavior. Influence of the opposite charged beams on each other results in increasing of betatron tune of particle oscillations. As far as electron beam is typically much stronger than the ion beam, it is enough for estimations to consider behavior of ion beam in the unchanged field of the electron beam (strong-weak model). For beam-beam interaction the linear part of the betatron tune shift ξ is a scale parameter, which defines the strength of the beam-beam interaction [2]:

$$\xi = \frac{r_p \beta_x \frac{Z}{A} N_e (1 + \beta_e \beta_i)}{4 \pi \gamma_i \beta_i^2 \sigma_e^2} \quad , \qquad (1)$$

where N_e is a number of electrons per bunch, β_x is a beta - function at the interaction point, Z/A is a charge to mass ratio of ion, β_e, β_i are the velocities of electrons and ions, γ_i is a reduced ion energy, σ_e is a half size of electron beam envelope and $r_p = e^2/ 4\pi\varepsilon_o mc^2 = 1.5\cdot10^{-18}$ m (classical radius of a proton). Limitation in ξ results in constraints of luminosity L, which is defined as follows:

$$L = \frac{f N_i N_e}{4 \pi \sigma_e^2} N_{bunch} \approx \frac{f N_i N_{bunch} \gamma_i}{2 \beta_x r_p} \frac{A}{Z} \xi \quad , \qquad (2)$$

where $f = \omega_s/2\pi$ is the particle revolution frequency in a ring, N_i is a number of ions per bunch and N_{bunch} is a number of bunches per beam.

2 NUMERICAL MODEL

Particle-in-cell model has been developed to study the details of beam-beam interaction. The double storage ring is supposed to have 2 colliding point, therefore, the simulations are done successively from one collision point to another one with the half values of betatron tune of the ring $Q_x/2$; $Q_y/2$. Modulation of betatron tune due to coupling with synchrotron oscillations is taken into account as well. Particles in every ring are advances by matrix multiplication from one collision point until another one:

$$x^{n+1} = x^n (\cos 2\pi Q_x) + p_x^n \left(\frac{R}{p_s} \frac{\sin 2\pi Q_x}{Q_x}\right) \quad ,$$

$$p_x^{n+1} = x^n \left(- \frac{p_s Q_x}{R} \sin 2\pi Q_x\right) + p_x^n (\cos 2\pi Q_x) + \Delta p_x \quad , \qquad (3)$$

$$Q_{x,y} = \overline{Q}_{x,y} + A_s \sin(2\pi Q_s n) \quad ,$$

analogously for y^{n+1}, p_y^{n+1} ,where (x,y) are the particle position, p_x, p_y, p_s the reduced momentum of particles (divided by mc), R is a radius of a ring, Q_s is a synchrotron tune and A_s is a modulation depth. In collision point every particle receive a beam-beam kick

$$\Delta p_{x,y}^{i,e} = \frac{q E_{x,y} d (1 + \beta_i \beta_e)}{mc^2 \beta_{i,e}} \quad , \qquad (4)$$

where electrostatic field $E_{x,y}$ is calculated directly from solution of 2-dimensional Poisson's equation on a grid 128 x 128 in Cartesian coordinates. For comparison, simulation with analytical expression of beam-beam kick is used as well:

$$\Delta p_x = \frac{2 r_p \frac{Z}{A} N_e (1 + \beta_i \beta_e)}{\beta_i} \frac{x}{r^2} \left[1 - \exp\left(- \frac{r^2}{2\sigma_e^2}\right)\right] \quad , \qquad (5)$$

similar for Δp_y.

3 NONLINEAR RESONANCES

Beam-beam interaction induces nonlinear resonances, which can overlap in phase space creating unstable particle motion [3]. Presence of betatron tune modulation increases the density of nonlinear resonances which obey the resonant condition (j, n, k and m are integer):

$$jQ_x + nQ_y + kQ_s = m \quad . \tag{6}$$

Stochastic threshold (Chirikov criteria [3]) for linear tune shift expresses condition that separation of resonant islands should be larger than width of resonances:

$$\xi \le \frac{Q_s}{4\,n\,M} \sqrt{\frac{M^{1/2}}{U''(J)\,V_n(J)\,J_k\!\left(\frac{nA_s}{Q_s}\right)}} \quad , \tag{7}$$

where n is the order of resonance, M is a number of collisions per ring, $J_k(x)$ the Bessel function of k-th order and functions U(J), V(J) are defined by equations:

$$U'(J) = \frac{2}{J}\left[1 - e^{-J/2}\ I_o(J/2)\right] \quad , \tag{8}$$

$$V'_n(J) = (-1)^{n/2+1}\ \frac{4}{J}\ e^{-J/2}\ I_{n/2}\left(\frac{J}{2}\right) \quad . \tag{9}$$

Fig. 1 illustrates the mechanism of the beam stochasticity. Parameters of the problem were chosen as follows: A/Z= 4, $\beta_i\gamma_i = 1.7$, $\beta_x = 10$ cm, $Q_s = 0.01$ and $A_s = 0.01$. Depending on the combination of parameters, behavior of particles can be either stable or chaotic (unstable). For small values of $\xi = 0.005$ the phase space trajectories are ellipses (see Fig. 1, a). With increasing of ξ until 0.03 the characteristic structure of a 7th and 14th order resonance at phase plane appears (Fig. 1 b, c). With the further increasing of ξ up to 0.05 the high order resonances overlap each other, which results in stochastic behavior of particles (Fig. 1, d). Analytical value of threshold, calculated from formula (7), shows the upper value of $\xi_{max}= 0.00625$.

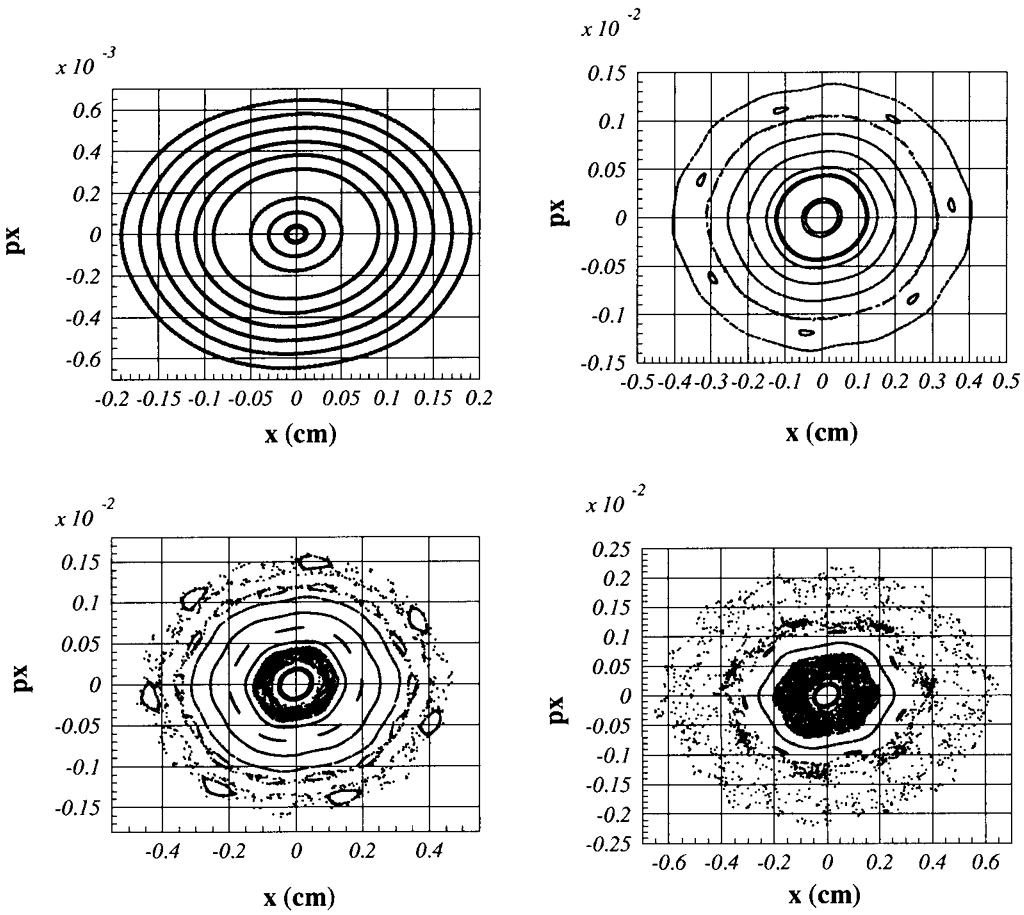

Fig. 1. Phase space trajectories in beam-beam interaction: Q = 8.1415, $Q_S = 0.01$, $A_S = 0.01$; (a) $\xi = 0.005$; (b) $\xi = 0.01$; (c) $\xi = 0.03$; (d) $\xi = 0.05$. Trajectories are plotted once per synchrotron period; the total number of turns is 200000.

4 NOISE BEAM-BEAM INSTABILITY

Without tune modulation the many-island structure appears, when the value of beam-beam tune is much larger than experimentally observed beam-beam threshold $\xi = 0.005$. Another mechanism of beam-beam instability is observed in numerical simulations, where electron beam size is a subject of noise (see also ref. [4]). In fig. 2 the unstable motion of particles appears when the value of standard deviation in beam-beam kick (5) is changes from turn to turn according to the expression

$$\sigma_e^{(n)} = \sigma_o (1 + u_n) \quad , \qquad (10)$$

where σ_o is fixed and u_n is randomly distributed within the narrow limits ±0.025. This kind of noise exists naturally in particle-in-cell simulations due to beam mismatch with the channel. Analysis shows that instability appears in rather simple conditions, where weak nonlinear beam-beam kick is accompanied by noise in parameter σ. The value of beam-beam kick, required to induce this kind of instability can be as small as $\xi = 0.005$, which is typical for experimentally observed beam-beam threshold for ion rings. Taking the value $\xi_{max} = 0.005$ as an upper limit for betatron tune shift, f = 1 MHz, the limitation in luminosity is as follows:

$$L \leq \frac{f N_i N_{bunch} \gamma_i}{2 \beta_x r_p} \frac{A}{Z} \xi_{max} \approx 10^{18} \cdot N_i^{total} \frac{1}{cm^2 sec} , \qquad (11)$$

where $N_i^{total} = N_i N_{bunch}$ is a total number of stored ions in a ring, limited by the lifetime of unstable ions and finite value of acceptance of the ring.

5 EFFECT OF ION BEAM ON ELECTRON BEAM

Realistic ion beam also affects electron beam due to space charge of ion beam. The influence can be estimated as an betatron tune shift caused by ion - e^- interaction:

$$\xi_e = \frac{r_e \beta_x Z N_i (1 + \beta_e \beta_i)}{4 \pi \gamma_e \beta_e^2 \sigma_i^2} \approx 10^{-9} \frac{Z N_i}{\gamma_e \beta_e^2} \approx 2 \cdot 10^{-13} Z N_i, \qquad (12)$$

where $r_e = e^2 / 4\pi\varepsilon_o m_e c^2 = 2.7 \cdot 10^{-15}$m is a classical radius of electron, σ_i is a standard deviation of ion beam size at the interaction point, γ_e is an electron energy. Influence of ion beam on electron beam can be characterized by the ratio of space charge density of ion beam with respect to electron beam $\eta = \rho_i / \rho_e$. From typical limitation of electron beam-beam tune shift $\xi_e <$ 0.05 the ion beam affects the electron beam starting with the value $\eta > 0.25$.

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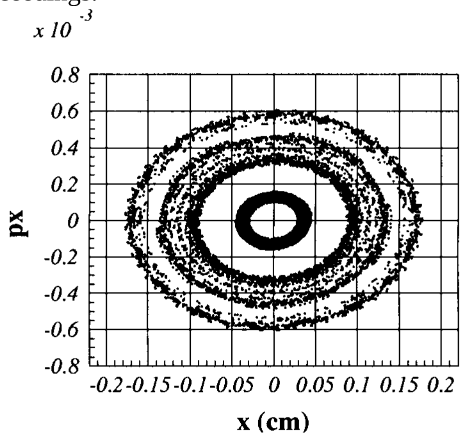

Fig. 2. Unstable particle trajectories in the case of noise in opposite beam size: Q=8.1415, Q_s=0, A_s=0, ξ = 0.005 (compare with fig. 1a).

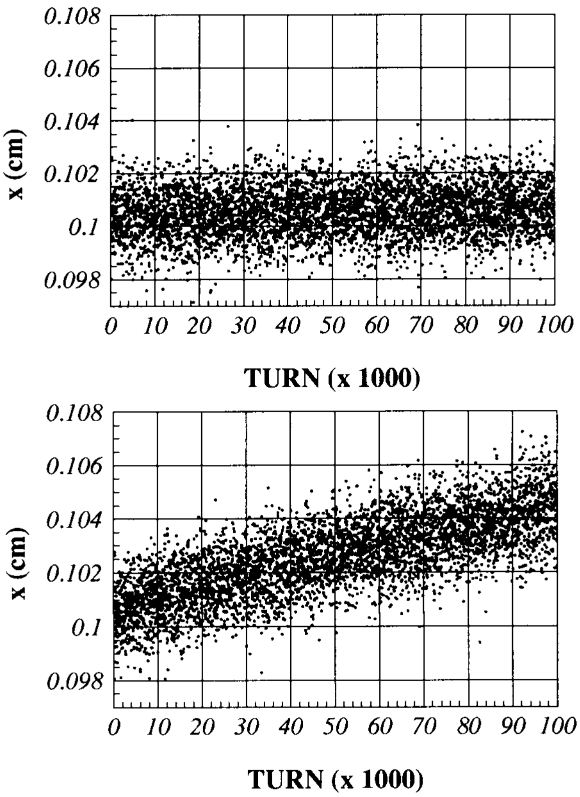

Fig.3. Conservation of ion beam envelope without noise in opposite beam size (up) and ion beam envelope growth in case of 5% noise in σ_e (bottom). The value of beam-beam tune is ξ =0.005.