Spin Dynamics Study for RHIC

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Abstract

At the RHIC project, polarized proton beams will be accelerated from 25 GeV to 250 GeV. To avoid the depolarization during the acceleration and colliding experiments, a pair of Siberian snake will be installed in each ring. To analyze the depolarizing phenomena, we set up the equations of motion of 9-D, spin(3D) and particle motion(6D) in magnetic system, for each kind of magnets. After the tracing of orbit motion and spin motion, we construct the second order transfer matrix in each magnet. The working points of Siberian snake are searched. For the arrangement of magnetic system of the lattice of RHIC, we obtain the depolarization resonance strength and frequency as a function of beam energy for the first and second order spin tracking. Both intrinsic resonance strength and imperfection resonance strength are calculated.

KEYWORDS: polarization, proton, resonance, storage-ring

1 INTRODUCTION

The Relativistic Heavy Ion Collider (RHIC) at Brookhaven, now under construction will have the possibility of polarized proton-proton collisions up to a beam energy of 250 GeV, with a luminosity of $2\times10^{32}\mathrm{cm}^{-2}\mathrm{s}^{-1}$ for BNL-RIKEN joint high energy polarized proton project. The RHIC collider is being constructed in an existing 3.834 km long tunnel, taking AGS as an injector. The collider consists of two rings of superconducting magnets. Each ring consists of three inner arcs and three outer arcs and six insertions joining the inner and outer arcs. Each arc is composed of 11 FODO cells. These kinds of periodic structure of setup will produce a periodic structure of the phase of beam, which will cause the intrinsic depolarization resonance of polarized beam. During acceleration process, the polarization may be lost when the spin precession frequency passes through a depolarizing resonance. These resonance occur when the number of spin precession rotations per revolution $G\gamma$ (G=1.793 is the anomalous magnetic moment of the proton, γ is the relativistic factor) is equal to an integer(imperfection resonance) or equal to $kP\pm\nu_y$ (intrinsic resonance), where k is an integer and P is superperiodicity of ring.

The motion of the spin vector $\vec{S}$ for proton, in a magnetic field, is a pure precession, that is described by BMT equation:

$$\frac{d\vec{S}}{dt} = \vec{\Omega}_{\mathbf{BMT}} \times \vec{S}$$

with

$$\vec{\Omega}_{\mathbf{BMT}} = -\frac{e}{m_o\gamma}[(1+G\gamma)\vec{B}_\perp + (1+G)\vec{B}_{||}]$$

At the same time, the motion of particle obeys the Lorentz force equation,

$$\frac{d\vec{v}}{dt} = \frac{e}{\gamma m}(\vec{v}\times\vec{B}+\vec{E})$$

The particle tracking, both spin and orbit, are given by the set of upper two equations.

This paper describes the spin transfer matrix of accelerating the polarized proton beam in the various magnets in RHIC, including a so-called $''$ Siberian snake$''$ magnetic field, a special kind of spin rotator that change spin direction by π around the axis lying in the horizontal plane. The Siberian snakes help to keep the polarization of beam when the beam are accelerated to cross the region of spin depolarizing resonance.

2 TRACKING IN INDIVIDUAL MAGNETS

To describe particle tracking through a beam transport line composed of drift space, bending magnets, quadrupoles and Siberian snake of helical magnets, or individual magnet, we should construct matrix of beam transport. The 9-D matrixes are given by result of solving 9-D equation set. For tracking in individual magnet, we found:

- The individual magnet is depolarized for polarized beam. The orbit trace affect spin trace, mainly by the velocity of x, y, at same time, the spin trace affect the orbit trace, mainly by positions. The spin direction can be maintained mainly through a magnets.
- The depolarization is produced by (a) the central part of magnet, where $S\times B$ is not zero. (b) the fringe part of magnet, where B_z, B_x are not equal to zero.
- The polarization lost by one magnet, but can be recovered another one in periodic magnet setup. There are many periodic structure in the ring, the actions of force and rotation strength are reverse between two half periodic phase [1].

3 WORKING POINTS OF SIBERIAN SNAKE

Siberian snake consists of four helical magnets. The perfect Siberian snake in one side of ring rotates spin 180^o and the Siberian snake in the other side of ring would restore the spin to the initial direction. Although the depolarization producing by one Siberian snake can be recovered by another Siberian snake, the inappropriate parameters affect seriously the acceleration process of polarized proton. One should search the parameters for a perfect working point[2].

3.1 The Analytic Tracking in Helical Magnet

The magnetic field near the axis of the perfect dipole helical magnet modules with the period λ and the field amplitude B can be written as

$$B_x = -B\sin kz; \quad B_y = B\cos kz; \quad B_z = 0$$

where wave vector $k = 2\pi/\lambda$. In [$\vec{i}, \vec{j}, \vec{k}$] coordinate system, the initial condition of spin at $z = 0$ are [$S_x\vec{i}$, $S_y\vec{j}$, $S_z\vec{k}$]. After two times coordinate transformation, we find a conservation system, in which spin direction rotate as a circle. We can deduce the spin motion along z in lab system as

$$\begin{aligned}
\vec{S} = &[A_1 \sin(\omega_k z + \theta_c)\cos kz + (-A_1\cos(\omega_k z + \theta_c)\sin\theta \\
&+(S_y\cos\theta + S_z\sin\theta)\cos\theta)\sin kz]\vec{i} \\
&+[-A_1 \sin(\omega_k z + \theta_c)\sin kz + (-A_1\cos(\omega_k z + \theta_c)\sin\theta \\
&+(S_y\cos\theta + S_z\sin\theta)\cos\theta)\cos kz]\vec{j} \\
&+[A_1\cos(\omega_k z + \theta_c)\cos\theta + (S_y\cos\theta + S_z\sin\theta)\sin\theta]\vec{k}
\end{aligned}$$

where

$$\omega = \frac{eB(1+\gamma G)}{m_o\gamma c v}, \quad \theta = tg^{-1}\|\frac{k}{\omega}\|.$$

Then the spin transport matrix at point z is given by the expression as,

$$\begin{vmatrix}
\cos\omega_k z & -\sin\theta\sin\omega_k z & \cos\theta\sin\omega_k z \\
\sin\omega_k z\sin\theta & \cos^2\theta + \sin^2\theta\cos\omega_k z & \sin\theta\cos\theta(1-\cos\omega_k z) \\
-\sin\omega_k z\cos\theta & \sin\theta\cos\theta(1-\cos\omega_k z) & \cos^2\theta\cos\omega_k z + \sin^2\theta
\end{vmatrix}$$

for the initial spin direction with same direction of magnetic field at the entrance point. The spin matrix of whole Siberian snake is obtained by times the four matrixes of helical magnet.

In the real helical magnet, the field is not only dependent with z (first order), but also with x, y (second order). The spin tracking in the second order expression can be calculated only by numerical method.

3.2 Search of Working Point of Siberian Snake

The search of working points are performed in the first order magnetic field by analytic analysis and by the numerical integral. Furthermore the second order calculation was done by the numerical method. The length of each helix modules is 2.4m, and the field direction of magnet field rotate 2π for each module with same helicity. $B_4 = -B_1$ and $B_2 = -B_3$, and the initial spin direction is same direction of magnetic field at the entrance of first module. The gap between two helixes is 0.4m. The matrixes are calculated various B_1 and B_3 by step 1Gauss in the range of 0-10 Tesla for beam energy at 25 GeV.

Requiring that the $S_y = 1$ changes into $S_y = -1$, the working points are three branch of curve line. But requiring that the S_y changes into $-S_y$, S_x into S_z and S_z into S_x, the working points are seven area, or seven points at high precision, that are given in Table 1.

Table 1 The perfect working parameter of Siberian snake, with B_2=-B_3, B_4=-B_1 at 25 GeV polarized proton, unit by Tesla, when the spin direction is same direction of magnetic field at entrance point.

		1	2	3	4
First order analytic	B_1	1.2319	3.2282	5.0892	5.3788
	B_3	3.9580	5.5300	2.3861	8.8272
First order numerical	B_1	1.2336	3.2265	5.0865	5.3769
	B_3	3.9570	5.5273	2.3836	8.8225
Second order numerical	B_1	1.0723	3.3872	5.1324	5.5050
	B_3	3.9328	5.6587	2.2711	8.9256
		5	6	7	
First order analytic	B_1	6.3878	8.0004	8.5256	
	B_3	6.6088	4.2691	5.1514	
First order numerical	B_1	6.3847	7.9988	8.5171	
	B_3	6.6115	4.2765	5.1463	
Second order numerical	B_1	6.4517	8.0139	8.4701	
	B_3	6.9018	4.4816	5.1915	

4 INTRINSIC DEPOLARIZED RESONANCE

4.1 First Order Tracking

MAD program was used to calculate the orbit transport matrix. The BMT equation is given by matrix formulation

$$\begin{vmatrix}
1-(B^2+C^2)c & ABc+Cs & ACc-Bs \\
ABc-Cs & 1-(A^2+C^2)c & BCc+As \\
ACc+Bs & BCc-As & 1-(A^2+B^2)c
\end{vmatrix}$$

with

$$\left\{ \begin{array}{l} c = 1-\cos\omega\delta s \\ s = \sin\omega\delta s \end{array}, A = \frac{P_x}{\omega}, B = \frac{P_y}{\omega}, C = \frac{P_s}{\omega}. \right.$$

in the terms of spin rotation strength P:

$$\begin{aligned}
P_x = &\frac{h}{(B\rho)}[(1+G\gamma)(-x'B_s + B_x) \\
&+(1+G)x'(x'B_x + B_s + y'B_y)] \\
P_s = &\frac{h}{(B\rho)}[(1+G\gamma)(-x'B_x + y'B_y) \\
&+(1+G)(x'B_x + B_s + y'B_y)] \\
P_y = &\frac{h}{(B\rho)}[(1+G\gamma)(-y'B_s + B_y) \\
&+(1+G)y'(x'B_x + B_s + y'B_y)],
\end{aligned}$$

The matrixes for each kind of magnet are given for the first order and second order approximationin ref. [3, 4].

Compared with spin tracking code SPINK [4], the following points are improved in our calculation: The orbit motion of y, y' two dimensions are changed into x, x', y, y' four dimensions; the spin tracking is affected by x and y in various magnets; the sextupole magnets are included; a spin tracking for Siberian snake is used instead of the perfect values; the focusing action of RF is used both velocity and mass parts. We get depolarized spectrum Fig 1 (a) for emittance 7 μrad-m and :

- There are some periodic structure in depolarized spectrum, the polarization approximately keep a constant in one period, if Siberian snake is not involved in spin tracking.

- Siberian snake should be set as $+B_1\odot$, $-B_2\odot$, $+B_2\odot$, $-B_1\odot$ for one and $-B_1\otimes$, $+B_2\otimes$, $-B_2\otimes$, $+B_1\otimes$ for the other Siberian snake. $\odot$, $\otimes$ indicate positive, negative helicity respectively.
- Imperfect working points of Siberian snake make impossible to accelerate large emittance of particle.
- The energy dispersion affects spin tracking.

4.2 Second Order Tracking

The second order terms of spin rotation strength equations are keeped in the second order tracking. x, x', y, y' four dimensions calculation are used inside of each magnet. The matrix of spin transport is multiples all local spin matrix.

Depolarized spectrum with second order calculation has more noise than that of the first order and larger strength of depolarizing peaks than the first order at same emittance value (Fig. 1 (b) for 7 μrad-m). It is difficult to track the spin with a larger emittance than 10 μrad-m, although the first order calculation can track the spin easily up to the emittance of 20 μrad-m.

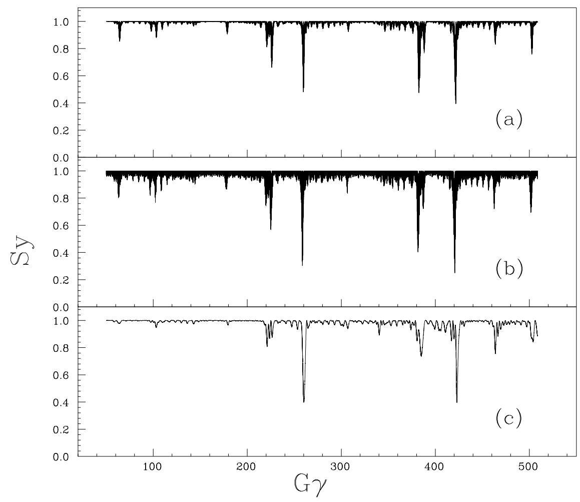

Figure 1: The intrinsic depolarization spectrum (a) for one order spintracking with the parameters of Siberian snake at $1.2336T$, $-3.9570T$, $3.9570T$, $-1.2336T$ and (b) for second order calculation with the parameters of Siberian snake at $1.0723T$, $-3.9328T$, $3.9328T$, $-1.0723T$ at a emittance 7 μrad-m through RHIC accelerator, compared with imperfection depolarization spectrum (c).

5 IMPERFECTION DEPOLARIZED RESONANCE

As the misalignment of magnets produce a imperfection depolarized resonance, the installation of each magnet at the correct position is very important for the acceleration of polarized particle. The uncertainty of installation of magnet is improved from one to two millimeter over the RHIC ring radius to 0.01 ppm by using new technology.

The magnet position tolerance at 4K are given by design book[5]: Dipole magnet orbit: 0.5mm rms; Quadrupole magnet orbit: 0.25mm rms; Sextupole magnet orbit: 0.13mm rms; magnets rotation: 1mrad rms; longitudinal error: 1mm rms. The COD was calculated by using one third of this tolerance, as a gaussion distribution of random value. The depolarized spectrum are given as Fig 1 (c). Imperfection depolarized resonance is independent with the emittance value.

6 REFERENCES

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