# **BEAM–BASED ALIGNMENT OF SEXTUPOLE MAGNETS WITH** $\pi$ **-BUMP ORBIT.**

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#### Abstract

Mis-aligned sextupole magnets tend to be main sources of large coupling and/or small dynamic aperture of circular particle accelerators. At KEKB whose vertical emittance is expected to be very small, precision of sextupole alignment should be better than 100  $\mu$ m. Although conventional alignment methods could meet this requirement, direct measurement of alignment errors using a stored beam is desirable. We have developed a new alignment method of sextupole magnets based on beam orbits in circular accelerators. This method applies a series of  $\pi$ -bumps at a position of a sextupole magnet and measures leakage orbits outside the  $\pi$ bumps. Magnetic center of sextupole is determined by analyzing the dependence of the leakage orbit amplitudes on heights of applied bumps in both horizontal and vertical directions. The method was examined at the TRISTAN main ring experimentally. The sextupoles were measured within a few 10  $\mu$ m accuracy using the beam position monitors of  $30 \,\mu m$  accuracy. Use of a position monitoring system which has better accuracy and quicker turn around time and of a steering magnet system which provides series of  $\pi$ -bumps quickly will enhance performance of this method.

### **1 INTRODUCTION**

The KEKB project [1], constructing an asymmetric  $e^+$ - $e^-$  collider for B physics in Japan, was officially approved and started in 1994. For a small emittance machine such as KEKB, alignment of optical components is important to keep beam emittance small. Computer simulation[1] shows that it is possible to recover emittances and dynamic aperture degraded by initial mis-alignment of sextupole magnets up to 150 $\mu$ m, if we can measure the magnetic centers of sextupole magnets in 75  $\mu$  precision.

We have developed a beam-base quadrupole error measurement method using  $\pi$ -bump orbits for a circular accelerator [2]. We applied this method to the TRISTAN-MR and proved the effectiveness of the method. Since offset of the beam in a sextupole magnet creates quadrupole (de)focusing force as the lowest order effect, it is possible to extend  $\pi$ bump technique to detect sextupole mis-alignment in a circular accelerator.

# **2 PRINCIPLE**

The central idea of  $\pi$ -bump method is to detect optical errors in the  $\pi$ -bump region as a orbit leakage outside the  $\pi$ -bump region. When we have a pair of orbit correctors, ST1 and ST2, separated by  $\pi$  in betatron phase, or a transfer matrix between two correctors has a form,

$$\left( egin{array}{cc} -1 & 0 \ * & -1 \end{array} 
ight)$$

 $\pi$ -bump orbit is created by applying same kick angle to these correctors. A quadrupole strength error within a  $\pi$ -bump region causes beam deflection depends on the height of the  $\pi$ -bump at the quadrupole magnet. Subtracting the orbit without excitation of these steering (reference orbit) from the orbit with this closed bump ( $\pi$ -bump orbit), a difference of these orbits is given by,

$$egin{array}{rcl} \Delta x(s) &=& \displaystylerac{1}{2\sin\pi
u} \int\limits_{\pi} ds' \sqrt{eta(s)eta(s')}\cos(\phi-\phi'+\pi
u) \ & imes \Delta k_1 \left(x_b(s')-x_r(s')
ight) \end{array}$$

Here  $\Delta k_1$  represents strength errors of quadruple magnets in the  $\pi$ -bump region and  $x_b(s)$  is a height of bump at the quadrupole. Integration is carried out over  $\pi$ -bump region. Subscripts *b* and *r* refer to a bump orbit and the reference orbit. A quadrupole strength error  $\Delta k_1$  is obtained as a gradient of deflection angle with respect to  $x_b$ .

Existence of a sextupole magnet within  $\pi$ -bump region causes a horizontal leakage orbit of which amplitude depends quadratically on a bump height at the sextupole magnet.

$$\begin{aligned} \Delta x(s) &= \frac{1}{2\sin \pi \nu} \int_{\pi} ds' \sqrt{\beta(s)\beta(s')} \cos(\phi - \phi' + \pi \nu) \\ &\times \left\{ \frac{k_2}{2} (x_b - x_c)^2 - \frac{k_2}{2} (x_r - x_c)^2 \right\} \end{aligned} \tag{2}$$

Here  $x_c$  is a horizontal coordinate of the magnetic center of the sextupole magnet. As easily observed from above equation, position of the magnetic center is determined as a points which gives minimum leakage orbit.

Vertical offset of sextupole magnet also causes a vertical orbit leakage of which amplitude depend linearly on the height of bump orbit at the sextupole magnet.

$$\Delta y = \frac{1}{2 \sin \pi \nu} \int_{\pi} ds' \sqrt{\beta(s)\beta(s')} \cos(\phi - \phi' + \pi \nu) \\ \times k_2 \left( x_b(s') - x_(s')r \right) \left( y_r(s') - y_c(s') \right) \quad (3)$$

Dependence of leakage orbit amplitude on the bump height gives us an information on vertical offset of magnetic center of sextupole magnet.



Figure 1: deflection by a sextupole magnet can be separated from strength unbalance of corrector magnets using phase information of betatron oscillation in a leakage orbit

### **3 ERROR SOURCES**

To apply the  $\pi$ -bump method to a real world, we need to consider possible error source.

 $\pi$ -bump method assumes linear optics between a couple of corrector magnets and the strength balance of these corrector magnets. Deviation from these assumptions also causes leakage orbit outside the  $\pi$ -bump region. These errors effectively shift a magnetic center of a sextupole magnet.

Fortunately we can directly measure the linear optics errors using  $\pi$ -bump method turning off sextupole magnets and can subtract an effect of quadrupole error from a measured leakage orbit.

Figure 1 is a graphical representation of deflection angles and phase differences between deflection sources, *i.e.* a pair of corrector magnets(ST1 and ST2), error of quadrupole magnet and a sextupole magnet. Leakage orbits caused by the sextupole magnet and by unbalance of corrector strength are linearly independent each other. The unbalance of corrector strength can be separated from deflection caused by sextupole magnet using phase information of betatron oscillation in the leakage orbit.

# 4 APPLICATION OF π-bumpMETHOD TO THE TRISTAN MR

We have tested the  $\pi$ -bumpmethod described in the previous sections to the TRISTAN main ring(MR) in 1994. We have applied this method to 80 sextupole magnets out of 240 sextupole magnets in MR. For the measurement, we used special optics to establish phase difference of  $\pi$  between corrector magnets.

We measured beam orbits with 10 different bump height at each sextupole magnets. We extracted the deflection angles  $\Delta p_x$  and  $\Delta p_y$  caused by sextupole magnet as a function of bump height from measured orbits using the accelerator modeling code SAD [3]

Figure 2 shows the example of the horizontal(left) and vertical(right) deflection angle as a function of bump height.

Deflection angles at sextupole magnets,  $\Delta p_x$  and  $\Delta p_y$ , are fitted by a quadratic function and a linear function respectively.

$$\Delta p_x = ax_b^2 + bx_b + c \tag{4}$$



Figure 2: Deflection angle(radian) at sextupole magnet as a function of a bump orbit at sextupole magnet(mm). Results of fitting by quadratic and linear functions are also shown as solid curves.

$$\Delta p_y = dx_b + e \tag{5}$$

These fitting functions are also shown as solid curve in Figure 2. Magnetic center of sextupole magnet can be obtained in terms of these coefficients.

$$x_c = -\frac{b}{2a} \tag{6}$$

$$y_c = y_r - \frac{d}{a} \tag{7}$$

The coefficient *a* gives a strength of the sextupole magnet  $k_2$ . If the ratio  $a/k_2$  is not close enough to one, the measurement of leakage orbits should be discarded.

We estimated the effect of beam position monitors(BPM) on the error in the estimate of magnetic center by computer simulation. In the simulation, we generated leakage orbits using a same setting of magnets as in the measurement. We added random errors on these orbit data to represent BPM reading error. Accuracy of the magnetic center calculated from these simulated orbits is about 50  $\mu$  r.m.s. when we assumes 100  $\mu$  r.m.s. as a precision of BPM.

# **5 VERIFICATION OF THE METHOD**

To demonstrate the validity of the  $\pi$ -bump method discussed above, we have measured betatron tunes as a function of excitation of a sextupole magnet. When there is a beam offset from magnetic center of sextupole magnet, change in excitation of this sextupole causes the change in the tune.

At first, we observed betatron tunes at full excitation of sextupole magnet and at zero excitation of sextupole magnets. Measured difference of betatron tunes are,

$$\Delta \nu_x (obs) = -5.24 \times 10^{-3} \tag{8}$$

$$\Delta \nu_y(obs) = 3.31 \times 10^{-2} \tag{9}$$

If we assume that sextupole magnets and beam position monitors are perfectly aligned on designed orbit, measured beam orbit data predict difference in betatron tunes at full and zero excitation as,

$$\Delta \nu_x (cod) = -3.39 \times 10^{-3}$$
 (10)

$$\Delta \nu_y(cod) = -1.60 \times 10^{-2}$$
 (11)



Figure 3: These figures shows betatron tune variation as a function of excitation of sextupole magnets. The left figure shows the result of measurement before beam orbit adjustment. The right figure shows the tune variation after beam orbit adjustment.

	Before		Moved		After	
Name	Н	V	Н	V	Н	V
SD2.2	-2.18 mm	1.24 mm	2.18 mm	-1.24 mm	0.14 mm	0.38 mm
SD5.2	-0.39 mm	1.87 mm	0	-1.87 mm	-0.51 mm	-0.81 mm
SF7.2	-0.26 mm	-0.72 mm	+0.26 mm	+0.72 mm	-0.11 mm	-0.62 mm

Table 1: Magnetic center of sextupole magnets before and after mechanical re-alignment of the magnets.

It suggest that there exist large mis-alignment of sextupole magnets or beam position monitors.

We chose a few sextupole magnets and measured magnetic center offset of these sextupole magnet using  $\pi$ -bump method. Then we steered beam so that resulted orbit go through measured magnetic center of sextupole magnet. After this orbit correction, we observed the reduction in the tune shift caused by On/Off of sextupole magnets.

$$\Delta \nu_x(cor) = 0.84 \times 10^{-3}$$
 (12)

$$\Delta \nu_u(cor) = -1.17 \times 10^{-2}$$
(13)

Reduction of measured tune shift after orbit correction supports validity of the  $\pi$ -bump method to measure magnetic center of sextupole magnet.

Secondly, we mechanically moved some of sextupole magnets and measured magnetic center of the sextupole magnets before and after re-alignment. Result of this measurement is shown in Table 1. The first column show the name of a sextupole magnet. Horizontal and vertical positions of a magnetic center before alignment are shown in the second and third columns. The following two columns are amount of movement for alignment. Measured magnetic center of a sextupole magnet are shown in the last two columns. It should be noted that these residuals include both errors of  $\pi$ -bump method and that of mechanical alignment method.

# 6 CONCLUSION

We analyzed the beam based sextupole alignment with  $\pi$ bump method. Application of the method to the TRISTAN main ring indicates usefulness of the method. Validity of the method checked by tune shift caused by sextupole excitation change and mechanical re-alignment of the sextupole magnet.

We measured ten leakage orbits at each sextupole magnet. It took several minutes in TRISTAN MR. If we measure more leakage orbits, error in result is reduced. Fast systems for both beam orbit measurement and corrector magnet control are required.

Distance between a sextupole magnet and its nearest beam position monitor adds another complication into the analysis of data. Shorter distance between these also reduces uncertainty of the measured magnetic center of the sextupole magnet.

#### 7 REFERENCES

- "KEKB B-Factory Design Report", KEK Report 95-7, August 1995
- [2] "Beam-based Measurement of strength error in quadrupole magnets with orbit bumps", H.Koiso et al., in these proceedings
- [3] SAD is the accelerator modeling program developed at KEK. Check http://130.87.74.156/SAD/sad.html for more information.