NEW SCALING LAWS FOR BEAM ENVELOPE IN OPTICS DESIGN

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Abstract

An equation to transform first order beam optics with space charge is presented. The equation allows to use a common approach to the beam matching problem in devices of various kinds. An invariant of the transformation is function coupling dependent variables of conventional envelope equations. Scaling optical axis the transformation keeps beam size. As a computer tool the new formalism is used to design matching optics for 433 MHz RFQ.

1. INTRODUCTION

The problems to design optics for required in size and slope exit beam at given entrance one are well-known, especially in case of space-charge-dominated beam. The direct methods of designing are developed for weak beam in several simple force field structures. Various modifications of the hit-and-miss method fall to others' fate lucky not always. The main reason is that designing deals with inversion of the cause-effect relations so with incorrectness inherent in it. Moreover the modifications in the most base on the optimization technique (fitting procedures, non-linear programming methods and the like) so can solve the task successfully being applied from the good start point. In other words, solution must exist and be known if only in outline. The offered approach intends to overcome this disadvantage.

2. SCALE EQUATION

Some benefits of the approach are possible to trace on the envelope equation in an axisymmetric electrostatic lens. Field of the lens is determined by the axial potential $U(z)$ only, the beam envelope so is by start conditions $r(0)=r_0$, $r'(0)=r'_0$ and the equation

$$r''U+0.5r'U'+0.25rU''=\frac{I}{4\pi\varepsilon_0\sqrt{2(e/m)U}\,r}+\frac{E_n^2}{r^3}. \quad (1)$$

Hence the system of lens and beam can be described by a space curve on the U,r,z coordinates. The conventional application of the equation (1) is to derive the space curve projection $r(z)$ from one on plane Uz, or conversely. To consider as a given the projection on the plane Ur is idea of the offered technique. Setting some function coupling potential with beam (i.e. the coupling function) we assign <u>in advance</u> all values both to beam size and to the lens potential, including maximal, minimal and boundary ones. The last is especially valuable, being applied to beam matching.

Another merits follow from form of the equations which relate the coupling function and an optical axis:

$$\begin{cases} \dot{M}=\dfrac{M}{f_1(\lambda)}\left(f_2(\lambda)-f_3(\lambda)M^2\right) \\ M=\dfrac{dz}{d\lambda} \end{cases}. \quad (2)$$

This form is valid for all the most frequently used lenses (full equations for quadrupole, solenoid, bending magnet see [1-3]) and invariant with respect to choice of new independent variable λ. In the considered lens the function of λ can be performed by the lens potential U as well as the beam size r if the coupling function to define in explicit form $r(U)$ or $U(r)$. Nevertheless in practice the parametric representation of the coupling function $\{U(\lambda),r(\lambda)\}$ with parameter λ is more convenient. For this case the characteristic function

$$f_1(\lambda)=0.25r(\lambda)\dot{U}(\lambda)+U(\lambda)\dot{r}(\lambda).$$

With z in place λ the expressions for $f_2(\lambda)$ and $f_3(\lambda)$ are the left- and right-hand parts of the equation (1) respectively. Obviously, the solution $M(\lambda)\equiv 1$ is possible when $f_2(\lambda)\equiv f_3(\lambda)$ only, i.e. when given parametrically coupling function coincides with any solutions of the equation (1). So implying λ being measured along the optical axis, we can treat the M as a differential coefficient of scaling. Such a parametrical representation can be considered as an approximate prescription to beam and to potential of desirable behaviour. For attaining this the equation (2) should rescale the axis to fit derivatives in exact conformity with conventional envelope equation, of course if it is possible.

Coupling function is realizable (there are solutions of the equation(1) with suitable projection on plane Ur) always unless respective integral curve of the equation (2) goes through zero or has a break. Such solutions of the equation (2) can be singular only and exist when $f_1(\tilde{\lambda})=0$. In number and location the singular points are different real roots of the equation

$$\tilde{M}\left(f_2(\tilde{\lambda})-f_3(\tilde{\lambda})\tilde{M}^2\right)=0.$$

In kind they are singularities of the linearized equation

$$\dot{M}-\dot{\tilde{M}}=a\left(M-\tilde{M}\right)/\left(\lambda-\tilde{\lambda}\right)$$

with the factors

$$\begin{cases} a_1=f_2(\tilde{\lambda})/\dot{f}_1(\tilde{\lambda}) & \text{for} \quad \tilde{M}_1=0, \\ a_{2,3}=-2f_2(\tilde{\lambda})/\dot{f}_1(\tilde{\lambda}) & \text{for} \quad \tilde{M}_{2,3}=\pm\sqrt{f_2(\tilde{\lambda})/f_3(\tilde{\lambda})}. \end{cases}$$

If $a\geq 0$ $\left(\dot{f}_1(\tilde{\lambda})\neq 0\right)$ the singular point is node. Otherwise a saddle will be. The case of $\dot{f}_1(\tilde{\lambda})=0$ requests studying the higher order terms[3].

From this analysis a necessary condition for the coupling function to be realized is

$$f_2(\tilde{\lambda}_i)\,f_3(\tilde{\lambda}_i)>0,\ \ i=1\div N \qquad (3)$$

When adjacent non-zero singularities ($\tilde{M}_i \neq 0$) are of different kind or nodes, the condition (3) is also sufficient. In the absence of the singular points there is a variety of the coupling function realizations to be given by initial conditions for integration of the equation (2). Each M_0 is the different optics axis scale at which both the beam and the potential have another derivatives but satisfy the specified coupling function as well as the conventional envelope equation (the ratio of first derivatives remains unchanged). To realize specified coupling function in vicinity of the saddle point the integral curve of the equation (2) must cat the saddle centre. Such a curve is only one and uniquely determines the proper initial value of M. In the neighbourhood of non-zero nodal point the specified coupled function can be realized on set of all the integral curves passing through the node. Unless $a_{2,3}=1$ the integral curves have in the node a common tangent line. This makes possible to joint perfectly any solutions of obtained at the left and the right from the singularity, so to solve matching problem formally by substituting the required boundary values of M for initial ones.

The properties of the parametric representation suggests to utilize instead of the coupling function the beam and the potential of lens designed previously or being under tuning, if to change the lens performance is need. When given parametrically coupling function is a solution of the conventional envelope equation and allows multiplicity of realizations, integration of the equation (2) with $M_0>1$ derives lens of greater length $L=\int M(\lambda)d\lambda$ but smaller first derivatives $d/dz=M^{-1}\,d/d\lambda$. The integration with $M_0<1$ does in contrast, since the integral curves do not meet except for singular points, so the value (M-1) saves sign. The same can be done for retuning (redesigning) lens on new current, emittance, kind of particles by simple substituting their new values in $f_3(\lambda)$. From this viewpoint the equation (2) is the generalized equation of scale transformation for first order beam optics with the coupling function as invariant.

3. GENERAL PRINCIPLES

As a result may be stated that application of the equation (2) reduces designing (tuning) lenses for required performance to a sufficiently free choice (correction) of the coupling function to form or not to form the appointed singularities of the main equation. From the analysis of the equation some principles general for lenses of various kinds follow (validity and comments see [1]).

Any coupling function provides required transformation of beam size when is realizable and has the proper boundary values.

The same coupling function is realizable over a wade range of currents, emittances, kinds of particles and the like.

The coupling function can be linearly scaled still staying the realizable.

The coupling function provides required beam size with any slope of the same sign when forms non-zero nodal point adjacent to the bound.

When the coupling function forms adjacent to the bound non-zero saddle point, there is the only slope of beam to realize this one. Some boundary conditions cause these discomforts unfailingly. Required beam slope then can be achieved by variation of the coupling function fixed on the edges. Another way is an optics piece of known in advance performance, inserted in the lens to transform the boundary conditions in any more comfortable.

To optimize lens for aberration, length, etc., the coupling function can be varied without changing beam in size and slope at the edges. For doing this handy the equation (2) as a special case of the Bernoulli equation can be reduced to linear inhomogeneus one with solution in quadratures.

The same coupling function guarantees the lenses of any length no less than a certain when forms two non-zero nodal points to be alongside.

4. APPLICATION

As a computer tool the new formalism was used to redesign matching optics for 433 MHz RFQ on more heavy ions in context of extending the possible application of the machine. Forming the 25 mA $D^{\pm}$ beam instead of the 20 mA $H^{\pm}$ one and matching it into the RFQ, the optics and injector as a whole should be in agreement with conventional standards on construction, vacuum, electrical strength. The specific requirement of the designs is the simplest power supplies to apply the voltage of two values only in compliance with energy of the beam extraction from the source (16 keV) and energy of injection to RFQ (60 keV). Because of this limitation the injector involves two optical elements: 1)bending magnet of rare-earth permanent magnets to put the beam from the slit ion source into near-axisymmetric form and to separate the beam; 2) a set of axisymmetric apertures at the alternating electrostatic potentials to transport this beam within the distance sufficient for required pumping, to accelerate and match it into the RFQ. In detail the design of the $H^{\pm}$ matching optics was considered in [4]. The requirements for the $D^{\pm}$ beam at RFQ entrance see [5]. Design of the bending magnet in case of the space-charge neutralization in the back ground gas is not presented here because of the paper limit. The entrance and exit beam parameters (in mm, mrad, mA) to redesign the matching optics are listed in the Table 1.

Table 1

	r_{en}	r'_{en}	r_{ex}	r'_{ex}	I	$E_{n\,ex}$
$H^{\pm}$	9.5	15.	1.5	-50.	20.	0.40
$D^{\pm}$	8.5	10.	2.0	-40.	25.	0.16

In principle, the axial potential for the new lens could be derived by one integration of the equation (2) (shape of electrodes from this potential can be reconstructed in various ways [1,6]). In effort to illustrate some scope for the approach the scaling was divided into three steps. For each of them the shape of electrodes and the beam envelope are shown on Fig.(1-3) in dash lines before scaling, in solid ones after. The scaling was started from the $H^{\pm}$ optics by changing the space charge factor and the emittance law in $f_3(\lambda)$ (emittance was assumed to growth just higher and be at exit as the required). The next modification (see Fig.2) was to have up the beam size needed at the entrance. For this doing the beam envelope was factored suitably and again substituted in the equation (2). For finishing the main equation was integrated from such initial values of M to obtain at the bounds the beam size of required slope (Fig.3). Boundary conditions with the diverged entrance beam and the converged exit one at vanish field of the lens make possible this always.

5. REFERENCES

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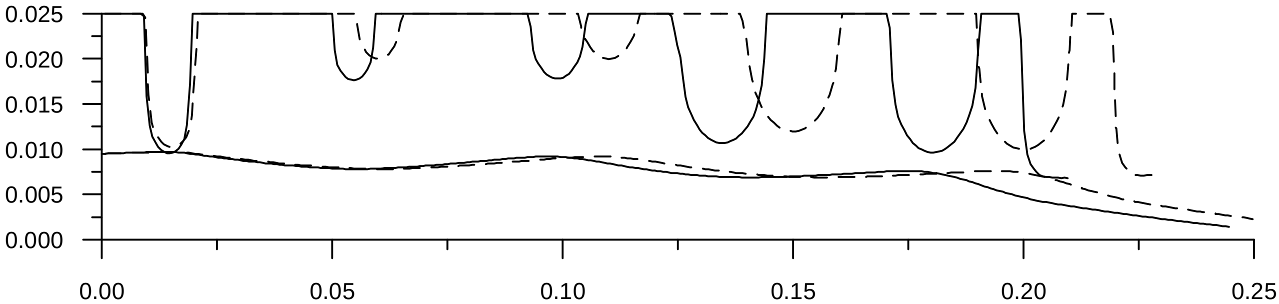

Fig.1 Beam envelope and electrode contours before and after first step of scaling.

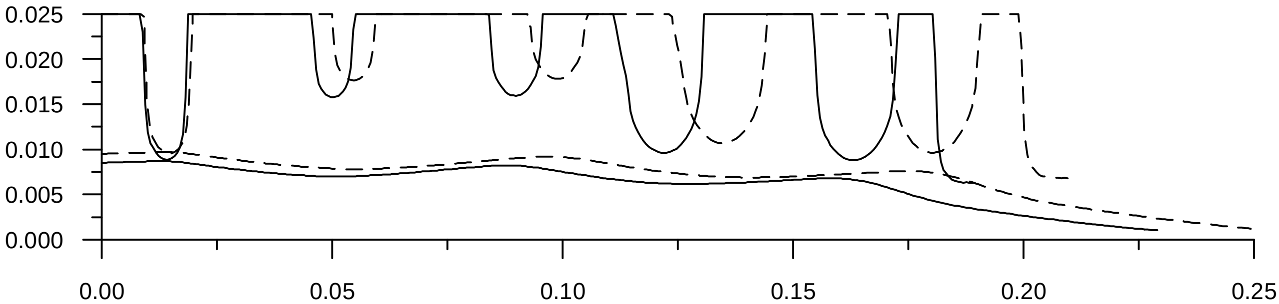

Fig.2 Beam envelope and electrode contours before and after second step of scaling.

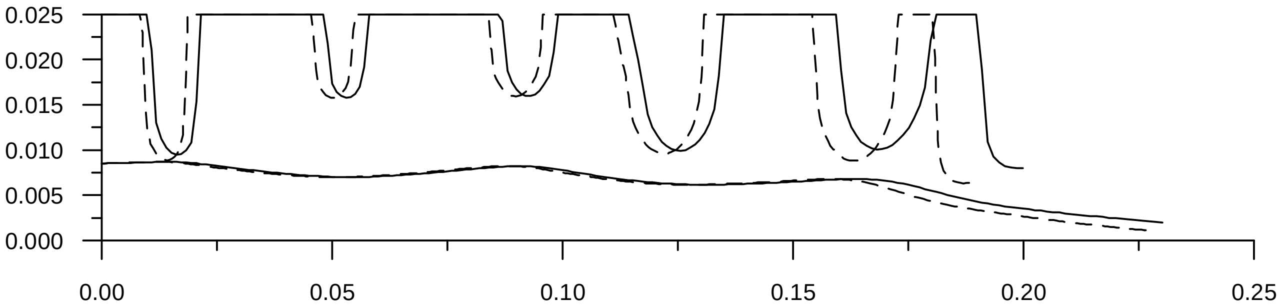

Fig.3 Beam envelope and electrode contours before and after third step of scaling.