Quadrupole Misalignment Determination at BESSY *

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1 INTRODUCTION

To increase the performance of a storage ring facility the alignment of beam to the center of the quadrupoles, under consideration of requirements resulting from the beamline construction, is necessary.

The measurement of the beam offset in the quadrupole magnets can be done by changing the strength of a single quadrupole. From the evaluation of the resulting difference orbit, the calculation of the beam offset is possible and often used.

But the resulting offset of this method is the result of the sum of closed orbit distortions from all steerer and other quadrupole offsets (as the main contributors) as well as the misalignment of the quadrupole magnet as such. So the beam-based alignment may be impossible if i.e. adjacent magnets have opposite misalignments. Therefore we programmed and applied the model from [1] which uses the same measurement procedure [2] but which allows to calculate the total offsets and the misalignment of the quadrupole magnets.

2 THEORY

An offset in a quadrupole magnet results in a new disturbed orbit resulting from the kick. The change of this disturbed orbit resulting from a change of the strength of a single quadrupole (and thus a change in the offset) is described with y_0 as original offset in this quadrupole [3]:

$$\Delta y''(s) - (k(s) - \Delta k(s))\Delta y(s) = \Delta k(s) y_0(s) \qquad (1)$$

From that one has

$$\delta\Theta_h = \Delta k(s_0) \cdot L_q \cdot x_0(s_0) \qquad (2)$$

and

$$\delta\Theta_v = -\Delta k(s_0) \cdot L_q \cdot y_0(s_0) \qquad (3)$$

if one changes the strength of a quadrupole at s_0 by Δk. Using the unperturbed linear lattice functions the respective equations are given in [2]. With the resulting kick equations and using the expression for the dipole kick distortions one can fit the kicks (and therefore the offsets) to the difference orbits respectively.

* funded by the Bundesministerium für Bildung, Wissenschaft, Forschung und Technologie and by the Land Berlin

To calculate the quadrupole misalignment vector we start with the ansatz of J. Corbett [1]:

$$\vec{x} = \vec{x}_{el} + \mathbf{C}\vec{\Theta} + \mathbf{Q}\vec{x}_q \qquad (4)$$

$\vec{x}, \vec{x_{el}}$ are the measured orbit and the electronic offsets of the orbit measurement; $\vec{\Theta}, \vec{x_q}$ are the vectors of all steerer kicks and the offsets (misalignments) in the quadrupoles. A further term which allows to get an unknown energy offset is neglected, because we know from the precise spin depolarisation energy measurement at BESSY this energy offset to be only 10^{-3}.

$\mathbf{C}$ und $\mathbf{Q}$ are the response- and the quadrupole matrices with the usual definitions:

$$C_{ij} = \frac{y(s^i)}{\Theta^j} = \Theta \cdot \sqrt{\beta(s^i)\beta_0} \cdot \frac{\cos(\pi Q - |\Psi(s^i) - \Psi_0|)}{2\sin(\pi Q)} \qquad (5)$$

$$Q_{ij} = \frac{y(s^i)}{k \cdot l \cdot \mathrm{x_q^j}} = \Theta \cdot \sqrt{\beta(s^i)\beta_0} \cdot \frac{\cos(\pi Q - |\Psi(s^i) - \Psi_0|)}{2\sin(\pi Q)} \qquad (6)$$

0 marks the place of the kick destortion. If one measures the orbit at k and at $k + \Delta k$, so one has from the difference orbit

$$\Delta\vec{x} = (\mathbf{C^k} - \mathbf{C^0})\vec{\Theta} + (\mathbf{Q^k} - \mathbf{Q^0})\vec{x_q} \qquad (7)$$

The indices k marks the response- and the quadrupole matrices at k-values $k + \Delta k$, the indices 0 the starting state at k-value k.

In the case of BESSY I with 36 quadrupole magnets and 16 BPMs the above system of equations is underdetermined. But if one repeats the measurement step for every quadrupole which means two orbit measurements representing the effect of the respective change of the k-value, results an overdetermined linear system of equations with 16*36 rows to get 36 unknowns which is solved with a least squares fit (also in the case of changing the k - values of the seven quadrupole **families**, the resulting system of equations to calculate $\vec{x}_q$ would be unequivocal).

If we define the vector $\vec{b}$ with np, ns, nq as number of BPMs, steerers, quadrupoles, we have

$$\begin{pmatrix} {}^{(1)}\Delta x_1 - \sum_{i=1}^{ns}({}^{(1)}C^k_{1i} -{}^{(1)} C^0_{1i})\Theta_i \\ \cdots\cdots\cdots\cdots\cdots\cdots \\ {}^{(1)}\Delta x_{np} - \sum_{i=1}^{ns}({}^{(1)}C^k_{npi} -{}^{(1)} C^0_{npi})\Theta_i \\ {}^{(2)}\Delta x_1 - \sum_{i=1}^{ns}({}^{(2)}C^k_{1i} -{}^{(2)} C^0_{1i})\Theta_i \\ \cdots\cdots\cdots\cdots\cdots\cdots \\ \cdots\cdots\cdots\cdots\cdots\cdots \\ {}^{(nq)}\Delta x_{np} - \sum_{i=1}^{ns}({}^{(nq)}C^k_{npi} -{}^{(nq)} C^0_{npi})\Theta_i \end{pmatrix}$$

and the matrix $\mathbf{A}$:

$$\begin{pmatrix} {}^{(1)}Q^k_{11} -{}^{(1)} Q^0_{11} & \cdots & {}^{(1)}Q^k_{1nq} -{}^{(1)} Q^0_{1nq} \\ \cdots\cdots & & \cdots\cdots \\ {}^{(1)}Q^k_{np1} -{}^{(1)} Q^0_{np1} & \cdots & {}^{(1)}Q^k_{npnq} -{}^{(1)} Q^0_{npnq} \\ {}^{(2)}Q^k_{11} -{}^{(2)} Q^0_{11} & \cdots & {}^{(2)}Q^k_{1nq} -{}^{(2)} Q^0_{1nq} \\ \cdots\cdots & & \cdots\cdots \\ \cdots\cdots & & \cdots\cdots \\ {}^{(nq)}Q^k_{np1} -{}^{(nq)} Q^0_{np1} & \cdots & {}^{(nq)}Q^k_{npnq} -{}^{(nq)} Q^0_{npnq} \end{pmatrix}$$

from that the overdetermined system of equations results

$$\mathbf{A}\vec{x}_q = \vec{b} \tag{8}$$

and from the condition of minimalization

$$r = |\mathbf{A} \cdot \vec{x_q} - \vec{b}| \tag{9}$$

one gets

$$\mathbf{A^T A}\vec{x}_q = \mathbf{A^T}\vec{b} \tag{10}$$

To get a quantitative information about the conditioning of the system of equations, here it is better to calculate the solution with the SVD. From that one has [4]:

$$\vec{x}_q = \mathbf{V} \cdot diag(1/\omega_i) \cdot (\mathbf{U^T} \cdot \vec{b}) \tag{11}$$

The $nq\omega_i$'s of the diagonal matrix are the eigen values of the matrix $\mathbf{A^T A}$. The matrix is ill conditioned if:

$$\frac{\omega_{min}}{\omega_{max}} < 10^{-6}, 10^{-12} \tag{12}$$

The two numbers are for the application of single or double precision accuracy.

3 APPLICATION

In the last years at BESSY a large closed-orbit application program package has been developed, which is based on three C++ libraries (optic, machine, graphic). The optical model is well established and has been improved with fits of the measured and calculated response matrices.

The measurement of the quadrupole offsets is done in measuring the orbit differences resulting from the change of a single quadrupole magnet strength of about 1 % for all quadrupoles around the ring [2].

The program POSQUAP calculates as a first step the actual total offset in the given quadrupole with a fit of the kick at the difference orbit using the beam-based alignment method of [2]. From the resulting trajectory, faulty BPM values are detected and excluded from further consideration.

The numerical solution is implemented as:

0.) Exclusion of faulty BPM values
1.) Calculation of the TWISS functions for k - value k
2.) Calculation of the response matrix for k - value k
3.) Calculation of the quadrupole matrix for k - value k
4.) $nq \cdot 2$ orbit measurements
    Difference orbit
    TWISS functions for k - value $k + \Delta k$
    Response matrix for k - value $k + \Delta k$
    Quadrupole matrix for k - value $k + \Delta k$
    Change of the effects of the steerer kicks in $\vec{b}$
    Expansion of the matrix $\mathbf{A}$
    with difference of quadrupole matrices
5.) Singular value decompositon of the matrix $\mathbf{A}$
6.) Control of the conditioning (typically 10^{-3}, ok)
7.) Offset vector $\vec{x}_q$

same procedure for the other direction
(horizontal) (from point 4)

8.) Graphics

Because the calculation of the offsets is based on the evaluation of the difference orbits, with the input of the calculated $\vec{x}_q$ in the equation (4), the calculation of the vectors of the electronic offsets $\vec{x}_{el}$ is possible.

As an intermediate result the fit of the kick from difference orbits is shown in fig. 1 and fig. 2:

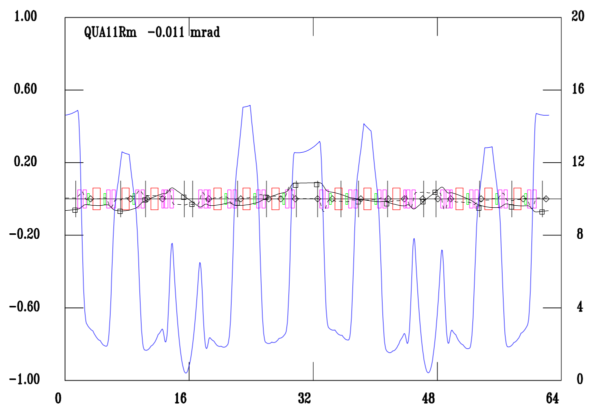

Figure 1: Fit of the vertical closed orbit distortion created by a 1 % k - value change of the first quadrupole, betas rhs, orbit lhs, squares measured difference orbit values.

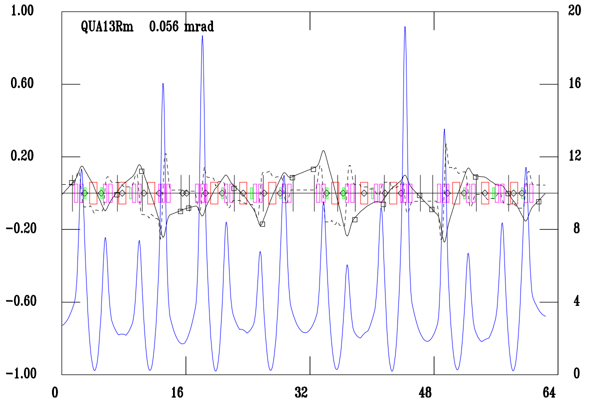

Figure 2: Fit of the horizontal closed orbit distortion created by a 1 % k - value change of the third quadrupole, betas rhs, orbit lhs, squares measured difference orbit values, solid: kick trajectory, beta function

As result the quadrupole misalignment values at BESSY are given in fig. 3, for the offset of the beam in the quadrupoles see [2].

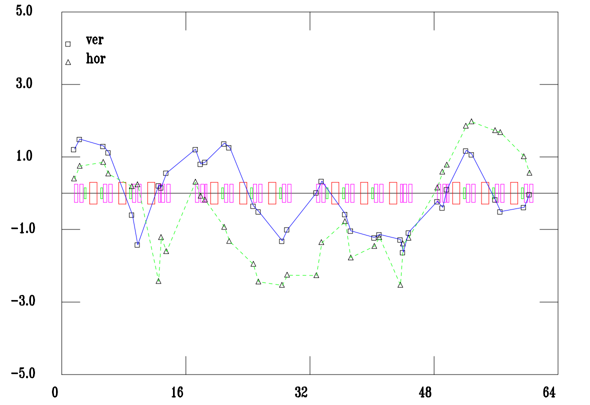

Figure 3: Vertical (solid) and horizontal (dashed) quadrupole misalignment at BESSY.

4 REFERENCES

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