# NUMERICAL SIMULATION OF 3-D FIELD OF SYSTEMS USING PERMANENT MAGNETS

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## 1 INTRODUCTION

Permanent magnets find extensive application in various electrophysical, electrical and electronic devices.

In context of sophistication of technologies and the advent of new high-coercivity materials with high specific energy, the demand is growing for effective algorithms for computing magnetic systems with permanent magnets. Taking into account the nonlinear dependence between external magnetic field and magnetic properties of materials is essential for such systems to be thoroughly optimized.

Lastly, the mutual consideration is required for systems including retentive and non-retentive structural components and conducting elements as well.

## 2 THE FINITE ELEMENT PACKADGE KOMPOT

The KOMPOT package [1] is intended for numerical simulation of 3D magnetostatic fields of magnetic systems containing nonomitlinear retentive materials. The KOMPOT package has been verified by correction with the analytical solutions, magnetic measurements data, results obtained with using other 3D and 2D programs developed later.

Magnetic field strength vector $\vec{H}$ can be determined using a unique continuous scalar potential throughout a calculation region (including 3D magnetostatic fields of magnetic systems and current subregions) as [2,3]: $\vec{H} = gradV + \vec{P}$, where an additional vector $\vec{P}$ must meet the condition $rot\vec{P} = \vec{j}$, j is current density vector. Using an additional gauge condition, $\vec{P}$ can be presented in a form to be convenient in practical use, for example, when minimizing the volume of region occupied by the vector $\vec{P} \cdot \vec{n} = 0[4, 5]$. An alternative method for constructing $\vec{P}$ is to define the only $\vec{P}$ component being normal to a shell of arbitrary shape spanned on a current thread, the population of those forms a real conductor with current [2,3].

In particular, by using the system of coordinate $n, \tau, b$, $\vec{P}$ can be presented as [6]

$$\vec{P} = P_n \cdot \vec{e}_n$$

where

$$P_n = \frac{1}{h_n} \int\limits_{\tau_*}^{\nu(\tau)} j h_n h_\tau d\tau$$

$$\nu(\tau) = \{\tau : \tau_* < \tau < \tau^*; \tau^* : \tau > \tau^*\}$$

Here vector $\vec{e_n}$ is normal to the shell which covers the current thread with the cross section $ds = h_\tau d\tau \times h_n dn$. The unit vectors $\vec{e_\tau}$ and $\vec{e_b}$ are tangent to the shell, and $\vec{e_b}$ is opposite to the vector of the thread current $\vec{\tau} = \vec{j} ds. h_n, h_{tau}, h_b$ are the Lame coefficients, $\tau_*, \tau^*$ determine the coil cross section. A generalization for the case of coils with finite cross section is apparent.

This method of constructing the vortex vector permits to develop a reasonably adaptable approach for defining field sources being coupled with coils of complicated geometrical shape.

Analysis of the field has been performed on the permanent magnet quadrupole (radius of aperture $r_0 = 1.41$ cm) to study the field impact on particle dynamics. Permanent magnets have been made of $SmCo$ alloy, magnetization curve of which is close to a straight line. Not only the field distribution but field gradient distribution has also been analyzed. The accuracy of magnetic field gradient value calculation was obtained by comprasion with the magnet measurement and was about 1%.

## 3 ANALYTICAL SOLUTION BASED ON THE DIAMOND AND COFIE PACKAGES

In general formulation the problem can be solved using the integral equation method [10]. A model of the magnetization vector $\vec{M}$ being constant throughout the volume of an element is employed for magnetic materials. In this case

$$\vec{H} = -\frac{1}{4\pi} \sum_{k=1}^{k} (\vec{n}, \vec{M}) \int_{S_k} \frac{\vec{r} ds}{r^3} \quad ,$$

where

- $S$ surface of the $K_{th}$ face;
- $k$ number of the face;
- $n$ external normal to S at the integration point;
- $\vec{r}$ radius vector.

The development of electromagnetic systems includes a system synthesis. Then it is desirable to use high performance programs based on the precise analytic expressions to be applied for less detailed models. For this purpose DIAMOND and COFIE packages were developed to calculate fields of permanent magnets and current systems of arbitrary form. Each of the packages is a library of program modules which can be integrated in problem-oriented computational programs.

DIAMOND package allows to calculate magnetic fields of uniformly magnetized polyhedrons. A polyhedron field is the sum of vectors being worked out for each edge of the polyhedron using following analytical expressions

$$H_\xi = \sum_{j=1}^{N}\sum_{k=0}^{1}(-1)^k \left(A_{\xi,j}\, sh^{-1}\frac{\eta_{j+k}+B_{\xi,j}}{C_{\xi,j}}\right)\frac{M}{4\pi}$$

$$H_\eta = \sum_{j=1}^{N}\sum_{k=0}^{1}(-1)^k \left(A_{\eta,j}\, sh^{-1}\frac{\xi_{j+k}+B_{\eta,j}}{C_{\eta,j}}\right)\frac{M}{4\pi}$$

$$H_\zeta = \sum_{j=1}^{N}\sum_{k=0}^{1}(-1)^k \left(-\varphi_{j+k}+sin^{-1}\frac{sin(\varphi_{j+k}-\psi_j)}{C_{\zeta,j}}\right)\frac{M}{4\pi}$$

where

| | |
|---|---|
| $j$ | edge index for appropriate face (j=1, ... N); |
| $N$ | number of edges for the face; |
| $\xi, \eta$ | coordinates lying in the plane of the face; |
| $\zeta$ | normal; |
| $A, B, C$ | coefficients for corresponding edges, being expressed using rational functions through coordinates of the edges ends. |

COIFE package provides field calculation for current systems consisting of rectangular and arched elements with polygonal cross sections. Arched elements field is found by using multiple precision procedure for calculation of elliptic integrals of 1, 2, 3 kinds. A polyhedron field is found in a similar manner

$$\vec{H} = \frac{1}{4\pi}\left[\vec{J}\times\sum_{i=1}^{N_s}\sum_{j=1}^{N_{e,i}}\sum_{k=0}^{1}(-1)^k\left[A_j\cdot lnB_j + d\cdot tg^{-1}C_j\right]\right]$$

where

| | |
|---|---|
| $J$ | current density vector for polyhedron; |
| $N_s$ | member of faces; |
| $N_{e,i}$ | number of edges for $i^{th}$ face; |
| $A_j, B_j, C_j, d$ | coefficient for corresponding edges being expressed using rational functions through coordinates of edges ends. |

The programs are compiled with pre- and postprocessors to minimize the bulk of the relevant data, as they automatically determine whether a point of observation is inside of the polyhedron.

Calculation of a magnetic field of a uniformly magnetized cylinder was taken as a test problem. The sizes of the cylinder were as follows:

radius — $a = 1.5cm$

length — $l = 2b = 6cm$, $\vec{M} = 80kA/m$

The vector of magnetization was directed along the cylinder's axis and was equal to 80kA/m. With an inscribed polyhedron in place of the cylinder, the following values of induction were obtained (the center of the cylindrical coordinate system coincides with the center of gravity of the cylinder):

Table 1:

| $Z$ | $\vec{M}$ | $\vec{B}$ | $\vec{H}$ |
|---|---|---|---|
| $(mm)$ | $(kA/m)$ | $(mT)$ | $(kA/m)$ |
| 0. | 80. | 90. | -8.4 |
| $b-\epsilon$ | 80. | 48.6 | -41.3 |
| $b+\epsilon$ | 0. | 48.6 | 38.7 |

These values were found to be in complete agreement with those cited in [11].

The DIAMOND package was used for field compensation for MRI [12] and for choosing parameters of a hexapole of the ECR ion source [9].

The hexapole is a set of sectional rings for which the vector of magnetization lies in a plane perpendicular to the longitudinal axis.

With the given length and inner radius depending on the length and a outer radius of an ionization chamber, the optimization of the hexapole's design was limited to varying both the outer radius of the magnetic rings and the number of permanent magnet sections. As an optimization criterion the efficiency in the use of materials for permanent magnets was chosen.

The hexapole consists of 4 rings of the same length with the number of sections $N$ =18. Sections are made of magnets $NdFeB$ with the residual induction $B_0$ =10.5 kgs. The system has been fabricated and measured. The deviation between the results of measurements and those of calculations does not exeed 1%, which results first of all from difference in properties of materials for permanent magnets.

## 4 CONSIDERATION OF NONLINEAR EFFECTS

The finite element sampling of the equation $div\mu_{PM}gradV = -div\vec{M}_0$ involves no serious difficulties, as the curves $\mu_{PM}(H)$ and $\mu_{FE}(H)$ are of the same nature.

Thus, it is possible to consider retentive structural elements integrally with non-retentive ones.

A system of algebraic equatious can be solved using the symmetrical successive overralaxation method (SSOR) combined with a polinomial acceleration of a convergence rate on the basis of Chebyshev's procedures [7] and $B$-T acceleration process [8].

An integral approach requires the system of algebraic equations to be solved with respect to vectors of magnetization which are associated with uniformly magnetized bulks. In this case non-retentive structural elements aredescribed by means of a model of uniformly magnetizad bulks as well $\vec{H} = A\vec{M} + \vec{H}_{ext}$ where $A$ impact matrix; $M$ column vector for solutions; $\vec{H}_{ext}$ column vector for external fields allowance was made for the dependence $\mu = \mu(H)$

In this algorithm magnetization for retentive materials is determined by a magnetic field component being parallel to the axis of magnetization, while that for non-retentive materials depends on a total field.

On the basis of COFIE and DIAMOND packages CLONDIKE package has been developed which allows to use these packages in combination with an interative procedure for selecting values of magnetization vector components.

Fig. 1 gives the scheme of the magnet system for a magnetic field concentrator. The magnetic system is built up as a cylindrical solenoid with ferromagnetic cover plates.

Field analysis for alternatives of the system design was made using KLONDIKE packadge. Field distribution for one alternative is presented in Fig. 2.3.

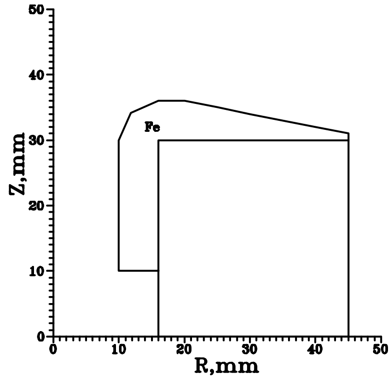


Figure 1: Concentrator model

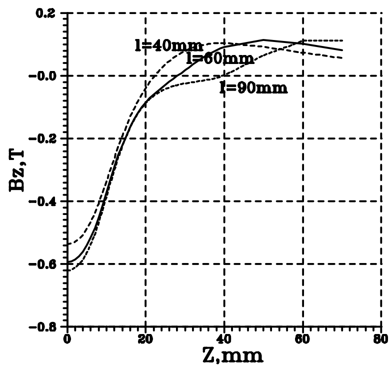


Figure 2: Distribution of magnetic field $B_Z$ along the concentrator's axis $O_Z$ for different sizes $l$ of the concentrator (along axis $O_Z$)

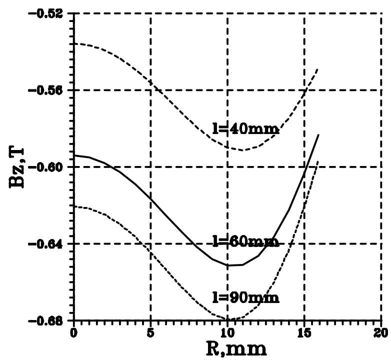


Figure 3: Distribution of magnetic field $B_Z$ along the concentrator's axis $O_R$ over the operating region for different sizes $l$ of the concentrator (along axis $O_Z$)

## 5 CONCLUSION

The presented modules are the major soft component for numerical simulation of 3D magnetostatic field. The described CAD/CAM subsystem allows to simulate a quite wide range of magnetic systems for electrophysical and electrical engineering devices under real design condition.

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