ALIGNMENT OF THE LINAC WITH THE HELP OF RADIATION FROM THE QUADRUPOLES OF THE FOCUSING SYSTEM

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Abstract

Focusing system as a FODO structure is, in principle, an undulator. Radiation from this undulator is split mainly in two frequency regions connected with the periodicity of the focusing system and with the betatron wavelength. Intensity of the radiation is connected with the transverse dimensions of the beam. This paper describes the properties of this radiation. An algorithm of misaligned element identification by using the spectrum analysis of the radiation is described also.

Some methods of the relativistic particle beam diagnostics and alignment in linear colliders using the radiation emitted by the particles in the external fields were considered in the papers: synchrotron radiation-[1], soft radiation in quadrupoles -[2] and strange radiation, hard radiation in undulators and quadrupoles -[3]. In [4] there was considered utilization of radiation from single quadrupole lens for the beam positioning. In [5] there was calculated an amount of radiation in the focusing system of high energy linear collider and there was proposed to use this radiation for alignment.

Some other point of interest for consideration of radiation from quadrupole wiggler connected with possible utilization of such a wiggler as a pick-up in the method of optical stochastic cooling [6]. In this case the energy is fixed and the description of the radiation properties is simpler.

We will use the solution of the equation of motion in periodic system in the form of Floquet function, which makes consideration more easy and clean. We will consider the radiation from a finite number of the periods by a particle moving in one plane. So the radiation is linearly polarized in the plane of oscillations.

A differential equation of a transverse particle motion in a quadrupole focusing system is determined by Hill's equation

$$\frac{d^2x}{d\vartheta^2}+g(\vartheta)x=0$$

where $g(\vartheta)=eG(\vartheta)L^2/4\pi^2mc^2\gamma$, $G(\vartheta)$ is the gradient of the quadrupole magnetic field, $\gamma=\varepsilon/mc^2$ is the relativistic factor, $\vartheta=2\pi z/L$, L is the period of the focusing system, z is the longitudinal coordinate. The function $g(\vartheta)$ is the periodic one, $g(\vartheta+2\pi)=g(\vartheta)$, and $\int_{-\infty}^{+\infty}g(\vartheta)d\vartheta=0$

The solution of the equation of motion has the form

$$x=2a|f(\vartheta)|Cos[\nu\vartheta+\chi(\vartheta)+b]$$

where a, b are real constants determined by initial conditions, $|f(\vartheta)|$ and $\chi(\vartheta)$ are the modules and argument of the 2π-periodic Floquet function, $\nu=\frac{1}{2\pi}\int_0^{2\pi}|f(\vartheta)|^{-2}d\vartheta$ is the betatron frequency of particle oscillation in the focusing system [7]. The value $\hat\beta(\vartheta)=|f(\vartheta)|^2$ is the envelope function, and $\chi(\vartheta)=\int_0^{\vartheta}d\vartheta/[\hat\beta(\vartheta)-\nu]$.

Let us represent the solution in the form

$$x=2a[\psi_1(\vartheta)Cos\,\nu\vartheta+\psi_2(\vartheta)Sin\,\nu\vartheta]$$

$$\psi_1(\vartheta)=|f(\vartheta)|Cos[\chi(\vartheta)+b],\ \psi_2(\vartheta)=|f(\vartheta)|Sin[\chi(\vartheta)+b]$$

and using the expansion $\psi_{1(2)}=\sum_{-\infty}^{+\infty}\psi_{1(2),n}exp(in\vartheta)$ we can represent it in the final form $x=x_\nu+x_s$, where

$$x_\nu=x_0Cos(\nu\vartheta-b_0),$$

$$x_s=(1/2)\sum_{n=1}^{\infty}x_n'Cos[(\nu+n)\vartheta+b_n']+x_n'Cos[(\nu-n)\vartheta+b_n''],$$

$$x_0=2a\sqrt{\psi_{10}^2+\psi_{20}^2},\quad b_0=arccos(\psi_{10}/\psi_{20}),$$
$$x_n'=2a|\psi_{1n}-i\psi_{2n}|,\quad x_n''=2a|\psi_{1n}+i\psi_{2n}|,$$
$$b_n'=arg(\psi_{1n}-i\psi_{2n}),\quad b_n''=arg(\psi_{1n}+i\psi_{2n}).$$

Usually $x_\nu>>x_s$. It means, according to equation of motion, that $d^2x/d\vartheta^2\cong g(\vartheta)x_\nu$. I.e. for calculation of the acceleration we use the smoothed trajectory and calculate the magnetic field as a function of the particle position in the quadrupole lens and, hence, acceleration.

Assuming that $g(\vartheta)$ is odd function and using the expansion $g(\vartheta)=\sum_1^{\infty}2g_nCos(n\vartheta)$, we can represent $d^2x_s/d\vartheta^2$ in the form

$$d^2x_s/d\vartheta^2\cong x_0\sum_1^{\infty}g_n[Cos(n+\nu)\vartheta+Cos(n-\nu)\vartheta]$$

where $n=2k-1, k=1,2,3...$

Spectral angular distribution of the energy emitted by the particle and falling on the area dS is determined by the expression

$$\frac{\partial^2 \varepsilon}{\partial \omega \partial S} = c\left|\vec{E}_\omega\right|^2,$$

where $\vec{E}_\omega = \frac{1}{2\pi}\int_{-\infty}^{+\infty} \vec{E}(t)\exp(i\omega t)dt$, $\vec{E}(t)$ is a vector of the electric field strength emitted by the particle

$$\vec{E}(t) = \frac{e}{cR}\frac{\vec{n}\times((\vec{n}-\vec{\beta})\times\dot{\vec{\beta}})}{(1-\vec{n}\vec{\beta})^3}\Big|_{t'=t-R(t')/c},$$

where e is a particle charge, R is the distance from the particle to the observation point, t' is the emission moment, t is the moment of observation.

In dipole approximation $\beta_{\perp max} << 1/\gamma$, which is in principle, the requirement for the undulatority factor K on the trajectory of the particle, $K = \beta_\perp \gamma = eGaL/(2\pi mc^2)$ is small, $K<<1$ ($\beta_{\perp max}$ is the maximal magnitude of $\beta_\perp$). Here Ga is the magnetic field value on the maximal deflection from the axis. The bending angle in the quadrupole lens $\varphi \cong x_0 Ga/(HR)$, where (HR) is the magnet rigidity of the particle. This angle one can compare with the angle of divergence of the radiation $\approx 1/\gamma$. In typical case $\gamma\varepsilon_b \cong 10^{-4}$ cm rad, $\gamma \cong 10^4 \div 10^5$, one can obtain, than $\gamma\varphi \approx 10^{-2} \div 10^{-3}$, depending of the envelope function value. So we can write

$$\vec{n}\times((\vec{n}-\vec{\beta})\times\dot{\vec{\beta}}) = (\vec{n}-\vec{\beta})(\vec{n}\dot{\vec{\beta}}) - \dot{\vec{\beta}}(\vec{n}(\vec{n}-\vec{\beta})) \cong$$
$$\cong -\dot{\vec{\beta}}(1-\vec{n}\vec{\beta}) \approx -\dot{\vec{\beta}}(1-\beta\vec{k}\vec{n})$$

$\vec{\beta} = \vec{\beta}_\parallel + \vec{\beta}_\perp \cong \vec{k}\beta_\parallel$, $\dot{\vec{\beta}} \cong \dot{\vec{\beta}}_\perp$ the values for the electrical field are

$$\vec{E}(t) = -\frac{e}{c\bar{R}}\frac{\dot{\vec{\beta}}_\perp}{\left[1-\beta\left(\vec{n}\vec{k}\right)\right]^2}, \quad \vec{E}_\omega = -\frac{e}{c\bar{R}}\frac{\dot{\vec{\beta}}_{\omega'}}{\left[1-\beta\left(\vec{n}\vec{k}\right)\right]^2}e^{i\varphi}.$$

where $\vec{k}$ is an unit vector along the longitudinal axis z, $\vec{\beta}_\perp, \vec{\beta}_\parallel$ are the transverse and longitudinal components of the particle velocity $\vec{\beta}$, $\bar{R}$ is an average distance from emission region to the observation point, $\varphi_i = \omega t_i$ is the phase of the wave emitted by a particle "i" in a moment t_i corresponds to the moment, when the particle enters a quadrupole wiggler. It was supposed that the longitudinal dimension of the emission region $L_{em} \cong ML << \bar{R}$, where M is the number of periods in the region, $\left(\vec{n}\vec{\beta}\right) << 1$, $\vec{n} = const$.

One can see that the time dependence of the electric field strength $\vec{E}(t)$ under conditions of dipole approximation copy the time dependence of the particle acceleration $\dot{\vec{\beta}}(t')$ in a time scale $1/\left(1-\beta\left(\vec{k}\vec{n}\right)\right)$. Time t' is connected with the longitudinal coordinate of the particle

$$t' = t_i' + z/\beta c = t_i' + L\vartheta/2\pi\beta c = t_i' + \vartheta/\Omega_0,$$

where $\Omega_0 = 2\pi\beta c/L$ is the frequency, connected with the periodicity of the focusing system, t_i' is the time of individual particle,

$$\frac{t-t_i}{t'-t_i'} = 1-\beta(\vec{k}\vec{n}), \quad \frac{dt}{dt'} = 1-\beta(\vec{k}\vec{n}), \quad \frac{\omega'}{\omega} = 1-\beta(\vec{k}\vec{n}).$$

That it is why the Fourier transform of the particle acceleration is

$$\dot{\vec{\beta}}_{\omega'} = (1/2\pi)\int_{-\infty}^{+\infty}\dot{\vec{\beta}}(t')\exp(i\omega' t')dt' = \left(L/2\pi\beta c\right)\dot{\vec{\beta}}_\Omega = \dot{\vec{\beta}}_\Omega/\Omega_0$$

where $\Omega = \omega' L/2\pi\beta c = \omega L\left[1-\beta\left(\vec{k}\vec{n}\right)\right]/2\pi\beta c \cong \omega/2\gamma^2\Omega_0$.

Let us consider the particle motion in the system of quadrupoles as a FODO structure, Fig.1 In this case taking

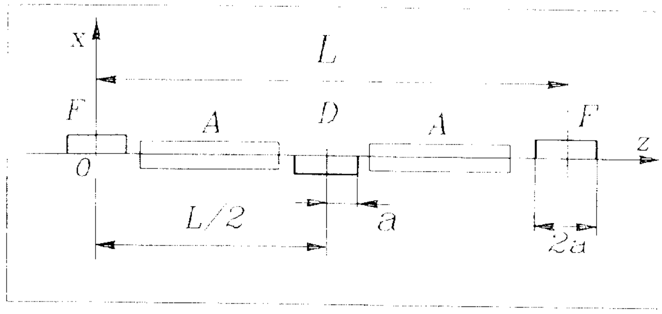

Fig.1 The focusing system. F -is the focusing lens, D-is the defocusing one, A -is the accelerating structure.

into account expression for $d^2x_i/d\vartheta^2$, one can obtain for a finite radiation region, containing M periods

$$\dot{\vec{\beta}}_{\omega'} = \frac{\Omega_0^2}{c}\left(\frac{d^2x}{d\vartheta^2}\right)_\Omega =$$
$$= -\frac{x_0\Omega_0\nu^2}{2c}\left\{\frac{Sin\left[2\pi(\nu-\Omega)M\right]}{\nu-\Omega} + \frac{Sin\left[2\pi(\nu+\Omega)M\right]}{\nu+\Omega}\right\} -$$

$$-\frac{x_0\Omega_0}{2c}\left\{\sum_{n=1}^{\infty} g_n\left[\frac{Sin\left[2\pi(n+\nu-\Omega)M\right]}{n+\nu-\Omega} + \frac{Sin\left[2\pi(n-\nu-\Omega)M\right]}{n-\nu-\Omega}\right]\right\}$$

where $\nu^2 = \left(\frac{eGa}{2mc\gamma}\right)^2\left(1-\frac{4a}{3L}\right)$, $g_n = G(2/\pi n)Sin(\pi an/L)$, for $n = 2k-1$, or $g_n = 0$, for $n = 2k$ [8]. The spectrum of the radiation is represented by spectral lines $(n \pm \nu - \Omega)\Omega_0$ around the harmonics of the main frequency up to maximal values in the wavelengths $\approx a/\gamma^2$. The main frequency Ω_0 which corresponds the period of the FODO structure and its harmonics have a sideboards satellites, connected with the betatron motion.

Here we supposed that the energy does not change in the wiggler.

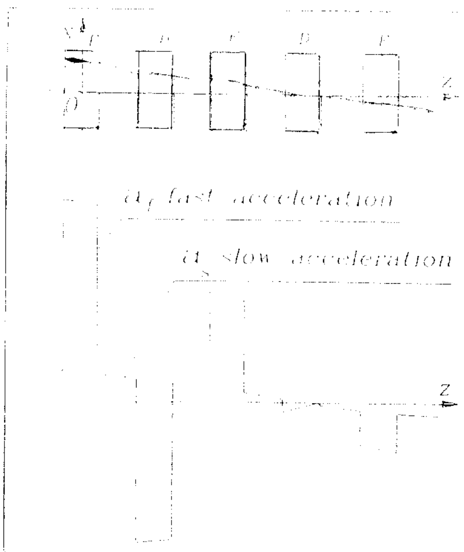

Fig.2. The qualitative structure of radiation from the quadrupole wiggler.

For estimation the ratio between the fast and slow acceleration, one can write

$$a_{fast} \cong \left|c\dot{\vec{\beta}}\right| \cong \left|e(\vec{k}\times\vec{H})/m\gamma\right| \cong x_0 Gc^2/(HR) = x_0 c^2/aF ,$$

where $F=(HR)/Ga$ is the focus distance of the lens. For the slow part we have $a_{slow} \cong x_0(2\pi\nu c/L)^2 = x_0\Omega_0^2$, so the ratio is $a_{fast}/a_{slow} \cong L^2/4\pi^2 Fa\nu^2$. As $F \cong L/\mu$, where μ is the phase shift per one period, $a_{fast}/a_{slow} \cong L\mu/(4\pi^2\nu^2 a)$. for some reasonable parameters $\mu \approx \pi/2$, $L/a \cong 10$, $\nu \cong 0.25$ the ratio $a_{fast}/a_{slow} \cong 10$. Thus one can see the importance of this input to radiation.

The formula for electric field $\vec{E}_\omega$ and for $\dot{\vec{\beta}}_\omega$ solves the problem. On the Fig.3 there is represented the structure of radiated field for different ratio of the betatron wavelength and the wiggler period. Generally if the particle has a betatron wavelength much higher, than the period of the wiggler, the spectrum looks the similar to the spectrum of radiation from the dipole wiggler for the field strength $H \approx Gx_0$ and period, equal to period of FODO structure.

Now we consider the way of identification of the element by the spectrum analyses, noticed briefly in [2]. One can see, that the radiation exists at the harmonics of the main frequency $\omega \cong 2\Omega_0\gamma^2$, which has a strong dependence of the energy. Around this frequency there are

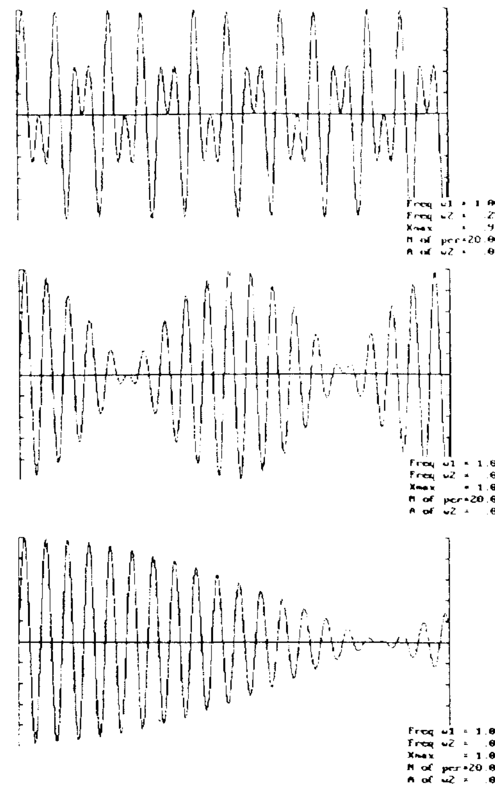

Fig.3. The structure of radiation from the quadrupole system for different ratio ν/Ω_0.

From up to down: 0.25; 0.05; 0.015

the satellites, connected with the betatron motion. If the wavelength of the betatron motion is keeping the same during the acceleration and the periodicity of the lenses are the same, then the identification becomes obvious. For typical energy $\gamma \cong 10^4 \div 10^5$, $L \cong 1m$, $\Omega_0/2\pi \cong 3\cdot 10^{10}$ 1/sec, one can obtain, than $\hbar\omega \cong \hbar 2\Omega_0\gamma^2 \cong 2.5\cdot 10^4 \div 10^5$ eV. If the radius of irises in accelerating structure is r, then the distance on which the radiation touches the surface, is $r\gamma$.

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