# Optimization of Beam Transport in a Long Recirculating Linear Accelerator

A. Savalle, J.L. Belmont, J.M. De Conto, J.M. Loiseaux
Institut des Sciences Nucléaires-CNRS/IN2P3-Université Joseph Fourier
53, Avenue des Martyrs F-38026 Grenoble FRANCE

*Abstract :*

Issues related to the beam transport in a recirculating linear accelerator are discussed. A simplified model for optical functions at second and higher passes is described. This model enables linac acceptances and sensitivity to misalignments to be obtained by use of an analytical approach. It is shown that the correction of the first pass central trajectory is not a guarantee for the trajectories at other passes. However, an optimization procedure exists. It leads to a local correction of each misalignment. Useful data could also be obtained by beam-based alignment methods.

## 1. INTRODUCTION

Linear accelerators enable the production of cw electron beams of high energy and high quality. Passing through the same linac (or linacs) several times reduces the linac (RF, cryogenics) cost. However, the beam quality is reduced by synchrotron radiation in recirculation arcs. In the ELFE (Electron Laboratory For Europe) project [1], the best compromize to get a 15 GeV beam is a 3 pass, one linac (5 GeV, 1 km) scheme.

In such an accelerator, the linac is a common structure for beams of different energies. Some particular questions arise :

- the decrease in focusing from pass to pass.
- beam line linearity / correction of the beam central trajectories.

In addition, for high beam intensities, regenerative beam break-up may be caused by recirculation. This is not true for the ELFE project, and this will not be considered here.

Analytical methods and simulations have been used to study the beam transport in such an accelerator. The question of the correction of the beam central trajectories has been emphasized.

## 2. SIMPLIFIED ANALYTICAL MODEL

The focusing along the linac is assumed to be achieved by FODO cells. In these cells, some well-known equations relate the values of the betatron function : $\beta_F$ (respectively $\beta_D$) in the focusing and the defocusing quadrupoles, and the betatron phase advance per cell ($\mu$) to the distance between lenses, d, and the focal length f.

In these equations, the focal length is supposed to be constant, so that the structure is periodic. In a recirculating linear accelerator, this may be true for one pass (with increasing magnetic gradient along the linac), but more generally, f varies from one pass to another, and from one cell to another. Let $f_p(i)$ be the focal length at cell i and at pass p (the variation over one cell is supposed to be negligible). $f_p(i)$ varies as the inverse of the beam energy (with the possible exception of the first pass). The beam energy at cell i and pass p is :

$$E_p(i) = E_0 - (i-1)g + (p-1)\,\Delta E \qquad (1)$$

where $E_0$ is the injection energy, $g$ is the energy gain par cell, $\Delta E$ the energy gain per pass.

From the well-known equations for FODO, the following ones are derived, for the cell i and the pass p :

$$\cos\mu_p(i) = 1 - d^2 / 2(f_p(i))^2 \qquad (2)$$

$$\beta_F(p,i) = \frac{d}{\sin(\mu_p(i)/2)}\sqrt{\frac{1+\sin(\mu_p(i)/2)}{1-\sin(\mu_p(i)/2)}} \qquad (3)$$

$$\beta_D(p,i) = \frac{d}{\sin(\mu_p(i)/2)}\sqrt{\frac{1-\sin(\mu_p(i)/2)}{1+\sin(\mu_p(i)/2)}} \qquad (4)$$

It is assumed, also, that the values of the $\alpha$ function ($\alpha = -\beta'/2$) in the middle of the quadrupoles are :

$$\alpha_F(p,i) = \alpha_D(p,i) = 0 \qquad (5)$$

These equations give some useful characteristics of the beam transport. The linac acceptances may be calculated this way. Indeed, with the additional hypothesis that the acceptance is assimilated to a straight ellipse, the acceptance area is :

$$a = \pi\, x_{e\max}\, x'_{e\max} \qquad (6)$$

$x_{e\max}$ and $x'_{e\max}$ are, respectively, the maximum position and angle of the particles tolerated at the linac entrance.

Eq. (6) becomes, with $A$ the maximum tolerated displacement of the particle trajectory inside the linac :

$$a = (\pi A^2 / T_{11\max} T_{12\max}) \qquad (7)$$

$T_{11\max}$ and $T_{12\max}$ are respectively the maximum of the $T_{11}$ and $T_{12}$ term of the transfer matrix from the linac entrance to any point in the linac. These terms may be calculated using the equations (2) to (5). Indeed, they may be written :

$$T_{11\max} = Max(\sqrt{\beta(p,0)/\beta(p,i)}\cos\left[\sum_{j=1}^{i-1}\mu_p(j)\right]) \quad (8)$$

$$T_{12\max} = Max(\sqrt{\beta(p,0)\beta(p,i)}\sin\left[\sum_{j=1}^{i-1}\mu_p(j)\right]) \quad (9)$$

Thus, the calculations of these terms resumes to a very simple numerical problem.

This method is a very fast and simple way of calculating the acceptances from different parameters : the pass number, the energy gains per cell and per pass, the injection energy, and the focusing at the first pass.

An example of the calculations of the acceptances is given fig. 1. This figure shows the variation of the acceptances with the betatron phase advance per cell at the first pass, which is constant along the linac. The following data are used (they correspond to the ELFE linac) :

three passes in a 1 km long linac of 5 GeV (mean accelerating gradient : 5 MeV/m). Focusing : 50 twenty-meter long FODO cells. $\mu_1$=120 degrees per cell (at first pass). Injection energy : 0.5 GeV. The maximum displacement A of a particle trajectory in the linac has been arbitrarily chosen to be 5 mm.

The results of the calculations described above are compared to results of simulations. These simulations are using a program written to simulate the beam transport in a recirculating linac. It uses first order transfer matrices for quadrupoles and accelerating sections. The acceleration is taken into account without approximations. The recirculation is simulated by injecting several beams at different energies in the same structure.

The two calculations give very similar results.

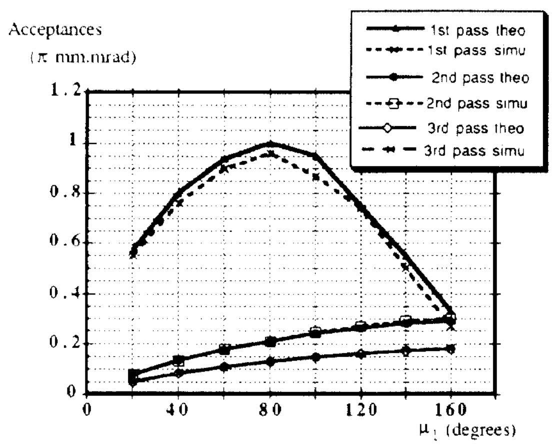


Fig. 1 : Acceptances versus the betatron phase advance at first pass, $\mu_1$, for A=5 mm. Simulation results for ELFE compared with analytical calculations.

## 3. CENTRAL TRAJECTORY CORRECTION

The focusing elements are misaligned with respect to a reference line (which is supposed to be a straight line). Steerers (one per quadrupole) are used to correct the beam central trajectory. Thus, after the first pass is completed, the beam central trajectory is supposed to be centered in the Beam Position Monitors (BPM). Unfortunately, the BPM are also misaligned. Thus, the kick $c_i$ related to the steerer i is :

$$c_i = -k_i + \left[\varepsilon_{i+1} - (2 + d/f)\varepsilon_i + \varepsilon_{i-1}\right]/d \qquad (10)$$

where $k_i$ is the kick related to the adjacent quadrupole misalignment $a_i$, $\varepsilon_i$ the misalignment of the BPM i, f the algebraic focal length (a negative f corresponds to a focusing quadrupole).

The kicks vary as the inverse of the beam energy : at pass p, the beam is deflected by a kick : $(E_1(i)/E_p(i))(c_i + k_i)$. It causes a displacement of the beam central trajectory at the linac end equal to $T_{12}(p,i)\,(E_1(i)/E_p(i))(c_i + k_i)$ where $T_{12}(p,i)$ is the $T_{12}$ term of the transfer matrix, at the pass p, from the cell i to the linac end.

It is easily shown, then, that the RMS value of the beam trajectory displacement at the linac end is :

$$\sigma_X = \sqrt{\sum_{i=1}^{N_Q}(a_i)^2}\ \sigma_\varepsilon \qquad (11)$$

where $N_Q$ is the number of quadrupoles, $\sigma_\varepsilon$ the RMS value of the BPM misalignments, and

$$\begin{aligned} a_i = \big[&(E_1(i-1)/E_p(i-1))T_{12}(p,i-1) \\ &-(2+d/f)(E_1(i)/E_p(i))T_{12}(p,i) \\ &+(E_1(i+1)/E_p(i+1))T_{12}(p,i+1)\ \big]/d \end{aligned} \qquad (12)$$

$\sigma_X$ has been calculated for different pass numbers, with the characteristics given above (ELFE linac) and an RMS value of the BPM misalignments of 0.2 mm (this study is extended to a large number of passes). Again, it may be calculated using the Eqs. (2) to (5) and (9), or by simulations.

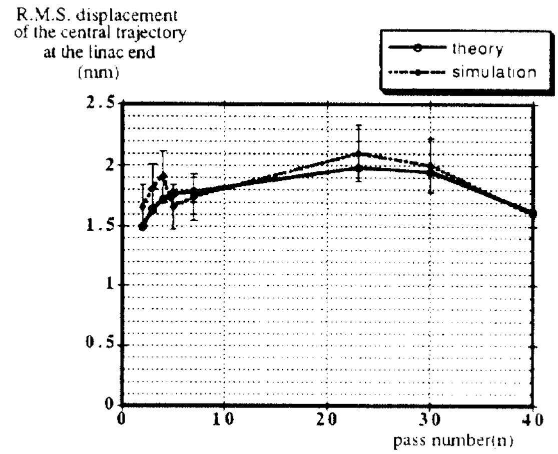


Fig. 2 : Beam trajectory displacement at pass n, *after a first pass correction* (R.M.S. BPM error = 0.2 mm).

It can be seen (fig.2) that the displacement of the beam central trajectory does not vary much, in a large range, with the pass number, and remains, in this case, of 2 mm RMS.

Let us suppose that the beam positions for each pass may be known independently. Then, a least-square optimization, which is described in details in [2] [3] of the corrections is shown to minimize the central beam trajectories at each pass (see fig.3).

A perfect correction corresponds to : $c_i + k_i = 0$ everywhere, or $c_i = -a_i/f$ where $a_i$ is the misalignment of the quadrupole number i, $f$ is the focal length at first pass.

In Eq. (10), we have :

$$c_i f = -a_i + (f/d)(\varepsilon_{i+1} - (2 + d/f)\varepsilon_i + \varepsilon_{i-1}) \quad (13)$$

The last term is the error on the correction of the quadrupole misalignment. Thus, it could be writen :

$$c_i f = -a_i + \delta a_i \qquad (14)$$

Let us suppose that the RMS values of both the quadrupoles and BPM misalignments are equal to 0.2 mm.

For $d / f = \sqrt{3}$ $(\mu = 120°)$, the RMS value of $\delta a_i$ is : $2.3 * 0.2 = 0.46$ mm.

After optimization, however, for the example given above, the RMS value of $\delta a_i$ is 0.11 mm for any quadrupole, and 0.07 mm for focusing quadrupoles (as the sensitivity is larger). Thus, this correction is closer to a local correction of the misalignments. This method appears then to be similar to a beam-based alignment.

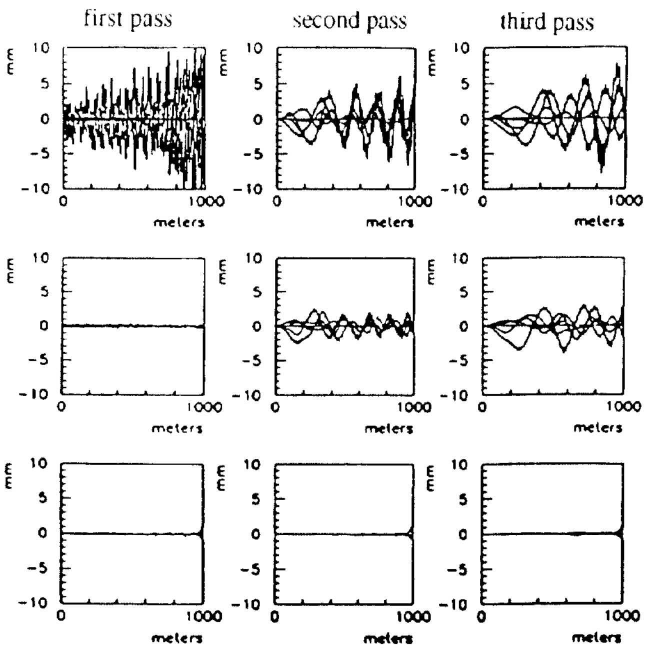


Fig.3 : Examples of beam central trajectories in the ELFE linac. Simulation results. Upper curves : without correction. Middle curves : After correction of the first pass. Lower curves : After optimization of the corrections. The trajectories are represented for 5 sets of quadrupole misalignments ($\sigma$=0.2 mm R.M.S.). The R.M.S. BPM total error is 0.2 mm. The R.M.S. BPM resolution is 50 $\mu$m. 1 BPM and 1 steerer per quadrupole.

## 4. DETERMINATION OF THE MISALIGNMENTS USING A SINGLE BEAM

The beam-based method of finding the misalignments which is used here is similar to that explained in [4]. The focusing structure is tuned successively to several different values of the betatron phase advance per cell, and the measurements of the beam central trajectory in each configuration are used to determine, by a least-squares fit, the misalignments of the quadrupoles. We have used almost the same procedure as in the precedent section. However, only one beam is present in the linac. Thus, no hypothesis is to be made for the measurement of several beams.

The quadrupole misalignments are obtained with a precision of 0.1 mm RMS (see fig.4 for an example of this determination). By the same procedure, the BPM misalignments may be found with the same precision.

This procedure should be used to check the straightness of the beam line, before recirculation is attempted. Indeed, the setting up of the first pass is in no way a guarantee for the circulation of several beams : a single beam may very well follow a broken line, but it is not possible for several beams of different energies, or a single beam at different tuning of the focusing structure.

This procedure is not a correction of the beam central trajectories, but provides data which should be known, in a recirculating linac, for this correction.

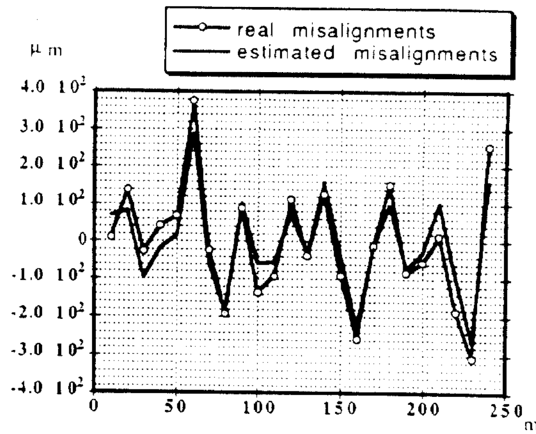


Fig. 4 : Example of an estimation of the misalignments of the quadrupoles along 250 m (1/4 linac). Simulation results. Quadrupole misalignment : 0.2 mm RMS. BPM non-systematic error : 50 $\mu$m RMS.

## 5. CONCLUSION

The beam transport in a long recirculating linear accelerator presents some particularities. The correction of the beam central trajectories must be considered with care. To be efficient and reproductible, the method should not control the beam central trajectory at one pass, but the beam line straightness. An optimization procedure using BPM data from all beams (with 1 BPM per quadrupole) is efficient. In addition, or as an alternative, beam-based alignment methods could be used to get precious data.

## 6. REFERENCES


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[4] C.E. Adolphsen et Al. Beam-based Alignment Technique for the SLC LINAC, in IEEE Particle Accelerator Conference Proceedings, Chicago, USA, 1989, p. 977-979.