The Cylindrical Cavity Geometry Optimization Over the Maximum Longitudinal Component of Electric Field

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Abstract

The excitation of a cylindrical finite Q-factor cavity by a train of charged particles bunches is considered. It is shown that cavity geometry optimization over the maximum longitudinal component of electric field for YerPhI Linac beam allows to generate $\cong$ 200 MV/m field in it. The number of efficient radiating bunches and maximum of accelerated current are determined.

1. INTRODUCTION

It should be noted that the process of a high charged electron bunch interaction with cavities must be considered only in a self-consistent formulation of the problem [1]. In some cases one may obtain results that allow to reveal the characteristics of excitation and optimize cavity sizes over the maximum strength of electric field longitudinal component, in the first stage disregarding the cavity field effect on the exciting bunches. We claim that some conditions exist to excite only one mode in the cavity by a train of the bunches structure break forces. Our investigations show the way for the self-consistent solution by the step-time and step-space consideration of the problem in given current and charge approximation.

2. THE FINITE Q-FACTOR CAVITY EXCITED BY THE TRAIN OF ELECTRON BUNCHES

It can be easily shown that the periodic train of the bunches with the repetition rate $f=v/d$ (where v-is the velocity of the buhch, d-is the interval between two neighbor bunches) will effectively excite a single mode in the cylindrical cavity when this train crosses the cavity along its axis if the radius of this cavity is [2] :

$$R_{0mk} = \mu_{0m} d/2\pi k \qquad (1)$$

and

$$2ak/d << 1 \qquad (2)$$

Above we suppose that the velocity of the bunches $v=c$, a is the cavity height, μ_{0m} is the m-th root of the zero order Bessel function, $k=1,2,3,...$ is the number of harmonics of the bunches repetition rate.

Then the longitudinal component E_z of electric field in the cavity is described by the expression [2] :

$$E_{z,m,k}(r,t) = -\frac{16qQ}{bd}\frac{J_1(\frac{2\pi kb}{d})J_0(\frac{2\pi kb}{d})}{\mu_{0m}^2 J_1^2(\mu_{0m})}\frac{\sin(\frac{\pi kd}{l})}{\frac{\pi kd}{l}} *$$

$$* \frac{\sin(\frac{\pi ka}{d})}{\frac{\pi ka}{d}}\left(1-e^{-\frac{\pi kN}{Q}}\right)\cos\pi k\left(\frac{2vt-L-a}{d}\right)e^{-\frac{\pi kv}{Qd}\left(t-\frac{L+a+(N-1)d}{v}\right)} \qquad (3)$$

where q is bunch charge, Q - is the cavity quality factor, N is the number of bunches in the train, b is the radius of the bunch and L is its length. The charge distribution in the bunch is assumed to be uniform.

We consider a beam has the parameters: repetition rate f=3 GHz, b = 0.5 cm, L = 1 cm, number of particles in a bunch $n = 5.10^9$, number of bunches in a pulse $N = 3.10^4$. To generate a field the optimal cavity parameters should be: at k=1,m=1: R=3.83 cm; a=2.75 cm and at k=2, m=1: R=1.92 cm; a=1.275 cm. For these parameters the calculated by (3) field's average value during the test bunch transit time equals.

at $k = 1$, $m = 1$, $Q = 13\ 000$ $\qquad$ $<Ez> \cong 250$ MV/m
at $k = 2$, $m = 1$, $Q = 9\ 000$ $\qquad$ $<Ez> \cong 280$ MV/m $\qquad$ (4)

Though the Q - factor in the first case is 1.5 times higher nevertheless, due to the fact that cavity radius at k = 1 is twice as large as when k = 2, the field value averaged during the transit time in case of k = 2 is somewhat larger than when k = 1

As follows from (3), the account of finite Q limits N number of efficiently radiating bunches. Thus at $N>3Q/\pi k$ the value of the field is practically independent of the number of bunches passing through the cavity (Fig. 1).

Thus in a beam consisting of N bunches, efficiently radiate

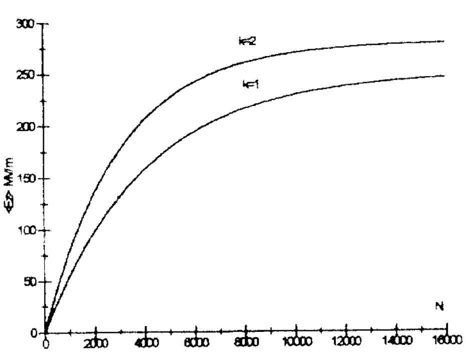

Fig 1

The research described in this publication was made possible in part by Grant N RYO 000 from the International Science Foundation.

only $\eta = (3Q/\pi kN)$ 100% of bunches. Therefore, the account of cavity Q-factor allows us to optimize the number of efficiently radiating bunches and decrease essentially the pulse width of the beam current.

3. Some Acceleration Problems

Let us find the energy accumulated in a cavity after N bunches having passed through it. Calculating the energy loss of each bunch element in the form of radiation in cavity, then suming up these losses over all bunches, we obtain:

$$\mathcal{E} = \int_0^L d\xi \int_0^{\tilde{N}} dt \int_0^b j(\xi) E_z(r,t) r dr \qquad (4)$$

where ξ is the bunch element longitudinal coordinate, $\tilde{N} = [(N-1)d + a + L]/v$.

The work of bunch trains is negative, i.e. they give energy to the cavity. Thus, when the resonance conditions are satisfied, each next bunch increases the energy accumulated in the cavity.

Let after a train of N bunches a test bunch of radius b_1 at the moment $t=d_1/v$ enter the cavity (where d_1 is the distance between the centers of the N-th and test bunches. Test-bunch length is negligible).

The energy accumulated in a cavity creates the acceleration gradient for the test bunch

$$\frac{d\mathcal{E}}{dz} = \frac{16qQ}{bd} \frac{J_1\left(\frac{2\pi k}{d} b\right)}{\mu_{0m}^2 J_1(\mu_{0m})} \frac{\sin^2 \frac{\pi ka}{d}}{\left(\frac{\pi ka}{d}\right)^2} \frac{\sin \frac{\pi kL}{d}}{\frac{\pi kL}{d}} \left(1 - e^{-\frac{\pi kN}{Q}}\right) \qquad (5)$$

if $d_1/2k$.

Being accelerated in a cavity a test charge carries part of the energy accumulated in the cavity. One can estimate the maximum charge value, which being accelerated takes away the hole energy accumulated in the cavity.

The train of bunches having the current $\vec{j}_1$ produces in the cavity the fields $\vec{E}_1 \vec{H}_1$ and the test charge having the current $\vec{j}_2$ crosses the cavity and induces the fields $\vec{E}_2 \vec{H}_2$ as well. One may write the energy balance equation in the form:

$$\frac{\partial \mathcal{E}}{\partial t} = -\left[\int_V \vec{j}_1 (\vec{E}_1 + \vec{E}_2) dV + \int_V \vec{j}_2 (\vec{E}_1 + \vec{E}_2) dV \right] \qquad (6)$$

This test-charge takes away part of the energy that has been accumulated in the cavity. If

$$\vec{j}_1 \vec{E}_1 > 0, \vec{j}_1 \vec{E}_2 < 0; \vec{j}_2 \vec{E}_1 < 0, \vec{j}_2 \vec{E}_2 > 0$$

the Eq. (6) takes the form

$$\frac{\partial \mathcal{E}}{\partial t} = -\left|\int_V \vec{j}_1 \vec{E}_1 dV\right| + \left|\int_V \vec{j}_2 \vec{E}_1 dV\right| - \left|\int_V \vec{j}_1 \vec{E}_2 dV\right| + \left|\int_V \vec{j}_2 \vec{E}_2 dV\right| \qquad (7)$$

It's obvious that when

$$\left|\int_V \vec{j}_1 \vec{E}_1 dV\right| = \left|\int_V \vec{j}_2 \vec{E}_2 dV\right| \qquad (8)$$

The test-charge will take away the whole energy accumulated in the cavity due to the previous train of bunches. However the cavity turns out filled up with energy

$$\mathcal{E} = \int_0 \left[\left|\int_V \vec{j}_2 \vec{E}_2 dV\right| - \left|\int_V \vec{j}_1 \vec{E}_2 dV\right| \right] dt$$

caused by the test-charge radiation. For that the test-charge must be pretty high. From (8) follows:

$$\left(q_{test}\right)_{\max} = \frac{qQ}{\pi k} \frac{J_1\left(\frac{2\pi kb}{d}\right)}{\frac{2\pi kb}{d}} \frac{\sin \frac{\pi kL}{d}}{\frac{\pi kL}{d}} \left(1 - e^{-\frac{\pi kN}{Q}}\right) \qquad (9)$$

which yield: $q_{\text{test max}} = 1354q$ for $k=1$ and $q_{\text{test max}} = 677q$ for $k=2$.

Thus, we can claim that the energy accumulated into the cavity by the train of bunches (number of particles in the bunch 10^9 , number of bunches 10^4) produces the acceleration gradient (5) for the test-charge (point-like bunch) with the number of particles more than $10^{11} \div 10^{12}$.

The transit time through the cavity is very small; therefore we can neglect the velocity change during the passage and suppose it constant. Indeed, it follows from the relativistic motion equation that

$$\frac{dv}{dz} = \frac{q < E_z >}{m_0 c \gamma^3} \qquad (10)$$

which for the considered cases makes less than 10^{-5}.

This fact allows us to use expression of current for the uniform motion of driving bunches. Then one can heuristically describe the velocity change in the sequence of cavities using the energy balance. This can be an approximation to the self-consistent problem.

The authors express their sincere gratitude to DESY Directorium and to Coordinator of DESY-YerPhI Collaboration Dr. G. Söhngen as well as to Prof. A.M. Sessler (LBL, Berkeley) for solidarity and great support, thanks to which the authors succeed in overcoming the difficulties their time.

4. References

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