

Design Study for the Accelerating Cavity of the Racetrack Microtron Eindhoven.

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Abstract

A 17-cell bi-periodic standing wave slot coupled structure will be used as accelerating structure for the RaceTrack Microtron Eindhoven. The numerical design of the structure has been performed with the computer codes SUPERFISH and MAFIA. Analytical calculations for a bi-periodic accelerating structure with full end cells including direct coupling are presented. Equations giving the detuning of the chain mode frequencies for known individual cell errors and coupling coefficients are derived. After numerical inversion these equations will be used for the final tuning of the cavity.

1 INTRODUCTION

The accelerating structure of the 10 – 75 MeV RaceTrack Microtron Eindhoven (RTME) [1] will consist of a 17-cell bi-periodic slot coupled structure terminated with full end cells. The numerical design of the cavity has been done with the computer codes SUPERFISH and MAFIA. This combination of codes is used the same way as described by Chang *et al.* [2]. Table 1 lists some parameters of the accelerating structure.

In literature most analysis is performed on bi-periodic accelerating structures terminated with half end cells. Theory for this is not well applicable in our case, since our structure is terminated with full end cells and moreover since it is relatively short. In section 2 the solution for the amplitudes and phases of the fields in a bi-periodic structure with full cell ends is given. Also the coupled resonator equations are given by means of matrix formulation. In section 3 an analytical derivation, including direct coupling (direct coupling is the second neighbour coupling in a bi-periodic structure), of the perturbations in the frequencies of the modes of a chain of cavities terminated with full end cells as a function of the frequencies of the individual cells is described. In section 4 a simulation of the influence of the finite tuning accuracy, losses and drives on the $\pi/2$ -mode vector of the RTME cavity is described.

2 BI-PERIODIC STRUCTURE WITH FULL END CELLS

An accelerating structure can be represented by a chain of coupled resonators, which in turn can be described by the coupled resonator equations [3]. The solution of these equations yields the amplitudes and phases of the standing wave in both accelerating and coupling cells. For an

Table 1: Accelerating cavity parameters

frequency	2.998 GHz
number of accelerating cells	9
accelerating voltage	5 MV
peak E-field at surface	0.9 Kilpatrick
wall losses	700 kW
transit time factor	0.77
shunt impedance R, T^2	79.9 M Ω /m
unloaded quality factor	17300
generator-cavity coupling β	1.5
next neighbour coupling	-4.5%
power source	EEV M5125, 2.2 MW magnetron

infinitely long chain the general solution is given by

$$\begin{cases} X_{2n} &= A^- e^{j2n\phi} + A^+ e^{-j2n\phi} \\ X_{2n+1} &= C^- e^{j(2n+1)\phi} + C^+ e^{-j(2n+1)\phi} \end{cases} \quad (1)$$

$n = 0, 1, \dots$ where X_{2n} and X_{2n+1} are the generalised amplitudes of the accelerating and coupling cells respectively. The ratio of the amplitudes of the fields in the accelerating and coupling cells, A/C , is given by:

$$\frac{A^-}{C^-} = \frac{A^+}{C^+} = \frac{-k_{ac} \cos \phi}{1 - \frac{\omega_a^2}{\omega^2} + k_{aa} \cos 2\phi} = \frac{1 - \frac{\omega_a^2}{\omega^2} + k_{cc} \cos 2\phi}{-k_{ac} \cos \phi}, \quad (2)$$

this ratio is not relevant for $\phi = \pi/2$, since the amplitudes of the fields in the coupling cells is zero for this mode. Here $\omega_a(\omega_c)$ is the resonance frequency of the accelerating (coupling) cells in the structure, k_{ac} represents the coupling coefficient for coupling between accelerating and coupling cells and $k_{aa}(k_{cc})$ the coupling coefficient for direct coupling between accelerating (coupling) cells. The phase shift ϕ per resonator is linked with frequency ω by the dispersion relation for a doubly periodic chain [3].

By terminating the infinitely long chain, boundary conditions are imposed on the general solutions. From the singly periodic chain we know [4] that the solution for a chain with full accelerating cells as end cells should be sine-like. To obtain this ideal solution the following boundary conditions have to be imposed on the generalised amplitudes (see figure 1 for numbering):

$$\begin{cases} X_{-1} = 0 & X_0 = -X_{-2} \\ X_{2N+1} = 0 & X_{2N} = -X_{2N+2} \end{cases} \quad (3)$$

After substitution of these conditions in the coupled resonator equations and disregarding drives and losses, the

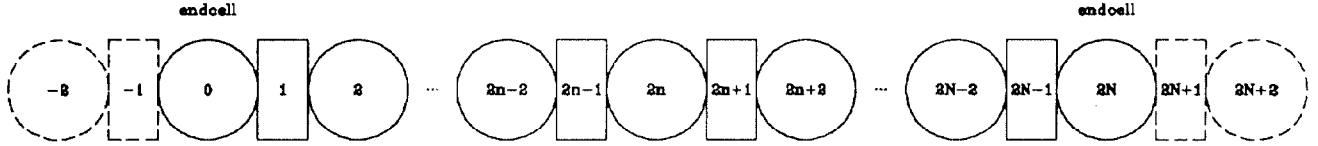


Figure 1: Schematic drawing of a $2N + 1$ cells long doubly periodic chain with two full accelerating cells as end cells.

solution is found to be

$$\begin{cases} X_{2n}^q = A \sin(2n + 1)\phi \\ X_{2n+1}^q = C \sin(2n + 2)\phi \end{cases}, \quad (4)$$

with $\phi = \frac{(q+1)\pi}{(2N+2)}$ and the mode number $q = 0, 1, \dots, 2N$.

It is convenient to write the coupled resonator equations for a chain terminated with full end cells by in matrix form:

$$M\bar{x} - \lambda T^2\bar{x} = \bar{0} \quad (5)$$

$$\bar{x} = \begin{bmatrix} X_0 \\ X_1 \\ X_2 \\ \vdots \\ X_{2N} \end{bmatrix}, \quad T = \begin{bmatrix} \omega_a & 0 & \cdot & \cdot & \cdot \\ 0 & \omega_c & 0 & \cdot & \cdot \\ 0 & 0 & \omega_a & 0 & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \omega_a \end{bmatrix}, \quad (6)$$

$$M = \frac{1}{2} \begin{bmatrix} (2 - k_{aa}) & k_{ac} & k_{aa} & 0 & \cdot & \cdot \\ k_{ac} & 2 & k_{ac} & k_{cc} & \cdot & \cdot \\ k_{aa} & k_{ac} & 2 & k_{ac} & k_{aa} & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & 0 & k_{aa} & k_{ac} & (2 - k_{aa}) \end{bmatrix}, \quad (7)$$

and λ represents the eigenvalues $1/\omega_q^2$.

By using the fact that T is a diagonal matrix, the problem is rewritten as an eigenvalue problem with $\tilde{M} = T^{-1}MT^{-1}$ and $\tilde{x} = T\bar{x}$:

$$\tilde{M}\tilde{x} - \lambda\tilde{x} = \bar{0}. \quad (8)$$

Matrix \tilde{M} is symmetric. This implies that the eigenvectors belonging to different eigenvalues for the eigenvalue problem with matrix \tilde{M} are orthogonal. Since there are as many different eigenvalues as equations all eigenvectors are orthogonal. Moreover the eigenvectors form a complete set. The orthogonality relations are

$$(\tilde{x}^q, \tilde{x}^r) = \delta_{qr} \frac{(N+1)(\omega_a^2 A^2 + \omega_c^2 C^2)}{2}, \quad (9)$$

under the condition that for the $\pi/2$ -mode the ratio A/C is given by $A/C = \omega_c/\omega_a$. This constraint is allowed as the ratio was not determined yet in the $\pi/2$ -mode. The Kronecker delta is denoted by δ_{qr} .

3 PERTURBATION CALCULATION

By linear perturbation of the coupled resonator equations, expressions for the frequency shifts of the cavity modes due to small errors in the resonance frequencies of individual cells can be found.

We start with the solution of the q -th mode of the eigenvalue problem:

$$M\bar{x}^q - \lambda_q T^2\bar{x}^q = \bar{0}, \quad (10)$$

$\lambda_q = 1/\omega_q^2$ is the eigenvalue belonging to mode frequency ω_q and \bar{x}^q is the corresponding eigenvector. Define $D^q = \lambda_q T^2$ and allow perturbations in the accelerating cell resonance frequencies $\omega_{a(2n)}^* = \omega_a + \delta\omega_{a(2n)}$ and coupling cell resonance frequencies $\omega_{c(2n+1)}^* = \omega_c + \delta\omega_{c(2n+1)}$. These perturbations lead to a shift in the resonance frequencies of the cavity chain $\omega_q^* = \omega_q + \delta\omega_q$ and a change in the eigenvectors $\bar{x}^{q*} = \bar{x}^q + \delta\bar{x}^q$.

If higher order terms are neglected, the perturbed matrix D^{q*} is given by

$$D^{q*} = D^q + \delta D^q = D^q + \frac{2}{\omega_q^2}(\delta C - \delta Q), \quad (11)$$

with

$$\delta C = \begin{bmatrix} \omega_a \delta\omega_{a(0)} & 0 & \cdot & \cdot \\ 0 & \omega_c \delta\omega_{c(1)} & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & 0 & \omega_a \delta\omega_{a(2N)} \end{bmatrix}, \quad \delta Q = \frac{\delta\omega_q}{\omega_q} T^2, \quad (12)$$

δC contains the perturbations of the frequencies of the individual cells and δQ the perturbations of the mode frequencies of the cavity.

After substitution of eq. 11 in eq. 10, insertion of the perturbed eigenvector \bar{x}^{q*} subtraction of the homogeneous equation and disregarding the higher order term, we get

$$M\delta\bar{x}^q = \delta D^q\bar{x}^q + D^q\delta\bar{x}^q. \quad (13)$$

The perturbations of the mode frequencies are isolated by taking the inner product with \bar{x}^q :

$$(\bar{x}^q, M\delta\bar{x}^q) = (\bar{x}^q, \delta D^q\bar{x}^q) + (\bar{x}^q, D^q\delta\bar{x}^q). \quad (14)$$

Using the symmetry of the matrices M and D^q and the homogeneous equation, it follows:

$$(\bar{x}^q, \delta D^q\bar{x}^q) = 0. \quad (15)$$

This expression relates the perturbations in the resonance frequency of mode q of a structure with the perturbations in the individual cell resonance frequencies. With eq. 13 and using the symmetry of T eq. 15 is written as

$$\frac{\delta\omega_q}{\omega_q} = \frac{(\bar{x}^q, \delta C\bar{x}^q)}{(T\bar{x}^q, T\bar{x}^q)}. \quad (16)$$

With the assumption that $\omega_a \simeq \omega_c$ this becomes

$$\frac{\delta\omega_q}{\omega_q} = \frac{2}{(N+1)(A^2 + C^2)}.$$

$$\left[\sum_{n=0}^N \frac{\delta\omega_{a(2n)}}{\omega_a} A^2 \sin^2 \left(\frac{(2n+1)(q+1)\pi}{2N+2} \right) + \sum_{n=0}^N \frac{\delta\omega_{c(2n+1)}}{\omega_c} C^2 \sin^2 \left(\frac{(2n+2)(q+1)\pi}{2N+2} \right) \right]. \quad (17)$$

This expression states that the relative frequency shift of a certain mode is equal to the sum of the relative frequency shifts in a cell times the stored energy in that cell, normalised to the total stored energy in the structure in this mode. When these equations are numerical inverted, they can be helpful for the tuning of an accelerating structure.

4 TUNING TOLERANCES

Tuning an accelerating cavity can be a tedious job, but an accelerating structure that is not perfectly tuned will generate unwanted phase shifts and will have varying field amplitudes in the accelerating cells and fields in the coupling cells. This means that it is important to make in advance an estimation of the permitted tolerances on the tuning parameters; $\delta\omega_a(n)$, $\delta\omega_c(n)$, stopband and coupling coefficients.

To investigate the influence of losses, drives and tuning parameters on the mode vector \bar{x}^q we start with the coupled resonator equations including losses and drives:

$$M\bar{x}^q - \frac{T^2}{\omega_q^2}\bar{x}^q - j\frac{LT}{\omega_q}\bar{x}^q = \bar{y}, \quad (18)$$

here \bar{y} represents the drives

$$\bar{y}^T = [Y_0, Y_1, Y_2, \dots, Y_{2N}] \quad (19)$$

and the term with j represents the losses. Matrix M is now given by

$$M = \frac{1}{2} \begin{bmatrix} 2 & k_{0,1} & k_{0,2} & 0 & \dots & \dots \\ k_{0,1} & 2 & k_{1,2} & k_{1,3} & \dots & \dots \\ k_{0,2} & k_{1,2} & 2 & k_{2,3} & \dots & k_{2,4} \\ \dots & \dots & \dots & \dots & \dots & \dots \\ \dots & \dots & 0 & k_{2N-2,2N} & k_{2N-1,2N} & 2 \end{bmatrix}, \quad (20)$$

where the indices indicate between which cells the coupling takes place. Matrix T is given by $T = \delta_{nm}\omega_n$, $n, m = 0, 1, \dots, 2N$, ω_n is the resonance frequency of cell n . Finally the matrix L is given by $L = \delta_{nm}/Q_n$, Q_n is the quality factor of cell n .

To find a solution for the mode vector \bar{x}^q first the homogeneous part of eq. 18 without losses is solved. The solution of this eigenvalue problem gives the ω_q . If we assume that the influence of losses and drives on the resonance frequency is negligible (large Q_n), the solution of

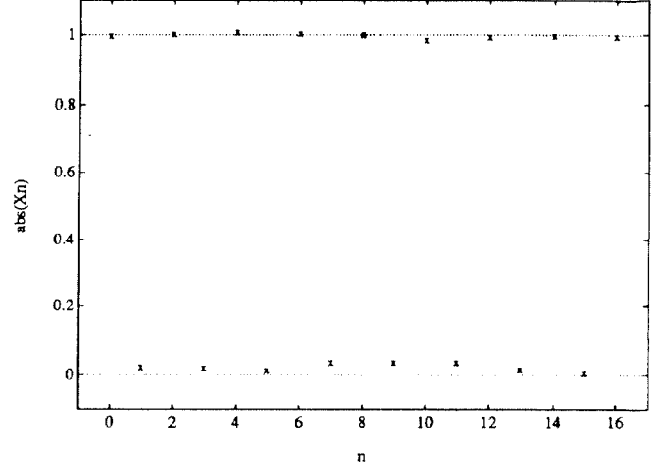


Figure 2: Absolute value of the mode vector in the $\pi/2$ -mode for the cavity of RTME ($\omega_a = 3,000$ MHz, $Q_a = 10,000$, $Q_c = 2,000$, $k = -0.05$, $N = 8$, drive in cell 8, $\delta\omega_a = 1$ MHz, $\delta\omega_c = 3$ MHz, $\delta k = 1\%$, stopband = 3 MHz).

the mode vector \bar{x}^q in eq. 18 is found by inversion:

$$\bar{x}^q = \left(M - \frac{T^2}{\omega_q^2} - j\frac{LT}{\omega_q} \right)^{-1} \bar{y}. \quad (21)$$

Now it is possible to investigate for each mode the separate or combined influence of the tuning parameters, drives and losses on the mode vector.

Figure 2 depicts the absolute value of the mode vector in the different cells for the $\pi/2$ -mode for reasonable values of achievable tuning accuracies, $\delta\omega_a(\delta\omega_c)$ is the maximum value of a random error in the accelerating (coupling) cells, δk is the error in the coupling coefficient. The phase shifts are not shown since the errors are negligible for the accelerating cells, and since the fields in the coupling cells are almost 0 their phases are not of interest.

5 REFERENCES

- [1] Webers G.A., *Design of an electron-optical system for a 75 MeV racetrack microtron*, Ph.D. Thesis, Eindhoven University of Technology (1994).
- [2] Chang C.R., Yao C.G., Swenson A., Funk L.W., *Computer simulation and cold model testing of CCL cavities*, Proc. of the 1993 PAC, vol 2 (1993), 812.
- [3] Nagle D.E., Knapp E.A., Knapp B.C., *Coupled resonator model for standing wave accelerator tanks*, Rev. of Sci. Instr. vol 38, no 11 (1967) 1583.
- [4] Coppens J.E., *Design study for the accelerating cavity of the racetrack microtron Eindhoven*, Eindhoven University of Technology, internal report VDF/NK 94-18 (1994).