THERMAL INSTABILITIES IN SUPERCONDUCTING RF CAVITIES

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Abstract

The methods of instabilities theory are applied to the system consisting of superconducting cavity, excited by rf generator and cooled by liquid helium. Three dimensional heat conduction equation as well as cavity excitation equation are used to derive the set of relationships, describing evolution of small perturbations in the system, being in steady state. Linear approximation is used as the first step of the analysis of thermal interaction of rf field with superconductor through the mechanism of temperature dependence of surface resistance. The expressions or the equations for the instability threshold have been obtain for some particular cases.

1. INTRODUCTION

Great efforts are undertaken in the field of rf superconducting science and technology to increase accelerating gradients in superconducting cavities to be used in new generation of accelerators and colliders. Among the other problems to be solved on the way to high gradients [1] (field emission, thermal break down, induced by local defects) the one of thermal stability being understood in general sense as an ability to damp any possible in the system thermal perturbation seems to be attractive for exploration too. Temperature dependence of BCS losses results in the feed back in the system rf generator - superconducting cavity - cooling bath. This feed back may be the main factor to determine the processes in the system at high field level when the dissipated rf power becomes appreciable to induce an instability resulting in thermal break down [2]. The problem under consideration is complicated significantly by non linear character of field surface interaction; the latter, in turn, may stabilise the system, forming new steady state.

The attempt of generalisation of the results that had been obtained in [2] is undertaken in present paper. The analysis below has the objective to cover the cases that can be treated analytically and thus to understand the main features of instabilities. Linear approximation is used to simplify the problem for the most general consideration.

2. THE EQUATIONS OF HEAT - RF FIELD INTERACTION

We shall consider the cavity of cylindrical form; l, R and L being the wall thickness, the cavity radius and its length respectively. It is assumed that the cavity is cooled by liquid helium outside. The mode TM_{010} is assumed to be excited by external rf generator as well. For simplicity, we exclude the end plates from thermal model. Thus, we have heat conduction equations

$$\frac{\partial T}{\partial t}=\chi\left(\frac{\partial^2 T}{\partial r^2}+\frac{1}{r}\frac{\partial T}{\partial r}+\frac{1}{r^2}\frac{\partial^2 T}{\partial\varphi^2}+\frac{\partial^2 T}{\partial z^2}\right) \quad (1)$$

where r, φ and z are the cylindrical co-ordinates, T and χ denote the cavity wall temperature and temperature conductivity [3], t is time, and cavity excitation equation

$$\frac{dH}{dt}+\left(\frac{\omega_0}{2Q}-i\Delta\omega\right)H=a\sqrt{P_g}\,. \quad (2)$$

Here, H is the complex amplitude of magnetic field in the cavity, ω_0 and Q are the cavity frequency and its quality factor, P_g is the power of rf generator and a is the constant, determined by coupling conditions, $\Delta\omega=\omega-\omega_0$, ω being the generator frequency. We shall assume $\Delta\omega=0$ throughout this work.

The equations (1) and (2) should be complemented by boundary conditions:

$$\begin{aligned}
&\lambda\frac{\partial T}{\partial z}=\alpha(T-T_{He}) \quad \text{at} \quad z=0\\
&\lambda\frac{\partial T}{\partial z}=-\alpha(T-T_{He}) \quad \text{at} \quad z=\mathrm{L}\\
&\lambda\frac{\partial T}{\partial r}=-P(T,H)=-\frac{1}{2}R_s(T)H_s^2 \quad \text{at} \quad r=R\\
&\lambda\frac{\partial T}{\partial r}=-\alpha(T-T_{He}) \quad \text{at} \quad r=R+l,
\end{aligned} \quad (3)$$

where $\alpha=1/R_K$, R_K is Kapitza resistance [4], R_s, H_s and P are the cavity surface resistance, surface magnetic field and rf power per unit area, dissipated on inner cavity surface. We shall assume, that $T(\vec r,t)=T_{st}(\vec r)+\vartheta(\vec r,t)$, $H_s(t)=H_s^{st}+h(t)$, where index "*st*" denotes the system steady state, also assumed to exist, and $\vartheta(\vec r,t)$ as well as $h(t)$ are small deviations from the steady state values. After the appropriate substitutions we have

$$\frac{\partial\vartheta}{\partial t}=\chi\left(\frac{\partial^2\vartheta}{\partial x^2}+\frac{1}{R}\frac{\partial\vartheta}{\partial x}+\frac{1}{R^2}\frac{\partial^2\vartheta}{\partial\varphi^2}+\frac{\partial^2\vartheta}{\partial z^2}\right) \quad (4)$$

$$\frac{dh}{dt}+\frac{\omega}{2}\left(\frac{1}{Q_0}+\frac{1}{Q_1}\right)h+\frac{\omega H_s}{2Q_0R_sS}\frac{\partial R_s}{\partial T}\int_S\vartheta\,dS=0 \quad (5)$$

Integration in (5) is performed over cylindrical cavity surface S; it is assumed, that $H_s=const$ on this surface. Q_0 and Q_1 are the cavity Q-value and the external Q-value respectively. We consider the case when rf power of the external rf generator is time independent as well as the cavity is tuned to the resonance.

The boundary conditions for small perturbations look like

$$\lambda\frac{\partial\vartheta}{\partial x}=-\frac{1}{2}H_s^2\frac{\partial R_s}{\partial T}\vartheta-R_sH_sh \quad \text{at} \quad x=0$$

$$\lambda\frac{\partial\vartheta}{\partial x}=-\alpha\vartheta \quad \text{at} \quad x=l$$

$$\lambda\frac{\partial\vartheta}{\partial z}=\alpha\vartheta \quad \text{at} \quad z=0 \tag{6}$$

$$\lambda\frac{\partial\vartheta}{\partial z}=-\alpha\vartheta \quad \text{at} \quad z=L,$$

where $x=r-R$. We assume that $r\approx R$ in the cavity wall. Our objective is the study of various particular cases of the development of small temperature perturbations in the course of time.

3. ABSOLUTE INSTABILITY

This particular case corresponds to the solution $\vartheta=\exp(kt)$ with real part of k $\mathrm{Re}k>0$, ϑ being the same for any point on the cavity surface. Such a case may be realised in the infinity cavity $L=\infty$, appropriate increment (or decrement) can be obtained from the equations, written above, if one put $\partial^2/\partial\varphi^2=\partial^2/\partial z^2\equiv 0$. We shall also assume that $l<<R$. It follows from the analysis [2], that k is the solution of the equation

$$k^2\tau_{rf}\frac{l\lambda^2}{\chi}+k\left\{\frac{l\lambda^2}{\chi}+\left[\lambda\alpha-(l\alpha+\lambda)\frac{\partial P}{\partial T}\right]\tau_{rf}\right\}+ \lambda\alpha(\lambda+\alpha l)\frac{dP}{dT}=0, \tag{7}$$

where τ_{rf} is the time constant of the loaded cavity and

$$\frac{\partial P}{\partial T}=\frac{1}{2}H_s^2\frac{\partial R_s}{\partial T},\quad \frac{dP}{dT}=\frac{Q_0-Q_1}{Q_0+Q_1}\frac{\partial P}{\partial T}. \tag{8}$$

Depending on relationship between thermal time constant $\tau_T=l^2/\chi$ and rf field time constant $\tau_{rf}=2Q/\omega$ the condition of instability varies from

$$\frac{dP}{dT}>\frac{\alpha\lambda}{\lambda+\alpha l} \tag{9}$$

for $\tau_{rf}<<\tau_T$ (rf field follows the change in the surface resistance without any delay) to

$$\frac{\partial P}{\partial T}>\frac{\alpha\lambda}{\lambda+\alpha l} \tag{10}$$

for the other extreme case $\tau_{rf}>>\tau_T$ (the regime of constant rf field). Fig.1 Illustrates the determination of the operating point location for the superconducting cavity, cooled by liquid helium and excited by rf generator. The straight line represents the dependence of inner surface temperature on specific rf power dissipated on it. Its intersection with the curve $P=P(T)$, originating from the dependence of cavity Q-value on the inner cavity surface temperature determines the location of operating point. For the case $\tau_{rf}<<\tau_T$ the points marked as 1,3 and 4 are stable, while the point marked as 2 is unstable.

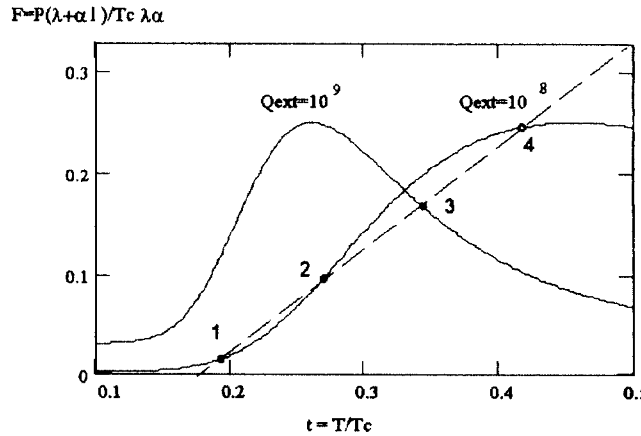

Fig1. The determination of the operating point location.

4. TWO DIMENSIONAL CASE

Let us assume, that $\tau_{rf}>>\tau_T=l^2/\chi$, but there is not any assumption concerning the relationship between rf time constant ant temporal characteristic of thermal process in z direction. For the analysis of such a case the following system might be used:

$$\frac{\partial\vartheta}{\partial\tau}=\frac{l^2}{L^2}\frac{\partial^2\vartheta}{\partial\eta^2}+\frac{l^2}{R^2}\frac{\partial^2\vartheta}{\partial\varphi^2}+ \frac{l}{\lambda}\left(\frac{1}{2}H_s^2\frac{\partial R_s}{\partial T}-\alpha\right)\vartheta+\frac{l}{\lambda}R_sH_sh=0, \tag{11}$$

$$\frac{\partial h}{\partial\tau}+\frac{l^2\omega}{2\chi}\left(\frac{1}{Q_0}+\frac{1}{Q_1}\right)h+\frac{l^2}{\chi}\frac{\omega}{2Q_0}\frac{H_s}{R_sS}\frac{\partial R_s}{\partial T}\int_S\vartheta dS=0 \tag{12}$$

Here, $\eta=z/L$, $\tau=t\chi/l^2$ are normalised coordinate and time respectively. The appropriate boundary conditions are:

$$\frac{\partial\vartheta}{\partial\eta}=\frac{\alpha L}{\lambda}\vartheta \quad \text{at} \quad \eta=0$$

$$\frac{\partial\vartheta}{\partial\eta}=-\frac{\alpha L}{\lambda}\vartheta \quad \text{at} \quad \eta=1 \tag{13}$$

Finding out the solution of (11)-(13) in the form

$$\vartheta=\exp(k\tau)F(\eta,\varphi) \tag{14}$$

one arrives at the equation

$$\frac{l^2}{L^2}\frac{\partial^2F}{\partial\eta^2}+\frac{l^2}{R^2}\frac{\partial^2F}{\partial\varphi^2}+\left[\frac{l}{\lambda}\left(\frac{\partial P}{\partial T}-\alpha\right)-k\right]F= \frac{1}{k+\kappa}\frac{\alpha l^3}{Q_0\chi\lambda}\frac{\partial P}{\partial T}\frac{1}{2\pi}\int_0^{2\pi}\int_0^1F(\eta,\varphi)d\eta d\varphi, \tag{15}$$

where $\kappa=l^2\omega/2\chi Q$. The solution of (15) satisfying the boundary conditions (13) is

$$F(\eta,\varphi)=\left(\sin s\eta+\frac{s\lambda}{\alpha L}\cos s\eta\right)\begin{Bmatrix}\sin n\varphi\\ \cos n\varphi\end{Bmatrix} \tag{16}$$

where $n \neq 0$ is integer, while s is a root of the equation

$$\cot s=\frac{s\lambda}{2\alpha L}-\frac{\alpha L}{2s\lambda}. \tag{17}$$

The minimum value of s lies in the interval $(0,\pi)$.

As it follows from (15) and also reflected in formulae (16) and (17), the temperature perturbations of sine type in azimuth direction do not result rf field perturbation (integral in (15) is equal to zero for such perturbations) and evolve at the constant level of rf field in the cavity. The instability takes place, if

$$\frac{\partial P}{\partial T}>\alpha+\frac{\lambda l}{L^2}\Omega^2 \tag{18}$$

where $\Omega^2=s^2+(\frac{L}{R}n)^2$. One can see that taking into account the distribution of temperature perturbations along the cavity surface results in increase of the instability threshold.

For the symmetrical temperature perturbations ($n = 0$) the increment of the instability is found as the minimum root of the equation:

$$\begin{aligned}&2\left[\Omega^2(\Omega^2-A)-\frac{\alpha L}{\lambda}A\right]\cos\Omega+2\frac{\alpha L}{\lambda}A+\\&\Omega\left[2A+(\Omega^2-A)\left(\frac{\alpha L}{\lambda}-\Omega^2\frac{\lambda}{\alpha L}\right)\right]\sin\Omega=0,\end{aligned} \tag{19}$$

where

$$\begin{aligned}\Omega^2&=\frac{L^2}{l^2}\left[\frac{l}{\lambda}\left(\frac{\partial P}{\partial T}-\alpha\right)-k\right]\\ A&=\frac{L^2 l\omega}{\chi\lambda Q_0}\frac{\partial P}{\partial T}\frac{1}{k+\kappa}.\end{aligned} \tag{20}$$

The condition $\mathrm{Re}\,k=0$ corresponds to the instability threshold. After substitution $\mathrm{Re}\,k=0$ in (19) one has the system of two equations relative $\frac{\partial P}{\partial T}$ and $\mathrm{Im}\,k$, which solution determines the instability threshold.

5. TREE DIMENSIONAL THERMAL PERTURBATIONS

The solution has been found for the case, corresponding to large rf field time constant, when one can neglect the field change in the cavity. It follows from heat conduction equation

$$\frac{\partial\vartheta}{\partial\tau}=\frac{\partial^2\vartheta}{\partial\xi^2}+\frac{l}{R}\frac{\partial\vartheta}{\partial\xi}+\frac{l^2}{R^2}\frac{\partial^2\vartheta}{\partial\varphi^2}+\frac{l^2}{L^2}\frac{\partial^2\vartheta}{\partial\eta^2}, \tag{21}$$

where $x=\xi l$, and from the appropriate boundary conditions

$$\begin{aligned}&\frac{\partial\vartheta}{\partial\eta}=\frac{\alpha l}{\lambda}\vartheta \quad \text{at} \quad \eta=0\\&\frac{\partial\vartheta}{\partial\eta}=-\frac{\alpha l}{\lambda}\vartheta \quad \text{at} \quad \eta=1\\&\frac{\partial\vartheta}{\partial\xi}=-\frac{l}{\lambda}\frac{\partial P}{\partial T}\vartheta \quad \text{at} \quad \xi=0\\&\frac{\partial\vartheta}{\partial\xi}=-\frac{\alpha l}{\lambda}\vartheta \quad \text{at} \quad \xi=1\end{aligned} \tag{22}$$

that the instability threshold is determined by the expression

$$\frac{\partial P}{\partial T}>\frac{\lambda}{l}\frac{\frac{l^2}{L^2}s^2+\frac{\alpha l}{\lambda}}{1+\frac{\alpha l}{\lambda}} \tag{23}$$

where s is the first root of the equation (17). The solution for the most general case, corresponding to the boundary conditions (6) has not been found so far.

6. CONCLUSION

Thermal interaction of surface resistance with rf field through the mechanism of temperature dependence of BCS losses of superconductor may be one of the dominating processes in high gradient superconducting cavities. Linear approximation that has been used in this paper to explore the main features of the instabilities development may be considered only as the first step of investigation of the processes through the mechanism mentioned. High gradient superconducting cavities are not the only object for the problem under discussion. The method developed might be used for the analysis of the microwave devices on the basis of high temperature superconductors as well.

7. ACKNOWLEDGEMENTS

The work has been done in the frame of collaboration between Lebedev Physical Institute and the University of Wuppertal (Germany). Author wants to thank H.Piel, G.Muller, R.Roeth, D.Reschke for fruitful discussions.

The work is supported by Russian Foundation for Fundamental Research, project No 94-02-04466.

7. REFERENCES

[1] W.Weingarten, "Superconducting Cavities", in Proceedings of the 15-th International Conference on High Energy Accelerators, Hamburg, Germany, July 20-24, 1992, pp. 678-685.

[2] V.G.Kurakin, "Thermal Stability of Defect Free Superconducting Cavity", in Proceedings of the 15-th International Conference on High Energy Accelerators, Hamburg, Germany, July 20-24, 1992, pp. 745-747

[3] L.D.Landau and E.M.Lifshitz, Fluid mechanics, Moscow: Nauka, 1988, 736 pp., in Russian.

[4] P.L.Kapitza, "The Investigation of the Mechanism of Heat Transfer in Helium-II", Journal of Experimental and Theoretical Physics, vol. 11, pp. 1-31, 1941, in Russian.