# RF Feedback Analysis for 4 Cavities per Klystron in PEP-II*

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*Abstract:*

Lattice changes in the PEP-II high energy ring have made the concept of driving four cavities with a single klystron an attractive option. This paper examines the topology from a RF feedback point of view. Sources of error are identified and their magnitudes are estimated. The effect on the performance of the longitudinal impedance reducing feedback loops is calculated using control theory and Mathematica.

## 1. INTRODUCTION

The original topology for the PEP-II high energy ring (HER) RF stations was based on driving two cavities with a single 1.2 MW klystron. Several RF feedback loops were planned to control longitudinal coupled-bunch oscillations driven by the interaction of the beam with the accelerating mode of each RF cavity[1][2]. There was concern that errors involving splitting the power from the klystron, cavity manufacturing tolerances, cavity tuning errors, and probe calibrations would degrade the performance of the RF feedback loops. A study was made for the relatively simple case of two cavities per klystron to place limits on the magnitude of the sources of error[3].

The new lattice changed the beam loading parameters such that one klystron could now supply power for four cavities. This change was attractive from a cost point-of-view since the number of RF stations and circulators would be reduced. It was decided to take a more careful look at the impact on the RF feedback loop performance.

## 2. SIMPLIFIED ANALYSIS

Using the topology defined by the schematic in figure 1, a feedback analysis was performed. Using Mason's gain rule [4] and Mathematica to perform the cumbersome algebra, an expression for the closed loop impedance seen by the beam was derived (equation 1).

$$\frac{4Z_c\left(1+GZ_c\left[2\Delta\gamma+\Delta^2(\Delta Z)\gamma-\Delta(\Delta Z)^2\gamma+\Delta^2\gamma^2+\Delta(\Delta Z)\gamma^2\right]\right)}{1+GZ_c\left[1+\Delta(\Delta Z)+2\Delta\gamma+(\Delta Z)\gamma+\Delta^2(\Delta Z)\gamma+\Delta^2\gamma^2+\Delta(\Delta Z)\gamma^2\right]}$$

Equation 1. Impedance seen by the beam based on figure 1.

Errors introduced in a two-way splitter or combiner are represented by $\Delta$ and $\gamma$ respectively. If we estimate that the maximum expected errors in combining the four cavity probe signals as 0.5 dB in amplitude and 5.0 degrees in phase we can calculate the corresponding value for $\gamma$ as listed in table 1.

| Specification | Corresponding $\gamma$ |
|---|---|
| 0.5 dB imbalance | 0.0144 |
| 5 degree error | 0.0218 |
| worst case | 0.0218 |

Table 1: Calculation of $\gamma$ based on estimated combining errors

In equation 1, all error terms are negligible when compared to 1 except for the denominator term $GZ_c 2\Delta\gamma$ . For a loop gain of 100 we can specify this term to be <<1 and specify a limit on $\Delta$ :

$$|\Delta| \ll \frac{1}{|2GZ_c\gamma|} \qquad |\Delta| \ll 0.229$$

Equation 2. Derived limit on splitting error term

If we allow the magnitude of $\Delta$ to be 1/5 of 0.229 (corresponding to a +-20% impedance variation) the tolerance for each 3 dB splitter is 0.8 dB or 5.2 degrees if the error is purely amplitude or phase.

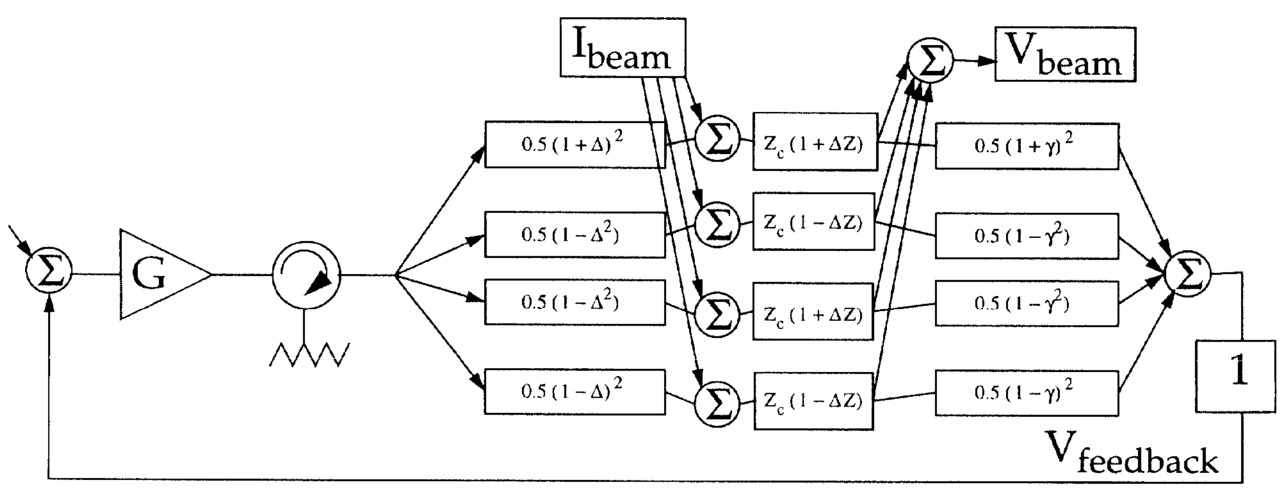


Fig. 1. Block diagram used to derive cavity closed loop transfer function seen by the beam (equation 1).

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## 3. A MORE COMPLETE CAVITY MODEL

Cavity impedances were next defined in terms of parameters which will vary as a function of manufacturing and operational tolerances. If one assume the cavity R/Q to be a fundamental constant while allowing for variations in unloaded Q, coupling coefficient and center frequency, the expression for cavity impedance is (equation 3):

$$Z_c = \frac{\frac{R}{Q}(\omega_c + \Delta\omega)\, s}{s^2 + \dfrac{(\omega_c + \Delta\omega)}{\dfrac{Q_0(1+\Delta Q_0)}{1+\beta(1+\Delta\beta)}}\, s + (\omega_c + \Delta\omega)^2}$$

Equation 3. Expression for cavity impedance with errors

The derivation proceeds to determine what effect the cavity variations would have on cavity reflected power. Differences in reflected power from each cavity lead to variations in transmitted power just as encountered by errors in the splitting network. If we model the cavity coupling factor to represent a transformer we can chose the turns ratio, N, to provide a perfect match between the source impedance $Z_0$ and the cavity at a given beam current (figure 2). The expression for N is equation 4.

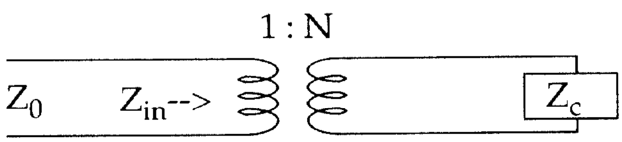


Fig. 2. Using a transformer to model the cavity coupling network

$$N^2 = \frac{Z_c|_{s=j\omega_c}}{Z_0\beta} = \frac{(\frac{R}{Q})Q_0}{\beta(1+\beta)Z_0}$$

Equation 4. Expression for the transformer ratio

If we assume that the transformer ratio is fixed based on non-perturbed $Z_c$ and the load $Z_c$ is perturbed, $Z_{in}$ becomes (equation 5):

$$Z_{in} = \frac{\beta(1+\beta)Z_0}{(\frac{R}{Q})Q_0}\left[\frac{\frac{R}{Q}(\omega_c + \Delta\omega)\, s}{s^2 + \dfrac{(\omega_c + \Delta\omega)}{\dfrac{Q_0(1+\Delta Q_0)}{1+\beta(1+\Delta\beta)}}\, s + (\omega_c + \Delta\omega)^2}\right]$$

Equation 5. Input impedance for a cavity with errors

For simplicity equation 4 is evaluated at $s = j\omega_c$ with $\Delta\omega = 0$. The reflection coefficient $\Gamma$ can be calculated from the defining expression shown in equation 6:

$$\Gamma = \frac{Z_{in} - Z_0}{Z_{in} + Z_0}$$

Equation 6. Definition of reflection coefficient

For a load impedance with reflection coefficient $\Gamma$, the power delivered to the load is (equation 7):

$$P_{load} = P_{in}(1 - |\Gamma|^2)$$

Equation 7. Expression for power delivered to a load with reflections

Based on equation 7, one can derive the expression for an insertion loss factor associated with power being reflected from the cavity. For our application, an expression for the variation in insertion loss from a cavity with no errors was developed. Equation 8 lists this new term which was added to each cavity in the model to evaluate the effect of cavity errors.

$$IL_{normalized} = \frac{IL}{IL_{ideal}} = \sqrt{\frac{(1+\beta)^3(1+\beta(1+\Delta\beta))(1+\Delta Q)}{(1+\beta(2+\beta+\Delta\beta+\Delta Q+\beta\Delta Q))^2}}$$

Equation 8. Normalized insertion loss term

Plotting this function (figure 3) assuming a $\beta$ of 4 and nominal values for $(R/Q)$ and $Q_0$ reveals that this term can introduce errors as large as the splitter/combiner errors. The new term was added and the closed loop impedance error was evaluated using the previous value for $\gamma$ while setting $\Delta = 0.03$ (figure 4). Results show that the closed loop impedance varies 30% in the worst case, which was considered acceptable.

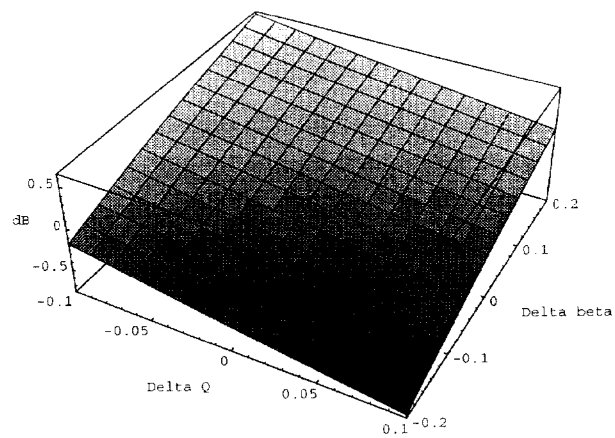


Fig. 3. Plot of normalized insertion loss vs. cavity parameters ($\beta$=4)

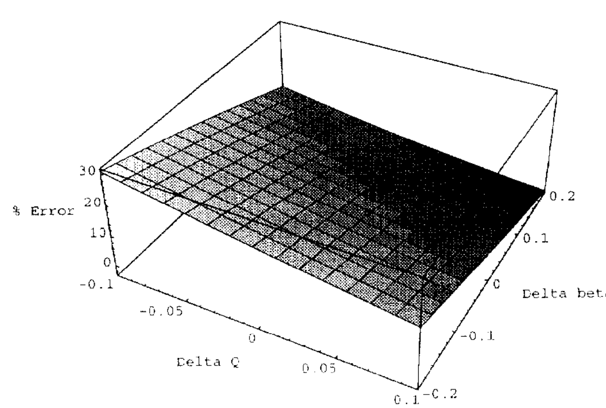


Fig. 4. Error in closed loop impedance for 4 cavities with errors

# 4. EFFECT OF CAVITY TUNING ERRORS

In order to determine the impact of cavity tuning errors on the insertion loss term, we follow the development of the complete cavity model but mistune the cavity and examine the resulting change in the insertion loss. The insertion loss varies only +/- .25 dB over the entire bandwidth of the cavity. Normal tuning errors would keep the cavity tuned well within this range. So for small tuning errors the deviation of the insertion loss is negligible. Recall that the nominal $\beta$ value is 4, therefore the system starts out with a fair amount of reflected power and the cavity tune deviations do not contribute a significant increase.

# 5. FULL ANALYSIS

Using Mathematica it is feasible to perform a full closed loop analysis of four cavities driven by a single klystron complete with both direct RF and equalized dual comb filter feedback loops. The expressions become too complicated to print out, but plotting the driving impedance for each mode is an easy ways to evaluate loop performance. The following analysis includes 350 ns of group delay consistent with expected values for transmission delay and klystron group delay. Phase margins were limited to 45 degrees throughout the analysis.

The procedure was to start with the model previously developed (figure 1) and add transfer functions for the direct RF feedback loop and the equalized dual peaked comb filter. The closed loop cavity impedance was then evaluated at the frequencies of the synchrotron sidebands corresponding to the first 20 longitudinal coupled-bunch modes. Driving impedances for each mode was then tabulated. The result for a system without errors is plotted below (figure 5). Positive terms represent unstable modes whose magnitudes must be kept below the 4 kW limit set by the power limitation of the bunch-by-bunch feedback system [5].

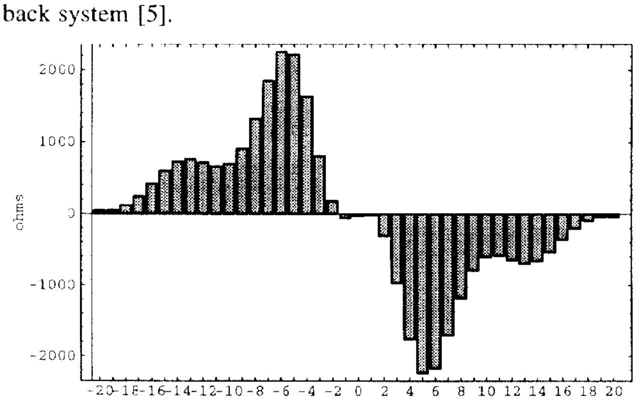


Fig. 5. Driving impedances (per cavity) for system without errors

Next the system errors were introduced to determine the effect on the driving impedances (figure 6). At first the plots look very similar but recall that the non-negligible error term $GZ_c 2\Delta\gamma$ is only large where the loop gain is high, within the bandwidth of the cavities. If we plot the difference in the closed loop impedances (figure 7), the effect is indeed evident.

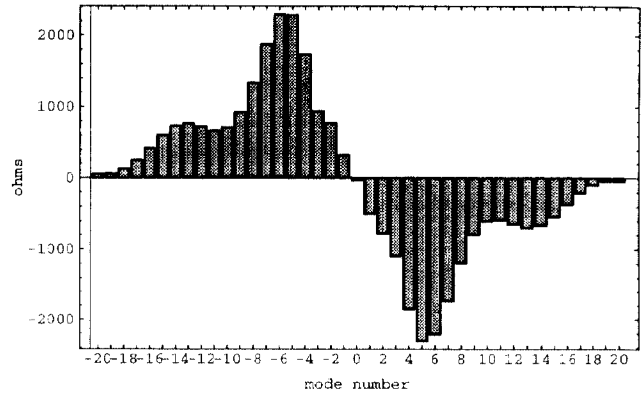


Fig. 6. Driving impedances (per cavity) for system with errors

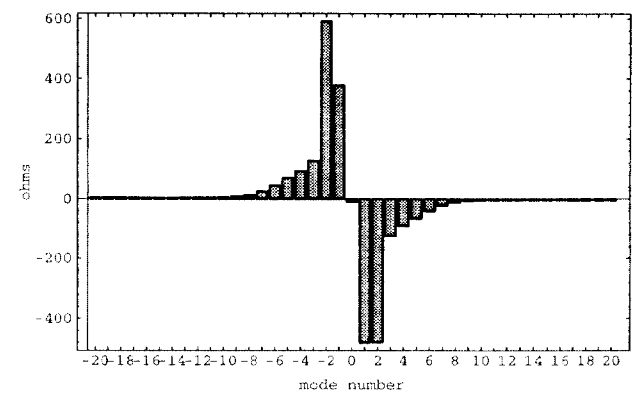


Fig. 7. Deviation in driving impedance - system with & without errors

# 6. CONCLUSION

Deviations in power splitting to the cavities, feedback signal recombining, and cavity parameters are taken into account. Using worst-case cavity parameter deviations it is shown that the closed-loop cavity transfer function is kept within 28% of an error free system. This is slightly above the guideline set by Pedersen [3]. It should be pointed out that the system would unlikely be built in such a way as to have worst-case error paths. By matching cavities, tuning waveguide lengths, and using a fully adjustable combiner network, system errors can be limited to an acceptable level.

# 7. REFERENCES


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