The Recovery of the Most Probable Longitudinal Profile and Duration of Bunches Using Transition Radiation Spectrum[1,2]

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Abstract

The method is based on the relationship between the spectrum of the transition radiation (TR) excited by a train of electron bunches traversing a set of waveguides and the charge distribution function in the bunch. The measured spectrum is compared with the corresponding standard distribution spectra, and a conclusion on the most probable charge distribution function is made.

1. INTRODUCTION

The well-known TR can be used to determine the longitudinal profile and duration of the electron bunches [1-4].

Below we'll consider the method of determination of the charge distribution along the bunch. The method is based on the measurement of TR energy spectral distribution in the set of rectangular waveguides. Their dimensions are chosen such that the k-th waveguide cannot excite lower harmonics of bunches repetition rate where k is a harmonic's number. The experimentally measured dependence of radiation energy distribution over harmonics k is compared with the TR spectrum for standard distributions of charge along the bunch. According to chosen criteria a conclusion is made on the most probable shape of the longitudinal profile and duration of the bunch.

2. TR ENERGY LOSS IN A WAVEGUIDE

It is known that charge q while flying at a velocity v through a waveguide or cavity leaves part of its energy in the form of a radiation field. In the waveguide (cavity) structures two radiation mechanisms may be of practical significance: the TR and the Cerenkov radiation. Further we'll consider the radiators wherein the Cerenkov radiation conditions are not satisfied.

Let a charge q moving parallel to the y axis traverses an infinite rectangular waveguide in a direction parallel to wall b and normal to wider wall a.

The particle entrance coordinates are : $x_0 = a/2$, $y_0 = 0$, z_0 and escape coordinates: $x_1 = x_0$, $y_1 = b$, $z_1 = z_0$.

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In such a waveguide all the E_{mn} and H_{mn} waves with the odd indexes m will be excited. It is shown in [2] that energy loss for the H_{mn} wave radiation will be

$$W_{mn} = \frac{8q^2 \varepsilon_m \pi^2 n^2}{a^3 b c^2 v^2 \lambda_{mn}^2} Re \int_{c\lambda_{mn}}^{\infty} \frac{\sin^2\left(\frac{\pi n}{b} - \frac{\omega}{v}\right)\frac{b}{2}}{\left[\left(\frac{\pi n}{b}\right)^2 - \left(\frac{\omega}{v}\right)^2\right]^2 \gamma_{mn}} \omega^3 d\omega \quad (1)$$

where ε_{mn} for $m \neq 0$ ($\varepsilon_{mn} = 1$), m and n are integers that determine the wave mode in the waveguide,

$$\lambda_{mn} = \pi\left(\frac{m^2}{a^2} + \frac{n^2}{b^2}\right)^{1/2} \quad \text{and} \quad \gamma_{mn} = \left(\frac{\omega^2}{c^2} - \lambda_{mn}^2\right)^{1/2}$$

As far as the waveguide is assumed to be empty, the Cerenkov radiation cannot arise in it. Then H_{mn} will be excited only by TR at the entrance and escape of the charge.

3. TR SPECTRUM IN A WAVEGUIDE

If the current carried by accelerated charge is written in the form of some time function $i_e(t)$, then the power radiated in the waveguide at frequency ω can be written as

$$P_\omega = I_\omega^2 R_{rad} \quad (2)$$

where I_ω is determined from Fourier expansion of function $i_e(t)$ and R_{rad} is "radiation impedance" of the waveguide [5].

It is quite evident that if $i_e(t) = i_e(t+T)$, i.e. represents a periodic train of bunches with a period T , then radiation will take place only at frequencies multiple of ω_0.

It is well known that if the complex spectrum (both of amplitudes and phases) is determined, then the time function shape can be recovered.

Function $i_e(t)$ describes charge distribution in a bunch; hence by measuring I_ω we can, in principle, solve the formulated problem. However the suggested method can be used to measure the amplitude spectrum only, so we can speak only about the most probable charge distribution function in a bunch and its length τ.

The value of R_{rad} can readily be found from (1). For H_{m0} it will be [5]

$$R_{rad} = \frac{4\pi\beta^2}{ab\omega} \sin\frac{\frac{\omega b}{v}}{2} \sum_{m=1}^{M} \frac{1}{\gamma_{m0}} \quad (3)$$

Now we consider a few standard charge distribution functions, using Fourier transformation.

a. Uniform distribution.

$$i(t)=\begin{cases}0\ldots\ldots at\ldots\ldots t\le\tau/2\\ I_0\ldots at\ldots-\tau/2\le t\le\tau/2\\ 0\ldots\ldots at\ldots\ldots t\ge\tau/2\end{cases}\qquad I(\omega\tau)=I_0\tau\,\frac{\sin\frac{\omega\tau}{2}}{\frac{\omega\tau}{2}}\tag{4}$$

b. Triangular distribution.

$$i(t)=\begin{cases}0\ldots\ldots at\ldots t\le-\tau/2\\ I_0(1+\frac{2t}{\tau})\ldots at\ldots-\tau/2\le t\le 0\\ I_0(1-\frac{2t}{\tau})\ldots at\ldots 0\le t\le\tau/2\\ 0\ldots\ldots at\ldots\ldots t\ge\tau/2\end{cases}\qquad I(\omega\tau)=\frac{I_0\tau}{2}\left(\frac{\sin\frac{\omega\tau}{4}}{\frac{\omega\tau}{4}}\right)^2\tag{5}$$

c. Cosinusoidal distribution.

$$i(t)=\begin{cases}0\ldots\ldots at\ldots t\le-\tau/2\\ I_0\ldots\ldots at\ldots-\tau/2\le t\le\tau/2\\ 0\ldots\ldots at\ldots t\ge\tau/2\end{cases}\qquad I(\omega\tau)=\frac{2I_0\tau}{\pi}\,\frac{\cos\frac{\omega\tau}{2}}{1-\left(\frac{2}{\pi}\frac{\omega\tau}{2}\right)^2}\tag{6}$$

d. Normal distribution.

$$i(t)=I_0\exp-\left(t/\tau_{eq}\right)^2\qquad I(\omega\tau)=I_0\tau\exp-\left(\omega\tau_{eq}/2\right)^2\tag{7}$$

From (4)-(7) it follows, that if $\omega\tau\ll1$, $I(\omega\tau)\cong$const and frequency-independent. Hence: if we want to determine wave-form of $i(t)$, we must measure the spectral function $I(\omega\tau)$ at high frequencies.

Any of the above-considered spectral functions can be expanded into a Taylor series

$$I(\omega\tau)\cong1-a_1(\omega\tau)^2+a_2(\omega\tau)^4-\ldots\tag{8}$$

If we now put $\omega=k\omega_0$, then (8) can be rewritten in the form

$$I(k)\cong1-\alpha_1(k\tau)^2+\alpha_2(k\tau)^4\tag{9}$$

Now assume that we measure the spectral function $I(k)$ at three frequencies corresponding to harmonics k_1,k_2,k_3. Restricting ourselves to three terms of expansion (9) we'll obtain a system

$$\begin{aligned}I_1&\cong1-\alpha_1(k_1\tau)^2+\alpha_2(k_1\tau)^4\\ I_2&\cong1-\alpha_1(k_2\tau)^2+\alpha_2(k_2\tau)^4\\ I_3&\cong1-\alpha_1(k_3\tau)^2+\alpha_2(k_3\tau)^4\end{aligned}\tag{10}$$

Here it should be recalled that the coefficients α contain information on spectral function shape, and τ is the bunch duration.

The system (10) has the following solutions

$$\alpha_1\tau^2=\frac{I_1(k_3^4-k_2^4)-I_2(k_3^4-k_1^4)+I_3(k_2^4-k_1^4)}{(k_3^2-k_2^2)(k_3^2-k_1^2)(k_2^2-k_1^2)}$$

$$\alpha_2\tau^4=\frac{I_1(k_3^2-k_2^2)-I_2(k_3^2-k_1^2)+I_3(k_2^2-k_1^2)}{(k(k_3^2-k_1^2)(k_3^2-k_1^2)(k_2^2-k_1^2)}$$

(11)

The parameter α_1^2/α_2 or (α_2/α_1^2) is independent of τ and determined only by the shape of the spectral function $|I(\omega)|$ or, what is the same, $|I_k|$.

In order to enhance the accuracy of determination of the spectral function, it is necessary to carry out measurements by triads of values of $I_1(k_1),I_2(k_2),I_3(k_3)$ on different parts of spectral function. So long as k_1,k_2,k_3 are integer (numbers of harmonics bunch running frequency ω_0), then expressions (11) can be simplified demanding that

$$k_3^2-k_2^2=k_2^2-k_1^2\tag{12}$$

and Eq.(12) can be solved in integers. The solution should be looked for in the interval of values of k from 5 to 60 in order to be able to use the industrial equipment. At the accelerating field frequency ≈ 3 GHz this corresponds to frequencies from 15 GHz to 180 GHz.

Then we'll obtain three triads of values of k:

1. $k_{11}=7$, $k_{12}=13$, $k_{13}=17$;
2. $k_{21}=17$, $k_{22}=25$, $k_{23}=31$;
3. $k_{31}=31$, $k_{32}=41$, $k_{33}=49$

on which we'll just carry out measurements of the spectral function (Fig.1)

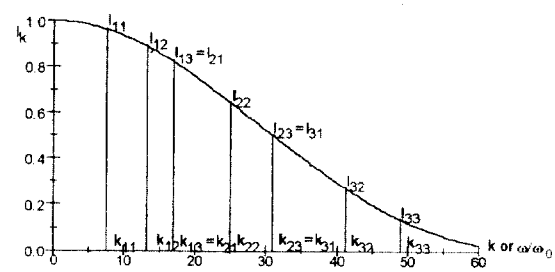

Fig. 1

Experimentally obtained spectral function $I(k)$, its first- and second-order derivatives are compared with the standard spectral functions. By chosen criteria the most probable charge distribution function is determined.

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