# On the Interferometric Diagnostic of Nanobunches in Electron Positron Colliders

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*Abstract.*

Small electron bunches passing through laser driven interference fringes can be diagnosed thanks to Compton scattering signals. It is shown that the amount of scattered light is simply related to the spatial Fourier transform of the transverse particle density distribution. Both real and imaginary parts of the Fourier transform can be recorded through a discrete scanning of the bunches across the interference pattern combined with variable dark fringe spacing. Thus, the even and odd parts of the profile of bunches whose size compares with or is slightly smaller than the optical wavelength can be reconstructed. The size of bunches transversally much smaller than the optical wavelength can be evaluated by scanning across the fringes while varying the optical intensity.

## 1. INTRODUCTION.

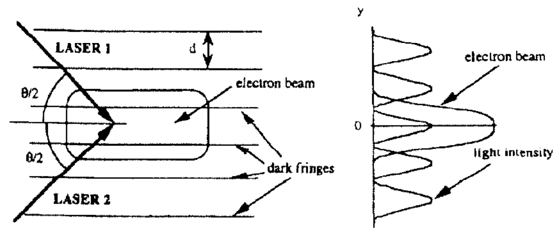
Recently, T. Shintake[1] proposed a new and attractive scheme to monitor the small transverse size (nanometer) electron bunches with rectangular cross-section required for future linear colliders [2]: an ultra relativistic electron bunch is passed through the plane of optical interference fringes. These are of the type usually set up in laser velocimetry [3]. The photons participating in the interference process undergo Compton scattering, giving birth to high energy quanta propagating almost colinearly with the incident electrons. Thanks to a scan of the electrons across the fringes, the scattered signal is modulated. An evaluation of the transverse size of the bunch can be inferred from the modulation depth. In the case of a gaussian beam, the accessible characteristic width is much smaller than the laser wavelength.

## 2. SCATTERED SIGNAL ANALYSIS.

The geometry of the interferometric system is shown on figure 1a: a laser beam with wavelength $\lambda_0$ is separated in such a way that the two beams recombine symetrically at the end of equal optical pathes. Denoting by $\theta$ the angle between laser beams in the interference region, the dark fringe interval is $d = \frac{\lambda_0}{2\sin(\theta/2)}$, with the wavenumber: $\kappa = \frac{2\pi}{d} = \frac{4\pi}{\lambda_0}\sin\frac{\theta}{2}$.

Foreseen bunch would contain $10^{10}$ particles. Their transverse size could be as small as 10 nm. The Compton process leading to the recorded signal is thoroughly described in [1]. It is shown that high counting rates can be expected using a moderate laser power (20 MW. in pulsed regime), a value accessible by many laser systems with optical wavelength $\lambda_0$ in the range 250-1000 nm.

As far as the bunch encounter with the photons is involved, the signal results from the superposition of the electron density profile to the light intensity in the fringe pattern (figure 1b).




a b

*Figure 1.*

Assume: i) the ultrarelativistic beam whose Lorentz factor is $\gamma_e$, has a rectangular cross-section with the y coordinate perpendicular to the direction of the fringes; ii) the beam is initially centered in the middle of the central bright fringe, iii) along the x direction, the fringe brightness is uniform and at any y, the electron distribution is the same, iv) both laser beams are linearly polarized with electric field in the z direction. The signal then reads

$$(2\text{-}1) \qquad S = H(\gamma_e, \lambda_0)\int_{-\infty}^{\infty} I_e(y)\, I_\varphi(y)\, dy\ ,$$

where $I_e = J_e\, f(y)$ is the electron transverse flux distribution in the y direction, normalized in such a way that $\int_{-\infty}^{\infty} f(y)\,dy = 1$, $I_\varphi(y)$ is the light intensity locally in the fringe pattern, and $H(\gamma_e, \lambda_0)$ contains all the relevant information associated with the kinematics of Compton scattering and the geometry of the apparatus. Actually, the integral in (2-1), is bounded at some maximum value $y_{max}$. Since f(y) decreases rapidly to zero at some value smaller than a few fringe intervals, the integral can be safely extended up to infinity.

The light intensity distribution $I_\varphi(y)$ in the fringe pattern depends upon the beam polarization. For a linear polarization with the electric field along the z axis, the fringe contrast is 100%. When scanning is used, the interference pattern is displaced by an amount $\eta$ in the y direction with respect to the bunch center. The intensity $I_\varphi$ is accordingly

$$(2\text{-}2) \qquad I_\varphi = I_0 \cos^2\{\pi[y - \eta(t)]/d\}\ ,$$

where the exact time dependance $\eta(t)$ may be ignored. Consequently, setting

$$(2\text{-}3) \qquad S_0 = H(\gamma_e, \lambda_0)\, J_e I_0/2\ ,$$

and

$$(2\text{-}4) \quad G(\kappa) = \int_{-\infty}^{\infty} f(y)\cos\kappa y\, dy\ ,\ F(\kappa) = \int_{-\infty}^{\infty} f(y)\sin\kappa y\, dy\ ,$$

and rearranging the formulas , the signal is

$$S = S(\kappa) = S_0\left[1+\int_{-\infty}^{\infty} f(y)\cos\frac{2\pi}{d}(y-\eta)\,dy\right] = S_0\left[1+\cos\frac{2\pi\eta}{d}\int_{-\infty}^{\infty} f(y)\cos\frac{2\pi y}{d}\,dy + \sin\frac{2\pi\eta}{d}\int_{-\infty}^{\infty} f(y)\sin\frac{2\pi y}{d}\,dy\right] \tag{2-5}$$

$$= S_0\left[1+G(\kappa)\cos\kappa\eta + F(\kappa)\sin\kappa\eta\right].$$

$G(\kappa)$ and $F(\kappa)$ appear as $2\pi$ times the real and the imaginary part respectively, of the spatial Fourier transform of $f(y)$.

## 3. PROBING THE TRANSVERSE STRUCTURE OF ELECTRON "MICROBUNCHES".

Suppose $G(\kappa)$ and $F(\kappa)$ are recorded for a suitable range of the variable $\kappa$. Then, applying an inverse Fourier transform:

- to the former, restitutes the even part of $f(y)$
- to the latter, the odd part.

Thus, the electron density distribution in the y direction can in principle, be completely determined after the knowledge of $G(\kappa)$ and $F(\kappa)$. The best sensitivity is expected from a *discrete* scan: changing the integer p step by step,

- $\eta = 0 + \frac{p\pi}{\kappa}$ yields $G(\kappa) = [S_{max}(\kappa) - S_{min}(\kappa)]/(2S_0)$;
- $\eta = \frac{d}{4} + \frac{pd}{2} = \lambda_0/\left(8\sin\frac{\theta}{2}\right) + p\,\lambda_0/\left(4\sin\frac{\theta}{2}\right)$ gives $F(\kappa)$ through the same formula.

The maximum of $\kappa$ is $4\pi/\lambda_0$ corresponding to $\sin(\theta/2) = 1$. Let the sampling interval be $\Delta\kappa = 4\pi/(N\lambda_0)$. Consequently $\Delta y = 1/(N\,\Delta\kappa) = \lambda_0/4\pi$ independant of N. This value is the resolution limit of the Fourier analysis. Then, the useful range in wavelength for a precise probing of the bunch structure is:

$$\pi a/4 \le \lambda_0 \le \pi a. \tag{3-1}$$

The size a should compare with the wavelength i.e. be in the micrometer range ("microbunches"). The high resolution situation for the distribution

$$f(y) = f(0)\left(e^{-y^4/a^4} + \mu\, y\, e^{-y^2/a^2}\right), \tag{3-2}$$

where $\mu$ is a small coefficient, is displayed on figure 2 together with the corresponding sampled signals (16 channels) associated with both $G(\kappa)$ and $F(\kappa)$. $\kappa$ is given equally spaced value in the interval $0{,}4\pi/\lambda_0$. Since $f(y)$ varies comparatively slowly, the spectrum is deprived of high spatial frequencies.

Figure 3 displays a reconstructed even part of the particle transverse distributions after an inverse Fourier transform of

$$S(\kappa) \propto 1 + \mathrm{sinc}(\kappa). \tag{3-3}$$

## 4. "NANOBUNCH" SIZE EVALUATION.

Below roughly one third of the optical wavelength, no precise analysis of the bunch transverse structure can be made. True, size estimates could be obtained after the Fourier spectra, down to the resolution limit $\lambda_0/4\pi$. Now, although high power lasers are commonplace in the visible and near U.V. spectra, a wavelength of 250 nm is a lower boundary for easy optics. Scanning the bunch across the interference fringes is the trick devised by Shintake [1] to diagnose so called "nanobunches" whose transverse size is around 10 nm, much smaller than the optical wavelength.

The original proposal deals exclusively with gaussian transverse profiles. Indeed, such profiles are expected to be actualy achieved in future accelerators. Let then

$$f(y) = e^{-y^2/a^2}/(a\sqrt{\pi}), \tag{4-1}$$

satisfying to the normalisation condition. This implies a constant number N of particles in bunches with similar shapes but different widths passing through constant intensity fringes. $J_e f(0)$ and $I_0$ are accordingly constants.

(4-1) being an even function, its Fourier transform has no imaginary part, so $F(\kappa) = 0$. The real part is also a gaussian:

$$G(\kappa) = e^{-a^2\kappa^2/4} = e^{-\pi^2 a^2/d^2}, \tag{4-2}$$

at given fringe interval d.

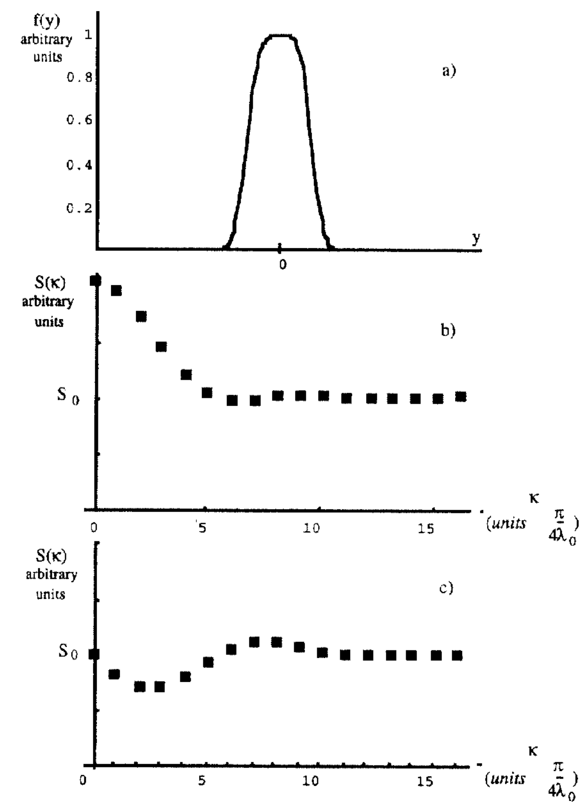



*Figure 2.*

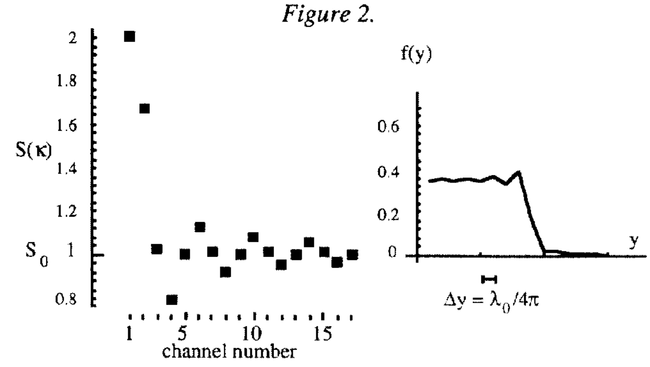



*Figure 3.*

Scanning induces a signal modulation proportionnal to $G(\kappa) = m(a)$ which is shown on figure 4. The reasonnably accessible values of a correspond to a variation of the modulation depth in the range 10%-90% of the maximum. At given laser wavelength, the lower limit for an evaluation of the bunch characteristic width a is $\lambda_0/20$, obtained with counterpropagating parallel beams perpendicular to the electron velocity and 90 % modulation of the Compton signal.

Now, since a general scaling law holds for Fourier transforms of scaled functions with constant value at the origin, the scanning method may be applied to other distributions for which a characteristic length can be defined.

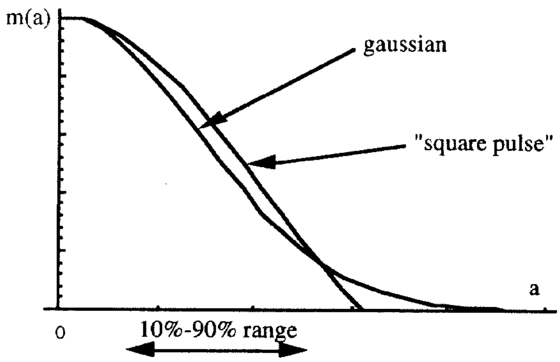



*Figure 4.*

However, in all cases but the gaussian, the Fourier transforms oscillate: small modulation depths might correspond to several values of a. Fortunately, in such conditions, the Fourier analysis works fairly well.

As an example, take the limit case of a distribution in the form of a constant height "square" pulse. Then one gets

$$\text{(4-2)} \qquad m(a) \propto \sin(\kappa a)\ /\kappa a\ .$$

The gaussian and the "square" are opposite limit cases. If the corresponding m(a) are drawn on the same plot, as shown on figure 4, they are quite close to each other in the 10%-90% range. The difference is at most 20 %. Only very precise measurements of the modulation depth thus enable one to discriminate between different electron density profiles across the bunch.

## 5. ADJUSTABLE LASER INTENSITIES.

If one relaxes the normalisation condition on f(y) and take

$$\text{(5-1)} \qquad f(y) = e^{-y^2/a^2}\ ,$$

then $J_e$ is a constant and

$$\text{(5-2)} \qquad G(\kappa) = g(a) = a\sqrt{\pi}\ e^{-\pi^2 a^2/d^2}\ ,$$

which, in the limit of small a's, is proportionnal to a. In order to keep $J_e$ constant, the signal should be lowered by the same amount as the bunch is squeezed in the y direction. This can be achieved by playing either with N or with the laser intensity. Since attenuation by a prescribed factor is commonplace in laser beam handling, the second solution is to be prefered. The smallest attainable value of a thus depends only upon the sensitivity of the measuring device.

Reducing the laser intensity is a major drawback for the use of g(a) in the diagnostic of nanobunches. It is shown in [1], that the minimum laser power to get a high counting rates is rather moderate: 20 M.W. in pulsed regime. Now, lasers exist in the optical range with power far exceeding this requirement. Then, one could use attenuated laser beams for a in the vicinity of $\lambda_0/4\pi$. At smaller a's, attenuators are removed so that the laser intensity grows proportionnally to the squeezing of the bunch. Large Compton signals and modulation depths h(a) can thus be obtained at small a, as displayed on figure 5 for the case of distribution (3-2).

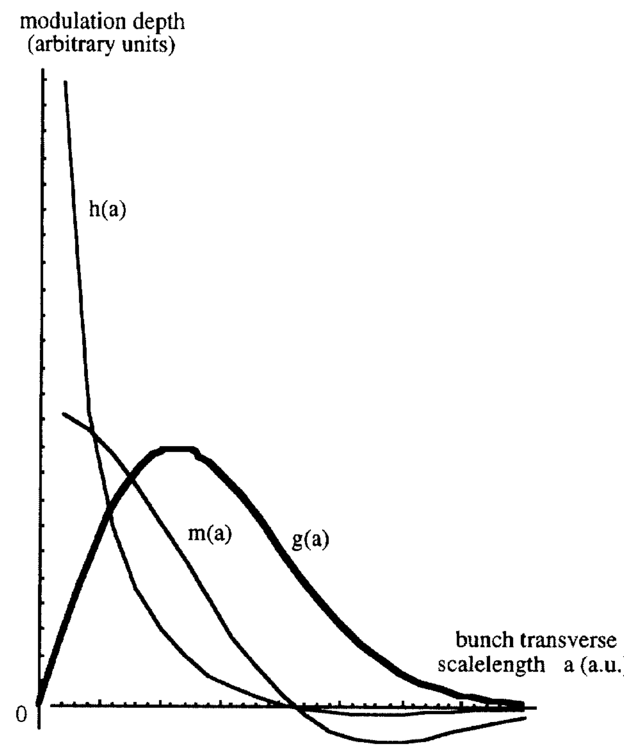



*Figure5.*

## 6. CONCLUSION.

The combination of optical interference and Compton scattering provides efficient diagnostic methods for electron bunches with rectangular cross section. Given the laser wavelength within the range defined by inequalities (3-1), by playing with the angle between the laser beams in the interference region [4] and the displacement $\eta$ of the bunch in the y direction, one gets both the real and imaginary parts of the Fourier transform of the transverse distribution. Inverse transforms restitute the even and odd parts of the structure which is completely resolved. This high resolution method cannot be applied to bunches with a size smaller than about a third of a wavelength (down to 80 nm for 250 nm light).

When the transverse size a is much smaller than the optical wavelength (i.e. a << 80 nm), only an evaluation of the transverse size of bunches is possible. This is obtained thanks to a scan of the electron bunch across the fringes. *Active* scanning, i.e. adjusting the laser intensity to the squeezing of the bunch, could be an improvement of the method when applied to very small a.

## REFERENCES

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2. see e.g. C. Rubbia in Proceedings of the European Particle Accelerator Conference, EPAC-88, Rome, ed S. Tazzari (1988).& H. Henke, in Proceedings of the European Particle Accelerator Conference, EPAC-90, Nice, eds P. Marin, P. Mandrillon (1990).
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4. J.L. Bobin, Optics Communications **105** (1994) 178.