Influence of Electrodes Shape on Emittance Growth in Periodic Axisymmetric LEBT

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Abstract

We report the results of design studies for electrostatic low-energy- beam-transport system(LEBT) of H injector for a 433MHz RFQ.The characteristic property of the LEBT is beam transport and matching by means of a series of axisymmetric apertures at potential alternation.The aim of our initial effort was to investigate the influence of varying electrodes shape on axial field and rms emittance growth from spherical aberration in ideal periodic system with K-V beam using numerical simulation.We found a simple analitic expressions for approximation of potential distribution on axis,derived from it an equation to calculate electrodes contour and optimized axial field in a manner to reduce excessive aberrations. Parameters of the optimal design are presented.

1 INTRODUCTION

The problem studying was initiated by need to design injector optics for radio frequency qudrupole (RFQ) operating at 433MHz so placing more stringent demands on the beam.The optics and injector as a whole, exept forming the beam and matching it to the RFQ, should meet the following requirements:(1) high conductance and sufficient lenght for effective pumping;(2) creating conditions to prevent accumulation of plasma due to beam and residual gas interaction;(3) simplicity of power supplies (employment of only two potential conforming to energy of beam extraction from the source and injection to RFQ);(4) placement of vacuum port and diagnostic equipment between injector and RFQ.All of these led to optical system in the form of set of axisymmetric apertures at alternating potential.The periodic electrostatic lenses are widly used for transportation and forming beams themselves as well as a part of more complicated systems,dispite introducing of perseptible,as a rule, aberrations[1].In our case the problem of emittance degradation,holding it at level suitable for successful injection into RFQ is pressing, bacause available reserve of emittance is very moderate.

Search of optical systems which should realize desired first order properties at minimum aberrations without additional corrective elements is enough difficult task, primarily in view of problem to describe adequately all kinds of electrode contours with limited number of parameters.However,spatial distribution of potential and consecuently the shape and potential of conducting electrodes which will give rise this field are uniquely determined by field functions on axis.For this reason we would like to optimize the axial functions rather than test an abundance of electrode configurations. A common properties and generality determing focusing and aberration characteristic of one system or another can be find out with help of simple analytic models of axial field containing several free parameters for variation.

The prime object of our effort is to study possibility to preserve emittance in electrostatic axisymmetric periodic system as a LEBT and identify the various control parameters.Therefore,at first we describe and substantiate mathematical model of the system;then present results of studying systems at symmetric and asymmetric period; at last discuss in brief the problem of electrode synthesis for optimal axial field.

2 MATHEMATICAL MODEL

There are several representations for potential off axis.The most frequently in use are:

$$\Phi(r,z) = \sum_{n=0}^{\infty}(-1)^n \frac{\partial^{2n} f(z)}{\partial z^{2n}(n!)^2}(r/2)^{2n} \qquad (1)$$

$$\Phi(r,z) = \sum_{n=0}^{\infty} I_0(\frac{2\pi n r}{L}) A_n cos(\frac{2\pi n z}{L} + \phi_n) \qquad (2)$$

where

$$f(z) = \Phi(0,z) = f_0 + \sum_{n=1}^{\infty} A_n cos(\frac{2\pi n z}{L} + \phi_n) \qquad (3)$$

Exact field determing with help of Eq.(1) is possible if the function f(z) is a polinomial of degree N (for periodical field it is unsuitable). For Eq.(2) axial distribution must be sum of N harmonics.Action of spherical aberrations consists in increasing beam transverse temperature due to radial nonlinearities of the field.Responsibility for them rests with the derivations of axial potential since the fourth.As a consequence our lense models intended to reduce aberrations are derived from analytic functions approximating behaviour of $f^{IV}(z)$.

The capability of each focusing system can best be evaluated by using the uniform K-V beam model with linear space charge force.Moreover, to avoid other effects connected with emittance but off our interest now, we consider the ideal system,i.e. matched beam and a negligibly low emittance(laminar flow,cold beam).For this reason parameters of the axial distribution $f(z)$ and initial conditions for beam envelope equation

$$R'' = \frac{1}{f}\left(\frac{I}{4\pi\varepsilon_0\sqrt{2e/m}\sqrt{f}}\frac{1}{R} - \frac{f'R'}{2} - \frac{f''R}{4}\right) + \frac{E^2}{R^3} \quad (4)$$

are chosen to realize required average radius

$$R_0 = \frac{1}{L}\int_L R(z)dz.$$

The validity of system is estimated by emittance growth ΔE on period L: $\Delta E = E(z+L) - E(z)$, where $E = 4\beta\gamma\sqrt{<r^2><r'^2> - <rr'>^2}$ - normalized rms emittance; r, r'-phase coordinates of test particles represent beam and evaluated by equation:

$$r'' = \left(\frac{\partial U(r,z)}{\partial r} - \frac{\partial U(r,z)}{\partial z}r'\right)(1+(r')^2)\frac{1}{2U(r,z)} \quad (5)$$

Here $U(r,z) = \Phi(r,z) + V(r,z)$. $\Phi(r,z)$ is potential in system without beam(solution of Laplace's equation written in form Eqs.(2) or (1) and computed through the model parameters).$V(r,z)$ is field of space charge (solution of Poisson's equation at $V(0,z) = 0$).For paraxial K-V beam it can be approximated as

$$V(r,z) = \begin{cases} \frac{I}{4\pi\varepsilon_0\sqrt{2e/m}\sqrt{f(z)}}(\frac{r}{R})^2, & r \le R, \\ \frac{I}{4\pi\varepsilon_0\sqrt{2e/m}\sqrt{f(z)}}(1 + 2ln(\frac{r}{R})), & r > R, \end{cases}$$

where I -beam current.

3 EMITTANCE GROWTH IN MODEL SYSTEMS

To find suitable model for optical system with symmetric structure of period we wiewed all real lense constructions designed by us before[2]. Necessary computations were performed by means of code FIELD[3].It was found that at all carried out modifications of apertures as well as a cylinders in bore size,thickness and curvature radius the fourth derivation of axial field may approximately be described by function:

$$f^{IV} = A(1 - Bsin^{2m}(kz))sin^{2n+1}(kz), \quad (6)$$

where $k = 2\pi/L$. It can easy be presented as a sum of m+n+1 harmonics and integrated to give exact analytic model of field in form of Eq.(2) .For real lenses with minimum aberrations of tested free parameters are $n = 0; m = 1,2; B > 0$. In model as these take place optimal B is just hagher unity and scurcely depends on period lenght and perveance.Fig.1 illustrates effect of transportation radius on emittance growth,envelope swing $\Delta R = Rmax - Rmin$, minimum necessary electrode voltage $\Delta U = U(Rmax) - U(Rmin)$.

In periodical system a field on axis as well as its derivations are periodic functions. Matched beam pulsates with period of system too.One would expect reducing aberrations when larger nonlinearity corresponds to lesser beam radius and conversely.Such conditions can be realized at asymmetric structure of period e.g. if the fourth derivation appears as

$$f^{IV} = A(B_m - cos^{2m}(kz/2)), \quad (7)$$

where $B_m = \frac{(2m)!}{(m!)^2 4^m}$. Effect of asymmetry factor m on emittance gain at various period length see in Fig.2.

The most preferable for emittance preservation is quadratic changing of axial potential.In this case all derivations, since the third, are zero,the radial force is linear at any distance of r,the equipotential surfaces are rotational hyperboloids. Close them properties may be realized at following axial function:

$$f^{IV} = A(1 - Bsin(kz))(1 + Csin(kz))^m, \quad (8)$$

if parameter C equal unity(in the general way $0 < C \le 1$). B is function of C, bacause $\int_L f^{IV}(z)dz = 0$. Value m limits maximum asymmetry of system.Fig.3 shows increasing emittance in this model and influence of perveance.Effect of E^2 in Eq.(4) is similar.One of the way to preserve emittance is extending of system period.It and its cost are demonstrated in Fig.4 for models Eqs.(6) and (8) at optimal free parameters .

4 SYNTHESIS PROBLEM

Seeking shape of field forming serfuces through the field on axis can be reduced to a Cauchy's problem for Laplase's equation which, as well known,is not correct.Even slight changing of axial function may give fields strongly different one another.For this reason more often than not resulting map of equipotential lines is very complicated for analytic approximating functions,contains poles, corners,other singularities,so realization of the electrodes may occour impossible[4].To produce more realizable shape of electrodes we replace exact solution in form of Eq.(2) by approximate Eq.(1).Due to instability of external boundary problem this approximation,as a rule, provides in the beam region electric field which for all practical purposes does not differ from one created by theoreticaly exact electrodes.All necessery derivation can easy be calculated from Fourier series of axial field Eq.(3),see Fig.5.

5 CONCLUSION

Curvature of field forming surfaces may measurably influence on emittance growth being placed in close proximity to beam.More sensible reducing of emittance degradation is possible by (1)decreasing transportation radius,(2)extending system period,(3)increasing average beam energy,(4) going to asymmetric structure.However,all this factors are accompanied by gain in applyed voltage.Obviously,minimum aberration belong to asymmetric system like set of ring lenses[5],but having near-hyperbolic electrode shape.

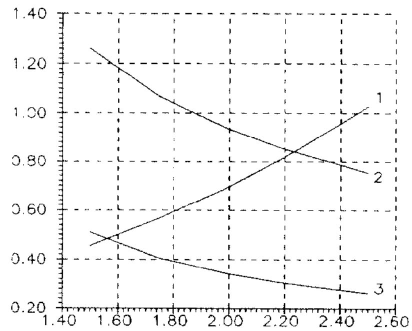

Figure 1. Effect of transportaton radius $R_0(mm)$ on:
1-emittance $(\Delta E/E_M)$,
2-voltage $(\Delta U/U_M)$,
3-beam ripple $(\Delta R/R_0)$
in symmetric model (Eq.(6):$m = 2, n = 0, B^{-1} = 0.75$) at
$I = 30mA$, $f_0 = 30kV$,$L = 40mm$,
$U_M = 1.5f_0$, $E_M = 0.045mm * mrad$

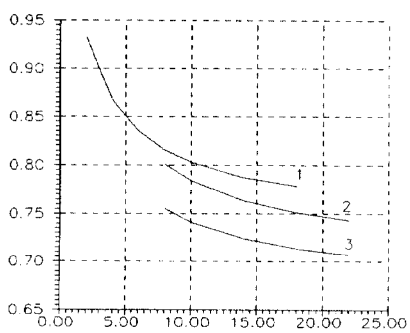

Figure 2. Emittance growth $(\Delta E/E_M)$ versus asymmetry of period (factor $2m$ in model Eq.(7)) at
$I = 30mA$, $f_0 = 30kV$, $R_0 = 2.5mm$,
$E_M = 0.050mm * mrad$
$1 - L = 40mm$, $2 - L = 50mm$, $3 - L = 60mm$

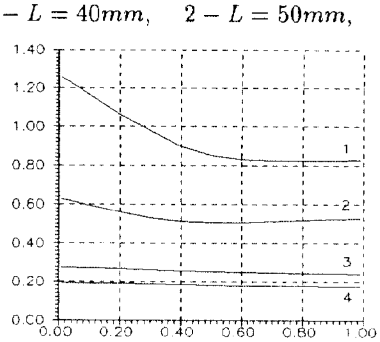

Figure 3. Influence of field form
(parameter C in Eq.(8), m=6) and perveance on
1,2- emittance $(\Delta E/E_M)$ and
3,4-beam ripple $(\Delta R/R_0)$ at
1,3-$I = 30mA, f_0 = 30kV$;
2,4-$I = 20mA, f_0 = 35kV$;
$L = 40mm$, $R_0 = 2.5mm$, $E_M = 0.040mm * mrad$

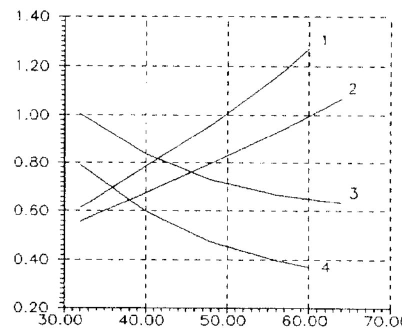

Figure 4. 1,2-Emittance $(\Delta E/E_M)$ and
3,4- voltage $(\Delta U/U_M)$
vs length of period $L(mm)$ for optimal field form in
2,3-symmetric $(Eq.(6) : m = 2, n = 0)$ and
1,4-asymmetric $(Eq.(8) : m = 6)$ models at
$I = 30mA$, $f_0 = 30kV$, $R_0 = 2.5mm$,
$E_M = 0.055mm * mrad, U_M = 1.5f_0$

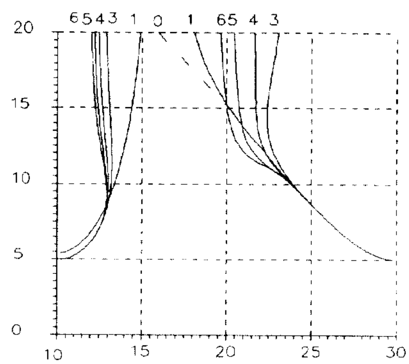

Figure 5. Electrode contours (mm) computed at different number (figures near the lines) of terms in Eq.(1)
without beam at $L = 40mm$
for model Eq.(8) $(C = 1, m = 6)$
Dash line is hyperbola

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