NUMERICAL CODE FOR THE RELATION DISPERSION OF CYLINDRICAL AND INHOMOGENEOUS PLASMA*

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Abstract

In a general formulation, starting from Vlasov–Maxwell equation, the dispersion relation for a radial inhomogeneous cylindrical plasma in a strong magnetic field is obtained. We develop a numerical algorithm, based in expanded Fourier - Bessel functions for the calculus of the relation dispersion of cylindrical and inhomogeneous plasma under electron beams interactions. The program operates as a Windows application for PC computers and it interacts directly with the "Mathematica" graphic mode package for Windows [1]. Besides, this program can calculate different profiles density for the plasma and the beam, that it is necessary in the relation dispersion calculation.

1. INTRODUCTION

The RELDIS program calculates the dispersion relation for an inhomogeneous electronic plasma, and the dispersion relation for an inhomogeneous electron beam that is interacting with an inhomogeneous electronic plasma that is placed in a strong magnetic field [2]. The theoretic frame is based in the kinetic theory. We will assume a fully ionized plasma. This case is related with laboratory problems for focusing of electron and positrons [3,4,5] and in nature as a possible cosmic ray acceleration. The space-charge interactions of an electron beam with a cold collisionless plasma in the presence of an infinite magnetic field in the direction of the beam velocity has been studied in the past by Bogdanov [6] and Vlaardingerbroek [7]; they considered the case of both the beam and the plasma filling the wave guide. A systematic review of the excitation of plasma oscillation by a radially inhomogeneous electron beam for a cold axisymmetric beam has been given by Mikhailovskii [8].

2. THEORETIC FRAME.

In order to study the dispersion of the waves in a inhomegeneous and cylindrical plasma we use Vlasov–Maxwell equation system:

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$$\frac{\partial f_\mu}{\partial t}+\boldsymbol{v}\bullet\frac{\partial f_\mu}{\partial \boldsymbol{r}}+\frac{q_\mu}{m_\mu}\left[\boldsymbol{E}+\frac{\boldsymbol{v}}{c}\times\boldsymbol{B}\right]\bullet\frac{\partial f_\mu}{\partial \boldsymbol{v}}=0 \quad (1)$$

$$\nabla\times\boldsymbol{B}=\frac{4\pi}{c}\sum_\mu q_\mu\bar{n}_\mu\int \boldsymbol{v} f_\mu(\boldsymbol{r},\boldsymbol{v},t)d^3\boldsymbol{v}+\frac{1}{c}\frac{\partial \boldsymbol{E}}{\partial t} \quad (2)$$

$$\nabla\bullet\boldsymbol{E}=4\pi\sum_\mu q_\mu\bar{n}_\mu\int f_\mu(\boldsymbol{r},\boldsymbol{v},t)d^3\boldsymbol{v} \quad (3)$$

$$\nabla\times\boldsymbol{E}=-\frac{1}{c}\frac{\partial \boldsymbol{B}}{\partial t} \quad (4)$$

$$\nabla\bullet\boldsymbol{B}=0 \quad (5)$$

Where $f_\mu(\boldsymbol{r},\boldsymbol{v},t)$ depends on the $\boldsymbol{r}$ position and $\boldsymbol{v}$ velocity in the t time and it is the distribution function of the identical particles characterized by the m_μ mass and the q_μ charge of the μ type. $\boldsymbol{E}$ and $\boldsymbol{B}$ are the electric and magnetic fields respectively and $\bar{n}_\mu$ is the medium density particles per unit of volume. The system of equations (1) to (5) taking into account the boundary conditions and using small perturbations is transformed in the following form:

$$\frac{\partial f_\mu^1(\boldsymbol{r},\boldsymbol{v},t)}{\partial t}+\boldsymbol{v}\bullet\frac{\partial f_\mu^1(\boldsymbol{r},\boldsymbol{v},t)}{\partial z}+g_\mu\frac{q_\mu}{m_\mu}E_z\frac{\partial F_\mu^0}{\partial v}=0 \quad (6)$$

$$\nabla\times\boldsymbol{B}^1=\frac{4\pi}{c}\sum_\mu q_\mu\bar{n}_\mu(0)\int v_z f_\mu^1 dv+\frac{1}{c}\frac{\partial E_z}{\partial t} \quad (7)$$

$$\nabla\bullet\boldsymbol{E}^1=\sum_\mu 4\pi q_\mu\bar{n}_\mu(0)\int f_\mu(\boldsymbol{r},\boldsymbol{v},t)d^3\boldsymbol{v} \quad (8)$$

$$\nabla\times\boldsymbol{E}^1=-\frac{1}{c}\frac{\partial \boldsymbol{B}^1}{\partial t} \quad (9)$$

$$\nabla\bullet\boldsymbol{B}^1=0 \quad (10)$$

Here $\bar{n}_\mu(0)$ is the particles density definite on the z axis, and g_μ is a radial function related with the inhomogeneity of the plasma and the beam. Since there are no perpendicular currents $J_\perp=0$, therefore

$$J_z(r,\theta,z,t)=\sum_\mu q_\mu n_\mu(0)\int v f_\mu(r,v,t)d^3v \quad (11)$$

The electric permitivity is obtained expanding the equations (6) to (11) in terms of the Fourier Bessel series given by:

$$\psi_{m\ell}^{(k)}(r,\theta,z)=\frac{J_m(P_{m\ell}r)}{2aJ_{m+1}}e^{im\theta}e^{ikz},\quad m=0,\pm1,\pm2,\ldots;$$
$$\ell=1,2,\ldots;\quad -\infty\le k\le\infty \qquad (12)$$

We use the Fourier transform for the spatial coordinates, for the temporary variable we use the Laplace transform, then the electric permitivity is given by:

$$\varepsilon_z(r,k,\omega)=1-\sum_\mu g_\mu\frac{\omega_{p\mu}^2(0)}{k^2}\int dv\frac{dF_\mu^0/dv}{v-\omega/k} \qquad (13)$$

Where $\omega_{p\mu}^2(0)$ the plasma frequency is definite as:

$$\omega_{p\mu}^2(0)=\frac{4\pi q_\mu^2\bar{n}_\mu(0)}{m_\mu} \qquad (14)$$

The wave equation obtained from the Maxwell equations for the electric field with dependence azimuth of the form $E_z(r,\theta)=E_{zm}(r)exp(im\theta)$ and with the boundary condition $E_{zm}(r=a)=0$, takes in cylindrical coordinates the form:

$$\left\{\frac{d^2}{dr^2}+\frac{1}{r}\frac{d}{dr}-\left[1-g_\mu I_\mu(k,\omega)\right]\kappa+\frac{m^2}{r^2}\right\}E_{zm}(r)=0 \qquad (15)$$

where:

$$\kappa^2=-\left(\frac{\omega^2-k^2c^2}{c^2}\right) \qquad (16)$$

$$I_\mu(k,\omega)=\frac{\omega_{p\mu}^2(0)}{k^2}\int_L dv\frac{dF_\mu^0/dv}{v-\omega/k} \qquad (17)$$

and L is the Landau contour. The homogeneous case is obtained making $g_\mu(r)=1$; in the inhomogeneous case, $g(r)$ is a function that can be adjusted according to the conditions that are being studied (see the RELDIS help). We used the Maxwelliam velocity distribution functions for the plasma and the beam, which yields:

$$F_p^0=\left(\frac{2\pi K_BT_p}{m_e}\right)^{-\frac{1}{2}}e^{-\frac{(m_ev^2)}{K_sT_p}} \qquad (18)$$

$$F_b^0=\left(\frac{2\pi K_BT_b}{m_e}\right)^{-\frac{1}{2}}e^{-\frac{(m_e(v-v_0)^2)}{K_sT_b}} \qquad (19)$$

Where, v_0 is the beam velocity and K_B is the Boltzmann constant. The equation (15) is solved by expanding $E_m(r)$ in Fourier-Bessel series. Finally we get the following dispersion relation in dimensionless values.

$$D_{mn}\,\underline{\mathbf{X}}=0 \qquad (20)$$

Which $\underline{\mathbf{X}}$ is a vector associated with the eigen functions for the electric field, and the matrix D_{mn} take the follow form:

$$D_{mn}=\frac{\bar{c}^2\bar{k}^2-\bar{\omega}^2}{\bar{c}^2\left(\bar{k}^2+X_{m\ell}^2\right)-\bar{\omega}^2}\left\{\frac{C_{mp}^{\ell\ell'}Z'(\beta)}{2\bar{\lambda}_{dp}^2\bar{k}^2}+\frac{C_{mb}^{\ell\ell'}Z'(\alpha)}{2\bar{\lambda}_{db}^2\bar{k}^2}\right\}-\Pi \qquad (21)$$

with:

$$C_{mp,b}^{\ell\ell'}=\frac{2}{a^2J_{m+1}(X_{m\ell})J_{m+1}(X_{m\ell'})}\int_0^a drrJ_m(rp_{m\ell})J_m(rp_{m\ell'})g_{p,b}(r) \qquad (22)$$

The values $X_{m\ell}=p_{m\ell}a$ are the zeros of the Bessel functions J_m, $\bar{k}=ka$, $\bar{\omega}=\frac{\omega}{\omega_p}$, $\bar{\lambda}_{d\mu}^2=\frac{K_BT_{p,b}}{m_{p,b}\omega_{p,b}^2(0)a^2}$, $\beta=\frac{1}{\sqrt{2}}\frac{\bar{\omega}}{\bar{\lambda}_{dp}\bar{k}}$, $\alpha=\frac{\bar{\omega}}{\bar{k}}\frac{1}{\bar{\lambda}_{db}\sqrt{2}}-\sqrt{\frac{E}{K_BT_b}}$ and $\bar{c}^2=\frac{c^2}{a^2\omega_p^2}$.

The p and b subscript's are related with the plasma and beam respectively, a is the wave guide radius, k is the wave number, Π the unity matrix, E is the beam energy, Z' is the derive of the dispersion function. Due to the boundaries conditions $E_z(z=0,z=L)=0$ for the electric field, we obtain the following condition for the wave number:

$$\bar{k}_+ + \bar{k}_- = \frac{2\tau a\pi}{L_0},\quad \text{with}\quad \tau=0,1,2,\ldots. \qquad (23)$$

Here $\bar{k}_{+(-)}$ is the wave number when the electromagnetic wave travels to right (left). The equation (20) is solved by using the Zerizawa method [9]. The convolution coefficients $C_0^{\ell\ell'}$ given by the expression (22) were calculated taking into account functions $g_\mu(r)$.

For a completed theoretic frame, you can see the Help of the RELDIS code.

3. CODE DESCRIPTION.

This code runs under Microsoft Windows 3.x or later; it needs 2 Mb of RAM memory and consumes 5% of system resources. For a major advantage for this code, must have access to Mathematica Package. The code calculates the convolution coefficients $C_0^{\ell\ell'}$ (see fig. 1) for three different density profiles, in the which can be changed the parameters that determinate the inhomogeneity; also ask for the matrix order for generate this coefficients and can generate null coefficients are used in the calculate of the dispersion relation for the beam and the plasma independent.
The principal calculation corresponds to the final frequency of

Figure 1.

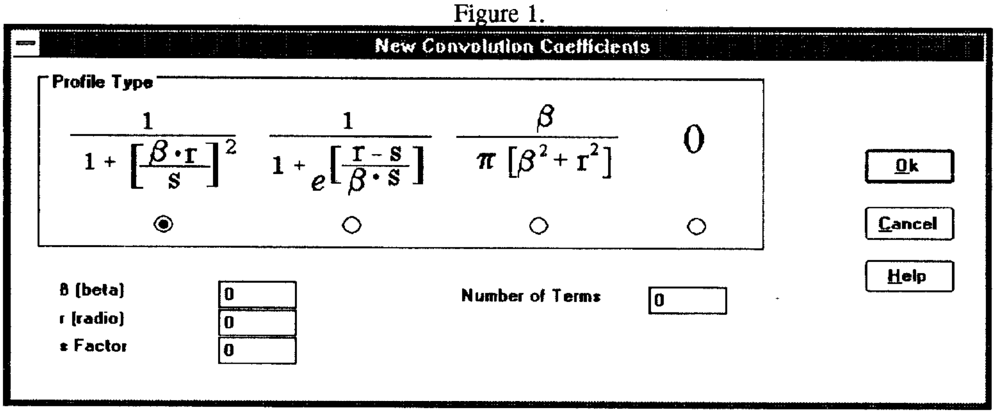
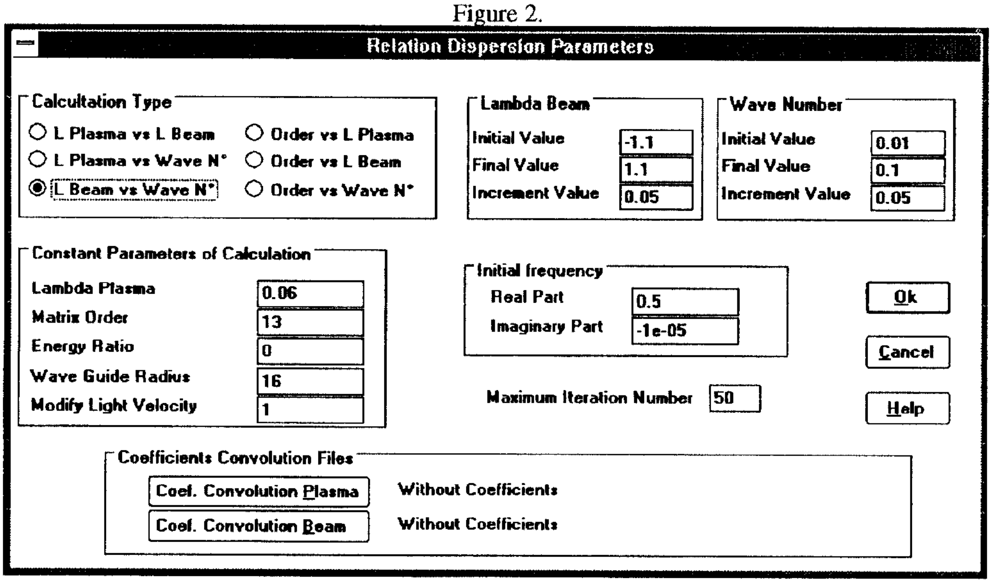


Figure 2.



the relation dispersion of cylindrical and inhomogeneous plasma interacting with electron beam; for this, the code ask for two types of variable parameters, for example Plasma Lambda vs Beam Lambda, or Plasma Lambda and Wave Number, etc.(see fig. 2), and fixed the others parameters as ratio kinetic energy and thermal energy, wave guide radius, modify light velocity, initial frequency values, maximum iteration number and files of convolutions coefficients for the plasma and the beam. This code has the possibility of make a file with the get data in a format of Mathematica Package, for to be used for this package.

4. CONCLUSIONS

We have developed a package for to calculate the relation dispersion for a radial inhomogeneous cylindrical plasma and inhomogeneous beam. The different parameters of the plasma and the beam can be changed easily, as it is illustrate in the fig. 2, this permit research different physical situations. An characteristic of the RELDIS is that the principal code is made in C programming language [10], and the results are plotted using Mathematica package.

5. REFERENCES

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