Effects of Thermal Noises on Electron Trajectories in the SPring-8 Storage Ring

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Abstract

Thermal noises are important and unavoidable ingredients in the density distribution of beam particles in accelerators. To understand their effects is especially important in achievement of low emittance in the SPring-8 storage ring. We treat those noises caused by quantum excitation and RF ripples in a systematic way and obtain criteria on the ripples for the SPring-8.

1. INTRODUCTION

Solving problems caused by noises in RF system in proton storage rings, where phase feedback between beam and RF is essential, is very important to keep long term stability against diffusion [1]. In an electron storage ring, they are negligible in view of stability because the stochastic process of quantum radiation smears out any history of motion. However, to obtain an equilibrium emittance or distribution, RF noises must be included by the same way as quantum excitation. Furthermore, when peaks in frequency spectrum of RF noises exist, they induce systematic oscillations in the longitudinal motion of beam bunches. These oscillations may affect the equilibrium bunch length significantly when a frequency of noises harmonizes with that of the synchrotron oscillation. In this paper, the equilibrium r.m.s. value of a longitudinal distribution is systematically derived when the RF phase noises are significant. These noises are mainly caused by a ripple of the acceleration voltage in klystrons. We obtain quantitative conditions on the ripple to suppress increases in the equilibrium bunch length due to the RF noises below a certain limit.

2. THE MODEL

We start with equations of the linearized synchrotron oscillation in a storage ring with an RF gap,

$$\begin{cases} \Delta\delta\phi_i = 2\pi\Gamma_s^{-1}\,\delta E_i \\ \Delta\delta E_i = -2\pi\Gamma_s \dfrac{\Omega_s^2}{\omega_s^2}\delta\phi_{i+1} - \dfrac{4\pi\alpha_s}{\omega_s}\delta E_i + Q_i \end{cases} \qquad (1)$$

where the variables $\delta\phi_i = \phi_i - \phi_s$ and $\delta E_i = E_i - E_s$ denote deviations from synchronous values of phase with respect to that of RF acceleration voltage and energy, respectively, at the i-th turn. It is noted that the synchronous values ϕ_s and E_s are fixed in our treatment and, therefore, $\delta\phi_i$ is the difference in arrival time of each particle and the synchronous particle at the RF gap in the i-th turn measured by a designed RF frequency. Δ stands for the change in a turn: $\Delta X_i = X_{i+1} - X_i$ for a variable X. ω_s denotes the revolution angular frequency and α_s stands for the radiation damping coefficient for the synchronous particle. Q_i denotes energy fluctuation due to quantum radiation in the i-th turn and we explain it in detail later. Ω_s is the angular frequency of the plain synchrotron oscillation given by

$$\Omega_s^2 = \frac{\omega_s^2}{2\pi\Gamma_s} eV_0(-\cos\phi_s)\,, \qquad \Gamma_s = \frac{\beta_s^2 E_s}{h\eta_s}\,,$$

where eV_0 is the peak acceleration energy of the RF, h the harmonic number and β_s stands for velocity of the synchronous particle in the unit of the light velocity. η_s is given by the momentum compaction factor α and the Lorentz factor γ_s of the synchronous particle as $\eta_s = \alpha - \gamma_s^{-2}$. Numerical values of the fundamental machine parameters for the SPring-8 [2] are listed in Table 1.

Table 1: Fundamental machine parameters for the SPring-8 storage ring. Values (in parenthesis) correspond to single (multi) bunch operation.

E_s	8 GeV
ω_s	$2\pi \times 0.20878 \times 10^6$ rad/sec
h	2436
α	1.460×10^{-4}
α_s	3.236×10^2 sec^{-1}
eV_0	1.7 (1.39) $\times 10^{-2}$ GeV
ϕ_s	2.324 (2.040) rad

The energy fluctuation term Q_i can be written as

$$Q_i = \mu_i\langle u\rangle + \sum_{k=1}^{N_i} f_{i,k}\,,$$

where $f_{i,k} = u_{i,k} - \langle u\rangle$ is the fluctuation in energy of the k-th radiated photon in the i-th turn and $\mu_i = N_i - \langle N\rangle$ is the fluctuation in the number of emitted photons in the i-th turn. Note that $\langle\cdots\rangle$ stands for the ensemble average. Using the fact that N obeys the Poisson distribution, we immediately obtain,

$$\langle Q_i\rangle = 0 \ \text{ and } \ \langle Q_iQ_j\rangle = \delta_{ij}\langle N\rangle\langle u^2\rangle\,. \qquad (2)$$

There are two ways to introduce RF noise in phase in Eq. (1) depending on whether we consider the noise in the reference phase or not. It is necessary to consider this effect when the phase of RF shifts continuously, such as in a case of the shift due to noise in a phase lock loop, and a set of values (ϕ_s, E_s) changes during motion of the synchronous particle. Here we consider, however, that the definition of the synchronous particle (ϕ_s, E_s) is fixed as mentioned previously. We therefore replace the factor $\delta\phi_{i+1}$ in the second line of Eq. (1) by $\delta\phi_{i+1} + \Phi_{i+1}$ with Φ_{i+1} being the noise.

We assume that Φ_i is composed of a white (thermal) noise part and a systematic noise part so that $\Phi_i = \Phi_i^{(w)} + \Phi_i^{(s)}$. The white noise component satisfies

$$<\Phi_i^{(w)}> = 0 \text{ and } <\Phi_i^{(w)}\Phi_j^{(w)}> = \delta_{ij}\,\Delta_\phi^2\,, \quad (3)$$

where the dispersion Δ_ϕ must be determined from characteristics of the RF. The systematic part corresponds to the contribution from peaks in the frequency spectrum of the noise. This drives a forced oscillation and it has a feature of no disappearing correlation between different turns. We write

$$\Phi_i^{(s)} = \sum_k S_k \sin(\omega_k^{(s)} t_i + \psi_k^{(s)}), \quad (4)$$

where $t_i = iT_S$ and $T_S = 2\pi/\omega_S$ is the period of a revolution. The sum is taken over all spectral components, each of which is characterized by its amplitude S_k, angular frequency $\omega_k^{(s)}$ and initial phase $\psi_k^{(s)}$. We consider $\psi_k^{(s)}$ as a randomly distributed variable.

3. SOLUTION TO THE EQUATION OF MOTION

Equation (1) with the introduction of Φ_{i+1} term and specifications of noises as given by Eqs. (2-4) leads to a first order linear Langevin equation and it is straightforward to solve the equation by means of the Green's function of the corresponding Fokker-Planck equation. For the purpose of obtaining second order correlation functions, it is however sufficient to write down a formal solution of Eq.(1). We refer [3] for the details.

4. RESULTS

Dispersions of the phase of revolving particles, σ_ϕ, and of the energy, σ_E, at the stationary state are composed of contributions from quantum emission, white and systematic noises. We write the dispersions in a form

$$\sigma_X^2 = \sigma_{XN}^2 \left(1 + R_X^{(w)} + R_X^{(s)} \right) \text{ for } X = \phi \text{ or } E,$$

where σ_{XN}^2 is the natural dispersion squared, namely the contribution from the quantum emission noise, $R_X^{(w)}$ and $R_X^{(s)}$ are the contributions from the white and systematic RF noises, respectively, normalized by σ_{XN}^2. Explicit forms of these quantities are given as

$$\sigma_{\phi N}^2 = \frac{1 - 2\pi\varepsilon\delta}{1 - \delta'^2 + \delta^2}\,\frac{1}{8\pi\varepsilon^3\delta}\,\frac{\langle N\rangle\langle u^2\rangle}{\Gamma_s^2}\,, \quad (5)$$

$$\sigma_{EN}^2 = \frac{\varepsilon^2\Gamma_s^2}{1 - 2\pi\varepsilon\delta}\,\sigma_{\phi N}^2\,, \quad (6)$$

$$R_\phi^{(w)} = R_E^{(w)} = R^{(w)} = \frac{1 - 2\pi\varepsilon\delta}{1 - \delta'^2 + \delta^2}\,\frac{\pi\varepsilon}{2\delta}\left(\frac{\Delta_\phi}{\sigma_{\phi N}}\right)^2, \quad (7)$$

$$R_\phi^{(s)} = \frac{1}{2}\sum_k\left(\frac{S_k\kappa_k}{\sigma_{\phi N}}\right)^2, \quad (8)$$

and

$$R_E^{(s)} = \frac{1 - 2\pi\varepsilon\delta}{2}\sum_k\left(\frac{S_k\kappa_k\mu_k}{\sigma_{\phi N}}\right)^2, \quad (9)$$

where the small parameters ε and δ are defined in Table 2 and $\delta' = \delta + \pi\varepsilon$. Resonance behavior of $R_X^{(s)}$, which is caused by the systematic RF noise, is conveniently described in terms of a parameter defined as $\mu_k = |\sin(\omega_k^{(s)} T_s/2)|/\pi\varepsilon$. The resonance factor κ_k in Eqs. (8) and (9) is given by

$$\kappa_k^{-2} = (\kappa_k)_{max}^{-2}\left[1 + \left(\frac{\mu_k^2 - \mu_{(0)}^2}{\mu_\pm^2 - \mu_{(0)}^2}\right)^2\right],$$

where

$$(\kappa_k)_{max} = \frac{1}{2\delta}\sqrt{\frac{1 - 4\pi\varepsilon\delta}{1 - \delta'^2}}\,, \quad \mu_{(0)} = \sqrt{\frac{1 - 2\delta\delta'}{1 - 4\pi\varepsilon\delta}}\,,$$

and

$$\mu_\pm^2 = \mu_{(0)}^2 \pm \frac{1}{\sqrt{1 - 4\pi\varepsilon\delta}\,(\kappa_k)_{max}}\,.$$

As it can be seen from these expressions, the contribution to σ_ϕ^2 from the white RF noise is given as $\sigma_{\phi N}^2 R^{(w)} \approx \Delta_\phi^2$, which means that the white RF noise directly vibrates the oscillation center ϕ_S. The amplitude of the systematic RF noise is, on the other hand, modulated by the resonance factor κ_k.

Table 2: Dimensionless small parameters. Values (in parenthesis) correspond to single (multi) bunch operation.

$\varepsilon = \Omega_S/\omega_S$	$0.9071\ (0.6664) \times 10^{-2}$
$\delta = \alpha_S/\Omega_S$	$2.720\ (3.700) \times 10^{-2}$
$\delta' = \alpha_S'/\Omega_S$	$5.569\ (5.795) \times 10^{-2}$

Numerical results in the SPring-8 Storage Ring

To estimate values of $\sigma_{\phi N}$ and σ_{EN}, we make use of relations [4]

$$<u> = \frac{8}{15\sqrt{3}}u_C,\ <u^2> = \frac{11}{27}u_C^2 \text{ and } u_C = \frac{3}{2}\frac{\hbar c\gamma_S^3}{\rho_S}\,,$$

where ρ_S is the bending radius of the design orbit. Substituting ρ_S = 39.2718 m, we have $u_C = 28.9 \times 10^{-3}$ MeV. Average radiation loss per tern, <N><u>, is 12.4 MeV in SPring-8 taking losses in insertion devices into account and we have

$$<N><u^2> = <N><u>\frac{<u^2>}{<u>} = 0.474\ \text{MeV}^2\,.$$

Adopting this value and taking values for other parameters in Tables 1 and 2, we obtain from Eqs. (5) and (6) that

$$\sigma_{\phi N} = 4.29\,[5.83]\times 10^{-2}\ (\text{rad})\,,$$
$$\sigma_{EN} = 8.76\,[8.76]\ (\text{MeV})\,,$$

where numbers (in square brackets) are values for the single (multi) bunch operation. We use this notation throughout the rest of this paper.

Behavior of $R_X^{(s)}$ in Eqs. (8) and (9) is dominated by κ_k^2 in the vicinities of resonance points given as z = 1.000[0.9995] in a coordinate defined by

$$z = \frac{|\omega_k^{(s)} - \nu\omega_s|}{\Omega_s}, \text{ where } \nu = \left[\frac{\omega_k^{(s)}}{\omega_s} + \frac{1}{2}\right]_G,$$

and $[\cdots]_G$ denotes the Gauss's symbol. Note that κ_k^2 as a function of $\omega_k^{(s)}$ in the whole region is obtained from κ_k^2 in the region $0 \le \omega_k^{(s)} \le \omega_S/2$. When we neglect the second and higher order terms of the small parameters, the resonance condition is given as $\omega_k^{(s)} \approx \pm\Omega_s + \nu\omega_s$. This result means that there are two classes of resonance points. One is in a low frequency region near $\omega_k^{(s)} \approx \Omega_s$ and the other is in high frequency regions near $\omega_k^{(s)} \approx \nu\omega_s$. The maximum height of the resonance peak is given as $(\kappa_k)_{max}^2 = 3.380[1.827] \times 10^2$ and the full width of the half maximum of κ_k^2 in $\omega_k^{(s)}$ as FWHM$[\omega_k^{(s)};\kappa_k^2] = 0.5452[0.7420] \times 10^{-1}\,\Omega_S$. $R_X^{(s)}$ is thus not negligible compared with the unity when one of the components k in the systematic RF noise has a frequency $\omega_k^{(s)}$ near a resonance region and the corresponding amplitude S_k is larger than or near to a value $\kappa_k^{-1}\sigma_{\phi N}$, which is $0.05[0.07]\sigma_{\phi N}$ at the peak.

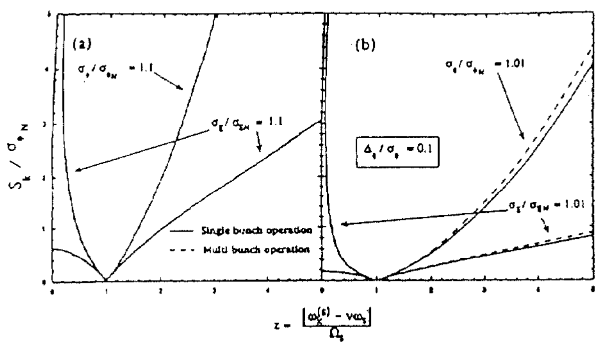

Fig. 1 Contour plots of (a) σ_X/σ_{XN} = 1.1 and (b) σ_X/σ_{XN} = 1.01 for $X = \phi$ and E in the space of $S_k/\sigma_{\phi N}$ and z. $\Delta_\phi/\sigma_{\phi N}$ is fixed to 0.1.

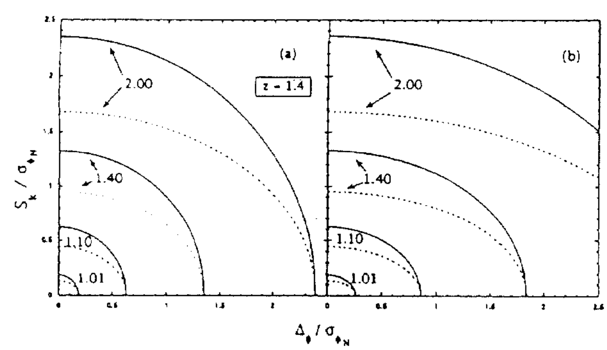

Fig. 2 Contour plots of $\sigma_X/\sigma_{XN} = p$ for (a) single and (b) multi bunch operations in the space of $S_k/\sigma_{\phi N}$ and $\Delta_\phi/\sigma_{\phi N}$. Solid (broken) lines correspond to $X = \phi$ (E). Values of p are shown in the figure. z is fixed to 1.4.

We now consider a condition that the increase of the equilibrium bunch length due to the effect of RF noises must be smaller than a certain limit. This condition can be written as

$$p > \sigma_X/\sigma_{XN} = \sqrt{1 + R^{(w)} + R_X^{(s)}}, \quad (10)$$

for a given value of $p > 1$. In this condition, $R^{(w)}$ is given from Eq. (7) as $R^{(w)} = 0.524[0.283] \times (\Delta_\phi/\sigma_{\phi N})^2$. Since $R_X^{(s)}$ is a function of S_k and $\omega_k^{(s)}$, we obtain criteria on the RF characteristics, Δ_ϕ, S_k and $\omega_k^{(s)}$ to satisfy the condition (10). Figs. 1 and 2 show boundaries of the condition (10) for different values of p in z-$(S_k/\sigma_{\phi N})$ and $(\Delta_\phi/\sigma_{\phi N})$-$(S_k/\sigma_{\phi N})$ planes, respectively. In these figures, we have assumed that there is only one component k in the systematic RF noise. In Fig. 1, $\Delta_\phi/\sigma_{\phi N}$ is fixed to 0.1. This figure shows that $S_k/\sigma_{\phi N}$ is limited to a small value to satisfy the condition (10) when $\omega_k^{(s)}$ is near a resonance region. The condition for $X = E$ gives a limit stronger (weaker) than that for $X = \phi$ in $z > 1$ $(z < 1)$ region. This is due to a factor μ_k^2 in $R_E^{(s)}$ in Eq. (9). The difference between the single bunch operation (solid lines) and the multi bunch operation (broken lines) is small for the present value of $\Delta_\phi/\sigma_{\phi N}$.

For a fixed $\omega_k^{(s)}$ (z = 1.4), the limit on $S_k/\sigma_{\phi N}$ is given as a function of $\Delta_\phi/\sigma_{\phi N}$ in Fig. 2. We observe that the condition for $X = E$ gives again a limit stronger than that for $X = \phi$. This is a general feature of the boundaries for $z > 1$ as we have mentioned above. Comparing Figs. 2a and 2b, we find that the limit is weaker in the multi bunch operation than that in the single bunch operation at large values of $\Delta_\phi/\sigma_{\phi N}$. Absolute maximum of Δ_ϕ for a given p corresponds to the boundary of the condition (10) when $R_X^{(s)} = 0$ and, therefore, is independent of X and z. Its value for p = 1.01 (1.10), for instance, is $(\Delta_\phi)_{max} = 0.84\ (2.71) \times 10^{-2}$ (rad) for the single bunch operation and $(\Delta_\phi)_{max} = 1.56\ (5.03) \times 10^{-2}$ (rad) for the multi bunch operation.

In SPring-8, the phase noise Φ_i is mainly caused by a ripple ΔV of an acceleration voltage V of a klystron. For a typical operation, they are related as [5]

$$\sqrt{\langle\Phi_i^2\rangle} = 20.94(\text{rad}) \times \frac{\Delta V}{V}.$$

The decomposition of the noise into the white and systematic parts is the result of a corresponding decomposition of the ripple. The r.m.s. value of each component of the ripple, $\Delta V^{(w)}$ and $\Delta V^{(s)}$, can be measured by using a power spectrum analyzer. The above equation together with Eqs. (3) and (4) gives

$$\frac{\Delta_\phi}{\Delta V^{(w)}} = \frac{\sqrt{\frac{1}{2}\sum_k S_k^2}}{\Delta V^{(s)}} = \frac{20.94(\text{rad})}{V}.$$

Considering again the case that only one component exists in the systematic noise, we can find boundaries for $\Delta V^{(w)}$ and $\Delta V^{(s)}$ from Figs. 1 and 2 to satisfy the condition (10). As we have observed in the figures, the boundary limits the ripple components stronger for the single bunch operation than for the multi bunch operation. For typical numbers to give criteria on the ripple components, we have for p = 1.01 (1.10) that

$$\Delta V^{(w)}/V < 0.401\ (1.30) \times 10^{-3},$$

when there is no contribution from the systematic noise component and

$$\Delta V^{(s)}/V < 0.29\ (0.94) \times 10^{-3} \times \sqrt{\sum_k 1/\kappa_k^2},$$

when we can neglect the contribution of white noise component.

5. REFERENCES

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