Limitations on Pumping Holes in the Thermal Screen of Superconducting Colliders from Beam Stability Requirements

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Abstract

The paper considers the choice of the shape, size and pattern of pumping holes in collider liners to minimize the coupling impedances while meeting vacuum and mechanical requirements. It summarizes results of analytical, numerical and experimental studies of this issue at the SSCL.

1 INTRODUCTION

Designs of modern high-energy superconducting colliders anticipate a thermal screen (liner) inside the vacuum chamber to screen cold chamber walls from synchrotron radiation. Pumping holes in the liner walls are required to keep high vacuum inside the beam pipe in order to provide for a long beam lifetime. For example, in the present design the LHC liner has more than 100 slots per meter, the total number is about $3 \cdot 10^6$ slots. The SSC liner had two options: 1300 holes or 350 short slots per meter. A thin copper coating of the inner liner walls is anticipated to slow down the resistive-wall instability.

However, the holes are the chamber discontinuities: fields diffracted by holes contribute to the beam-chamber coupling impedances and, therefore, effect beam stability. Due to the large number of holes their contribution to the total impedance of the collider can be significant. The coupling impedances should be minimized to have the stability safety margin large enough and allow for a future upgrade, e.g. higher beam current. A reasonable choice of the hole shape and size, of the number of holes per unit length, and of their distribution pattern has to ensure a compromise between beam stability, on the one hand, and vacuum, mechanical strength and production requirements.

The main concern in high-energy proton colliders is the coupling impedance at low frequencies, below the chamber cutoff, since a typical bunch length is a few times larger than the chamber radius. However, resonances at higher frequencies can cause multibunch instabilities because wake fields excited by a bunch-current perturbation will reach following bunches.

2 LOW-FREQUENCY IMPEDANCE

An analytical calculation of the longitudinal and transverse coupling impedance of small holes in the perfectly conducting walls of the vacuum chamber at low frequencies has been carried out in Ref. [1] for an arbitrary-shaped hole in the chamber with a circular cross section, using the Bethe theory of diffraction by small holes [2] and an expansion over waveguide eigenmodes. The paper [3] gives an alternative derivation and includes effects of wall thickness. In these papers the impedance is expressed in terms of hole polarizabilities, which are purely geometrical factors at low frequencies and can be found by solving a corresponding electro- or magnetostatic problem, e.g. [4]. The longitudinal impedance of a hole in the chamber with the circular cross section of radius b is inductive:

$$Z(\omega) = -iZ_0 \frac{\omega}{c} \frac{(\alpha_m + \alpha_e)}{4\pi^2 b^2} , \qquad (1)$$

where $Z_0 = 120\pi\ \Omega$, and α_e, α_m are electric and magnetic polarizabilities of the hole, respectively. The transverse impedance of the hole is

$$\vec{Z}_\perp(\omega) = -iZ_0 \frac{\alpha_m + \alpha_e}{\pi^2 b^4} \vec{a}_h \cos(\varphi_h - \varphi_b) , \qquad (2)$$

where $\vec{a}_h$ is the unit vector directed to the hole in the chamber transverse cross section containing the hole, φ_h and φ_b are azimuthal angles of the hole and beam in this cross section. It is worth noting that both the longitudinal and transverse impedances are proportional to the sum of polarizabilities, $(\alpha_m + \alpha_e) > 0$.[1] A generalization to an arbitrary chamber cross section [5] shows that the same is valid in any chamber.

For a circular hole with radius a in a thin wall, when thickness $t \ll a$, polarizabilities are [2]:

$$\alpha_m = 4a^3/3 , \quad \alpha_e = -2a^3/3 ,$$

and Eqs. (1) and (2) have very simple form. For the hole in a thick wall, $t \geq a$, the sum $(\alpha_m + \alpha_e) = 2a^3/3$ should be multiplied by a factor 0.56 [3].

There are analytical expressions for polarizabilities of elliptic holes in a thin wall, see [4], and recent study [6] gives thickness corrections for this case. Surprisingly, the thickness factor for $(\alpha_m + \alpha_e)$ exhibits only a weak dependence on ellipse eccentricity ε, changing its limiting value for the thick wall from 0.56 for $\varepsilon = 0$ to 0.59 for $\varepsilon = 0.99$.

For a longitudinal slot of length l and width w, $w/l \leq 1$, in a thin wall useful formulae are obtained in [7]:
for a rectangular slot

$$\alpha_m + \alpha_e = w^3(0.1814 - 0.0344w/l) ;$$

and for a rounded end slot

$$\alpha_m + \alpha_e = w^3(0.1334 - 0.0500w/l) ;$$

substituting of which into Eqs. (1) and (2) gives the impedances of slots. Fig. 1 compares impedances, calculated analytically, for different shapes of pumping holes. Numerical computations which include thick-wall effects give a similar picture, cf. [8].

[1] In fact, it is rather a difference because α_e and α_m have opposite signs.

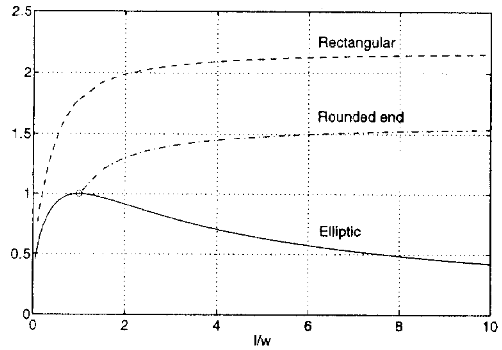

Figure 1: Slot impedance versus slot length l for fixed width w in units of the impedance of the circular hole with diameter w.

Taking into account pumping area of holes, one can conclude that elongated elliptical slots are the best choice. Rounded-end slots are good also, and they are much easier in production. However, very long slots are unacceptable in superconducting colliders due to low mechanical strength of the liner with long slots, and because of their high-frequency impedance, see below.

The low-frequency impedances given by (1) and (2) are in good agreement with simulations, e.g. [8, 9, 11], and measurements [10].

Due to additivity of the impedances at low frequencies, analytical results give reliable estimates of the liner coupling impedances in this frequency range, see Table 1. Estimates [7] are for two versions of the SSC liner: 1300 holes of diameter 2 mm ($M = 16$ holes in one transverse cross section of the chamber) or 350 rounded-end short slots 2×6 mm^2 ($M = 4$) per meter, $b = 1.5$ cm, $t = 1.25$ mm, and pumping area is 4% of the liner surface. For the LHC, the parameters are: 130 slots 1×10 mm^2 per meter (about 2% of the surface), $M = 8$, $t = 1$ mm, effective radius b is taken to be 1.5 cm, and the thickness correction factor 0.6 is used for the estimate. For the SSC the figures should be compared with the Collider impedance budget (without holes): $|Z/n| = 0.68\ \Omega$ and $|Z_\perp| = 40$ MΩ/m, and instability thresholds: 3.7 Ω and 240 MΩ/m.

Table 1: Impedances Produced by Pumping Holes

	$\lvert Z/n \rvert/\ \Omega$	$\lvert Z_\perp \rvert/$ (MΩ/m)
SSC holes	0.13	16
SSC slots	0.05	6.4
LHC	0.003	0.1

3 HIGH-FREQUENCY IMPEDANCE

3.1 Near Cutoff: Trapped Modes

It has been demonstrated recently [12] that a small discontinuity, such as an enlargement or a hole, on a smooth waveguide can result in the appearance of trapped electromagnetic modes with frequencies slightly below the waveguide cutoff frequencies. These trapped modes produce narrow resonances of the coupling impedance near the cutoff. This phenomenon for a waveguide with many small discontinuities, which is a good model for the vacuum chamber with a liner, is studied in [13]. Using results [12, 13], one can estimate the resonance impedance of a liner near its cutoff frequency. As an example, we will refer to the LHC liner. The "effective" area A due to $M = 8$ slots in one transverse cross section is [13]

$$A \to M\psi/(4\pi b) = Mw^2 s/(4\pi^2 b) = 0.135 \text{ mm}^2,$$

where we use transverse magnetic susceptibility $\psi \equiv 2\alpha_m = w^2 l/\pi$ for a narrow long slot in the thick wall, e.g. in [7]. The length of the region which would be occupied by the field of the trapped mode for a single discontinuity with this area is $l_1 = b^3/(\mu_1^2 A) = 4.32$ m. Since it is much longer than the longitudinal separation between adjacent cross sections with the pumping slots, which is $g = 6$ cm, discontinuities strongly interact each other. According to [13], the number of discontinuities, which work as a single combined one, is $N_{eff} = \sqrt{2l_1/g} \simeq 12$, and the new "effective" length of interaction $L = \sqrt{l_1 g/2} = 36$ cm. The frequency shift for the trapped mode, Eq. (15) of [13], is $\Delta f/f_1 = 1.5 \cdot 10^{-4}$, i.e. $\Delta f \simeq 100$ kHz for the cutoff frequency $f_1 \simeq 7$ GHz. The gap between the trapped mode frequency and the cutoff is rather small, but still larger then the resonance width due to the energy dissipation in the walls: $\gamma_1/\omega_1 = \delta/(2b) \simeq 2.5 \cdot 10^{-5}/\sqrt{RRR}$, where δ is skin-depth and RRR is the ratio of the copper conductivities at cryogenic and room temperatures, which is usually 30–100. The radiation width $\gamma_{rad}/\omega_1 \propto \psi_{ext}^2$, see [12], and it is very small, since the external magnetic susceptibility ψ_{ext} is exponentially small compared to the internal one, ψ, due to the thick wall, e.g. [6]. So, the resonance width is small compared to the frequency gap, and the trapped mode exists.

Should discontinuities be far separated, $g > l_1$, the total impedance of the ring would be just a sum of contributions $R_1 = 4Z_0\mu_1 A^3/(\pi J_1^2(\mu_1)\delta b^5)$ [13] from all $N = 2\pi R/g$ discontinuities on the ring (R is the machine radius):

$$\frac{Re\, Z}{n} = \frac{N R_1}{n} = \frac{2\pi b}{g\mu_1} R_1 = \frac{8 Z_0 A^3}{J_1^2(\mu_1)\delta b^4 g} . \qquad (3)$$

Since $g \ll l_1$, the interaction of discontinuities should be taken into account. One should consider each group of N_{eff} discontinuities as a single combined one, and the number of such group on the ring is $N_g = N/N_{eff} = \pi R/L$. Then the estimate follows from Eq. (3) with replacements $N \to N/N_{eff}$ and $R_1 \to N_{eff}^3 R_1$:

$$\frac{Re\, Z}{n} = N_{eff}^2 \frac{2\pi b}{g\mu_1} R_1 = \frac{16 Z_0 A^2}{\mu_1^2 J_1^2(\mu_1)\delta b g^2} , \qquad (4)$$

that gives $Re\, Z/n \simeq 5.5\ \Omega$ for the narrow-band impedance produced by the trapped modes in the LHC liner ($RRR = 100$ is taken). This value for the narrow-band coupling impedance is acceptable.

One can improve and generalize these estimates, considering that the pumping holes are not quite identical, since they have some distribution of areas. It causes a frequency spread of resonances produced by different discontinuities. One can take account of the resonance overlapping using a weighted sum in calculating the total impedance of the ring, e.g. [14]: $Z_{tot}(\omega) = NZ(\omega) \to N\int dAw(A)Z(\omega, A)$, where $w(A)$ is the area distribution, $\int dAw(A) = 1$, and $Z(\omega, A)$ at frequencies near the resonance, i.e. when $\omega \simeq \omega_1 - \Delta\omega_1(A)$, is

$$Z(\omega, A) \simeq i\frac{\delta}{2b}R_1(A)/\left(1 - \frac{\omega_1}{\omega} + \frac{\Delta\omega_1(A)}{\omega} + i\frac{\delta}{2b}\right),$$

where $R_1(A)$ and $\Delta\omega_1(A)$ are the resonance impedance and frequency shift for the trapped mode caused by a discontinuity with area A. If the dissipation is small enough, $\delta/(2b) \ll \Delta\omega_1(A)/\omega \ll 1$, the integral over areas can be treated like a dispersion integral to get

$$Im\int dA \frac{F(A)}{1 - \frac{\omega_1}{\omega} + \frac{\Delta\omega_1(A)}{\omega} + i\frac{\delta}{2b}} \simeq -i\pi \frac{F(A_*)}{\left|\frac{d}{dA}\frac{\Delta\omega_1(A_*)}{\omega}\right|},$$

where $A_* = A_*(\omega)$ is the solution of $\omega = \omega_1 - \Delta\omega_1(A_*)$.

In this way, we obtain two impedance estimates. For far separated discontinuities, i.e. $g \geq l_1$,

$$\frac{Re\,Z}{n} \simeq \frac{4\pi Z_0}{\mu_1^2 J_1^2(\mu_1)} \frac{w(A)A^2}{bg}, \quad (5)$$

with A being the averaged area per discontinuity. This estimate is applicable instead of Eq. (3) only when $\frac{\delta}{2b} < \frac{\mu_1^2 A}{\pi b^4 w(A)}$, otherwise it would give higher value than (3), that is unacceptable because spreading of resonance frequencies reduces the impedance.

For interacting discontinuities, $g \ll l_1$, the estimate is

$$\frac{Re\,Z}{n} \simeq \frac{8\pi Z_0}{\mu_1^2 J_1^2(\mu_1)} \frac{w(A)A^2}{bg}. \quad (6)$$

Surprisingly, it is just twice the result of Eq. (5). For a specific distribution one should take $\max w(A)$ to get maximal impedance estimates (5) and (6). Say, for a Gaussian distribution of areas with standard deviation σ_A, it is $1/(\sqrt{2\pi}\sigma_A)$. If we assume $\sigma_A/A = 0.1$ and apply Eq. (6) for the LHC liner, it gives $Re\,Z/n \simeq 3.5\ \Omega$. This estimate is lower than that from Eq. (4), and it is independent of the wall conductivity.

3.2 *Above Cutoff*

There are two potential sources of impedance resonances due to holes at high frequencies. First, for long enough slots resonances with wavelength $\lambda = 2l$, where l is the slot length, arise. It is reasonable to use relatively short slots in order to move this resonance frequency higher. Moreover, some distribution of the slot lengths could reduce the strength of these resonances.

Resonances of another kind are related to the periodicity of the hole distribution along the liner. This issue was studied in [15] using an analytical model. An exactly periodic structure would have narrow and high resonances (up to $300\sqrt{RRR}\ \Omega$ for the SSC liner with 2 mm holes). Fortunately, the periodicity of pumping holes in the liner is violated by various irregularities like interaction and utility regions, etc. It reduces these resonances significantly (to 8–12 Ω for the above example). Some intentional additional violation of the hole periodicity can further reduce high-frequency resonances (in the extreme of a "random" hole distribution for the SSC liner, these resonances disappear in the background 0.2 Ω). In fact, even small "random" longitudinal displacements (a fraction of radius) of holes from their positions in an exactly periodic array reduce the resonances by the orders of magnitude. The numerical comparison of periodic and "random" hole distributions [9] is in favor of the last one.

It should be also noted that the impedance estimates for the trapped modes in a liner as in a periodic structure would be much higher than those in Sect. 3.1, see in [13]. However, since even small periodicity distortions drastically reduce the resonance coupling impedance of the structure, estimates (4) and especially (6) are more appropriate.

4 CONCLUSIONS

There is a good understanding of the low-frequency coupling impedances of pumping holes in liners. The analytical methods are confirmed by simulations and measurements, and give accurate and reliable impedance estimates in this frequency range. They dictate narrow pumping slots as the best choice.

The impedance behavior at high frequencies depends on hole distribution patterns. In an optimal design one should avoid exact longitudinal-periodic patterns. It is also recommended to introduce additional spreadings both in the hole longitudinal positions and in their sizes.

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