Calculation of Electrostatic and RF Fields in UNDULAC-E with Plane Electrostatic Undulator

E.S. Masunov, A.P. Novikov
Moscow Engineering Physics Institute
Kashirskoe Shosse, 31
Moscow 115409 Russia

Abstract

The results of approximate and numerical calculation of fields in an electrostatic undulator linear accelerator (UNDULAC-E) designed for acceleration of ribbon ion beam are presented. Round and rectangular cross-sections of electrodes are considered. Simulation results obtained by solving the Laplace equation in 2D region allow to estimate the accuracy of analytical models.

1. INTRODUCTION

One of the novel and promising methods to accelerate and focus ion beams with low energy is the use of linear undulator accelerator [1]. Application of electrostatic undulator field along with radiofrequency structures forms a separate class of such accelerators - UNDULAC-E (called earlier lineondutron). For acceleration of ribbon beams UNDULAC-E is based on the plane electrostatic undulator combined with the RF-system. In paper [2] a principle of acceleration in UNDULAC-E is described and the main motion equations are given. Further in [3] a relevant accelerating accelerating structure based on a resonator with transversely located half-wave vibrators is suggested and experimentally tested. The designed specifications of a proton accelerator project are also presented. This paper deals with approximate and numerical calculations of RF and electrostatic fields in UNDULAC-E with a plane electrostatic undulator and transverse RF field for two simple shapes of electrodes: round and rectangular.

2. RF STRUCTURE AND UNDULATOR

A schematic cross-section of UNDULAC-E is depicted in Fig. 1. Here an electrostatic undulator is combined with the RF system forming the transverse electric field. The required field distribution is provided by a system of electrodes, mounted in a resonator and dc-isolated between each other. Showed in Fig. 1 are electrodes in the form of cylindrical rods.

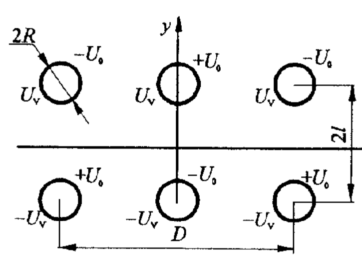

Figure 1. Schematic cross-section of UNDULAC-E

Let us assume that the rods are infinitely long in the x direction (out of the page), i.e. the undulator field is independent of the coordinate *x*. The axis *z* is directed along the beam propagation, the axis *y* characterizes its transverse deviation, *D* is the undulator period. A periodic undulator field is provided by the electrostatic potential differences, imprinted across the adjacent pairs of electrodes. Simultaneously RF-potentials $\pm U_v$ are applied to the electrodes of the upper and bottom row respectively. So the same electrodes are used to generate both the fields. This principal feature of UNDULAC-E imposes some specific requirements on the choice of the electrode shape.

In a periodic system involved the electrostatic and radiofrequency field components can be represented as the sums of space harmonics

$$
\begin{aligned}
E_{y0} &= E_0 \sum_{m=1}^{\infty} g_{2m-1} \cosh(2m-1)ky \cos((2m-1)kz + \theta_{2m-1}),\\
E_{z0} &= -E_0 \sum_{m=1}^{\infty} g_{2m-1} \sinh(2m-1)ky \sin((2m-1)kz + \theta_{2m-1}),\\
E_{yv} &= E_v \sum_{n=1}^{\infty} f_{2n} \cosh 2nky \cos(2nkz + \gamma_{2n}) \sin\omega t,\\
E_{zv} &= -E_v \sum_{n=1}^{\infty} f_{2n} \sinh 2nky \sin(2nkz + \gamma_{2n}) \sin\omega t,
\end{aligned} \tag{1}
$$

where $g_1=1$, $k=2\pi/D$ - the wavenumber, E_0 - the amplitude of the first electrostatic field harmonic, E_v - the amplitude of the zeroth RF-field harmonic, f_{2n}, g_{2m-1} the normalized amplitudes, and γ_{2n}, θ_{2m-1} the phases of the higher harmonics ($n\geq 1$, $m\geq 2$) respectively. Given the symmetry conditions are valid phases γ_{2n} and θ_{2m-1} are 0 or π. The fundamental space harmonics with the amplitudes E_0 and E_v are responsible for the mechanism of ion acceleration and focusing. So the required distribution of electrostatic field should have harmonic character, while RF-field should be purely transverse and uniform over the cross-section. Consequently, on the one hand, the "ideal" electrodes should have a hyperbolic profile, tending to infinity. On the other hand, they should form the field similar to that of a plane condenser. It is evident that these requirements are contradictionary and cannot be realized simultaneously. In practice it seems convenient to use simple profile of electrodes, easy in fabrication, for example, round or rectangular. Then the higher space harmonics will inevitably appear in the harmonic spectra of resultant fields. The values of field harmonics depend on the shape and geometric dimensions of electrodes. From the previous analysis [2] it follows that the higher space harmonics may strongly effects the beam dynamics. So the electrode configuration and positioning should provide both the desired distribution of fundamental electrostatic and RF field harmonics and the appropriate spectrum of higher harmonics. In general case it is possible to obtain numerical solution to the field by directly solving the Laplace equation. However, such a procedure will generally require a large amount of CPU. For preliminary choice of main geometrical parameters it seems useful to develop analytical methods of field determination, including evaluation of harmonic spectrum.

3. CYLINDRICAL ELECTRODES

Let us see a periodic array of infinitely long parallel cylindrical rods of the radii *R* (see Fig. 1). Both the electrostatic and RF potentials are supplied to these rods. We will consider electrostatic and RF fields independently of each other, the latter being treated in quasi-static approximation. The field distribution in the given system can be evaluated with the help of a method of image charges. As it known, the method is based on the concept of imaginary point or line charges, so located that the field of these charges within the region of field evaluation is identical with that of the induced charge on the boundary of conductors. Next the field distribution is calculated subsequently due to resultant system of image charges.

In the simplest analytical model, often used for multiconductor configuration, each rod is represented by a line charge $\pm\tau$ at its center. This model is reasonable accurate, when the distances between the individual rods are large compared with their radii ($l>>R$, $D>>R$). In this case for one pair of rods the transverse field component is straightforwardly determined:

$$E_{y0,v}=\frac{\tau_{0,v}}{2\pi\varepsilon_0}\left(\frac{y-l}{(y-l)^2+(z-D/2)^2}-\frac{y+l}{(y+l)^2+(z-D/2)}\right), \quad (2)$$

where τ is the charge per unit length on the rod, ε_0 is the dielectric constant, l is the transverse coordinate of the rod center. For our case all rods have identical value of τ. Summing up the fields of all rods in our system, one may find the total superimposed field:

$$E_{y0,v}=\frac{\tau_{0,v}}{2\pi\varepsilon_0}\sum_{m=-\infty}^{\infty}c_m\left(\frac{y-l}{(y-l)^2+(z-mD/2)^2}-\frac{y+l}{(y+l)2+(z-mD/2)^2}\right), \quad (3)$$

where $c_m=1$ for RF-field and $c_m=(-1)^m$ for undulator field. Subscripts "0" and "v" denote undulator and RF fields correspondingly.

Let us use the expression

$$\sum_{q=-\infty}^{\infty}\frac{1}{(\alpha+q\lambda)^2+\eta^2}=\frac{\pi}{|\eta|\lambda}\left[1+2\sum_{m=1}^{\infty}e^{-\frac{2\pi m|\eta|}{\lambda}}\cos\frac{2\pi m}{\lambda}\alpha\right] \quad (4)$$

Then after some transformations we may get at $-l<y<l$

$$E_{y0}=-\frac{\tau_0}{2\pi\varepsilon_0}\frac{8\pi}{D}\sum_{m=1}^{\infty}e^{-k(2m-1)l}\cosh k(2m-1)y\cos k(2m-1)z, \quad (5a)$$

$$E_{yv}=-\frac{\tau_v}{2\pi\varepsilon_0}\frac{4\pi}{D}\left(1+\sum_{n=1}^{\infty}e^{-2nkl}\cosh 2nky\ \cos 2nkz\right). \quad (5b)$$

In order to determine the charge of the rods, let us define the potential difference between the pair of rods by integrating (5a) and (5b) at $z=0$ from $y=-(l-R)$ to $(l-R)$:

$$\frac{\tau_0}{2\pi\varepsilon_0}=\frac{2U_0}{\ln\frac{(a-1)(b+1)}{(a+1)(b-1)}}, \quad \frac{\tau_v}{2\pi\varepsilon_0}=\frac{2U_v}{\ln\frac{2a-1}{2b-1}} \quad (6)$$

where $a=k(2l-R)$, $b=kR$. At the limiting transition $D\to\infty$ equations (6) yield the well-known formula for two conductor line. The sums of expansions in (5a) and (5b) can be also reduced to the finite form.

In the necessity to specify the solutions of electrostatic field problem, taking into account the finite radii of rods, it is convenient to apply the method of successive images [4]. This method is an extension of the above approximation. In accordance with that, a series of image line charges which are images of each conductor in the given one is introduced. Rather accurate estimate of fields and individual harmonics can be made considering five neighboring rods. In a similar way as it was done above one may derive the following expression for the transverse component of electrostatic undulator field:

$$E_{y0}=\frac{8\pi}{D}\frac{2U_0}{\ln\left|\frac{AB}{C}\right|}\sum_{m=1}^{\infty}\left(e^{-k(2m-1)h}+2e^{-k(2m-1)l}\cos k(2m-1)z_0-\right. \quad (7)$$
$$\left.-2e^{-k(2m-1)p}\cos k(2m-1)z_2\right)\cosh(2m-1)y\cos k(2m-1)z,$$

where

$$A=\frac{\cosh kR+\cos kz_0}{\cosh kR-\cos kz_0}\,\frac{\cosh k(2l-R)-\cos kz_0}{\cosh k(2l-R)+\cos kz_0},$$

$$B=\frac{\cosh k(h-l+R)+1}{\cosh k(h-l+R)-1}\,\frac{\cosh k(h+l-R)-1}{\cosh k(h+l-R)+1},$$

$$C=\frac{\cosh k(p-l+R)+\cos kz_0}{\cosh k(p-l+R)-\cos kz_0}\,\frac{\cosh k(p+l-R)-\cos kz_2}{\cosh k(p+l-R)+\cos kz_2},$$

$$h=l-R^2/2l; \quad z_0=\frac{R^2}{D/2} \quad p=l-\frac{2R^2l}{4l^2+D^2/4} \quad z^2=\frac{R^2D/2}{4l^2+D^2/4}$$

4. RECTANGULAR ELECTRODES

Let us see next a periodic system of electrodes with rectangular profile (see Fig. 2).

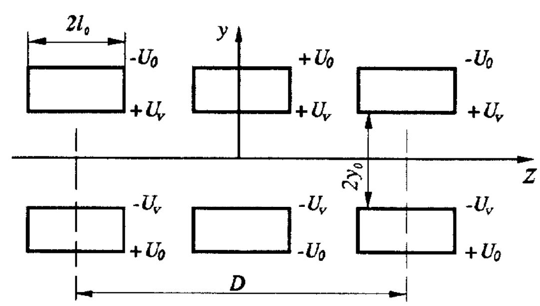

Figure 2. Geometrical dimensions of rectangular electrodes.

We will neglect the rounding of electrodes needed from the viewpoint of electrical rigidity. In such case the longitudinal distribution of electrostatic potential on the electrode boundary (at $y=y_0$) can be approximated by a linear law. Expanding the potential distribution into the Fourier series, one may find the electrostatic field harmonics on the axis:

$$(E_{2m-1})_0=\frac{4U_0}{\pi(2m-1)}\frac{\cos k(2m-1)l_0}{(D/4-l_0)\sinh k(2m-1)y_0}, \quad (8)$$

where l_0 is the half-size of the electrode in the longitudinal direction, y_0 is the half-size of the ribbon aperture.

Let us investigate how the field harmonics vary versus the gap width between the neighboring electrodes. Tending the gap width to zero, we get:

$$\lim_{l\to D/4}(E_{2m-1})_0=\frac{8U_0}{D}\frac{(-1)^{m+1}}{\sinh k(2m-1)y_0}. \quad (9)$$

Therefore in this limiting case if the values of D and y_0 are fixed the amplitude of the fundamental harmonic increases by $\frac{2\pi(D/4-l_0)}{\cos kl_0}$ times, and the ratio of the third and the first harmonics is

$$g_3=\frac{E_3}{E_1}=-\frac{\sinh ky_0}{\sinh 3ky_0}$$

In this case the phase value of the third harmonic in (1) is π.

As it seen from (8), within some range of l_0, $\gamma_3 = 0$. Similarly, from the qualitative considerations it is clear that at $l_0 \to D/4$ the distribution of transverse RF-field is tending to the uniform and $f_2 \to 0$. It is important to note, that in UNDULAC-E with the plane undulator the phase γ_2 is always equal to zero.

5. NUMERICAL RESULTS

Numerical calculation of fields in UNDULAC-E for round and rectangular electrode profiles was carried out using the specific program based on the solution of the Laplace equation in 2D region by the mesh method. At first the potential distributions at the boundary the operating region were calculated. Then the amplitudes and phases of harmonics given by relation (1) were determined having expanded these distributions into the Fourier series on the interval $[-D/2, D/2]$. A number of series members taken was defined by the accuracy of a reverse calculation of the potential on the region boundary. From the comparison of analytical and numerical results one may come to the following conclusions. In practice the error in determining the fundamental harmonic amplitudes for round profile of electrodes using formulas (5a), (5b), (6) is about 10%. A use of the next approximation such as (7) allows to reduce the error up to a few percent. The first electrostatic field harmonic amplitude for rectangular electrode profile given by the expression (8) is calculated with an error $1 \div 5\%$ within the wide range of values D, y_0 and l_0. The accuracy of this formula increases when the ratio of the gap width to the period decreases. An estimate of harmonic spectrum using (5), (6), (7) and (8) is not accurate enough and should be specified numerically. Fig. 3 shows the dependences of relative harmonic amplitudes and phases $f_2\cos\gamma_2$ and $g_3\cos\theta_3$ on the ratio l_0/D at some values y_0/D for electrodes of rectangular profile calculated using the program package. It follows from Fig.3 that the electrodes with rectangular profile are capable of producing the acceptable level of higher harmonics[2]. As it is seen, numerical results are in good agreement with qualitative estimates and conclusions made above.

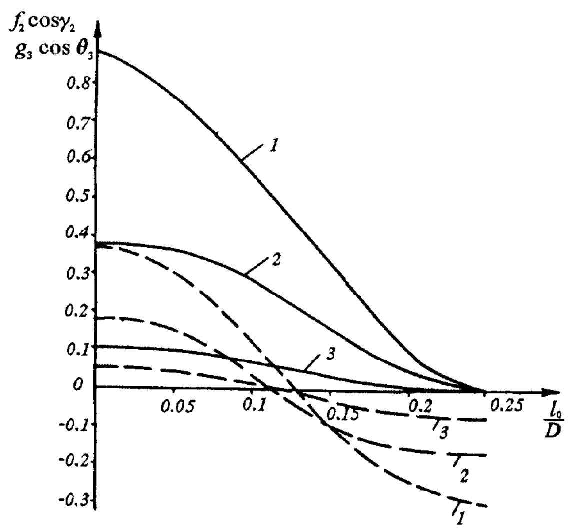

Figure 3. Dependences of $f_2\cos\gamma_2$ (solid line) and $g_3\cos\theta_3$ (dashed line) on the ratio l_0/D at different y_0/D: 1 - 0,05, 2 - 0,1, 3 - 0,2.

6. CONCLUSION

A use of approximate methods of field determination in UNDULAC-E described here makes it possible to choose quite easily and quickly the main geometrical parameters of the accelerating channel. Analytical formulas derived allow to calculate the electrostatic undulator and RF fields for electrodes with round or rectangular profile, including evaluation of relevant harmonic spectra. In a system of cylindrical electrodes both fields were determined using a method of image line charges. For electrodes of rectangular profile some estimates were made assuming a linear variation of electrostatic potential on the electrode boundary. The expressions obtained are convenient in forming the required field harmonic distribution along the accelerator. Numerical simulation results are in good agreement with analytical estimates.

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