RF Field Generation in a Coaxial Cavity by a Micropulsed Electron beam

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Abstract

We have investigated the possibility of generating the radio-frequency (RF) field of an accelerating coaxial cavity by means of a micropulsed electron beam injected and slowed-down into the cavity like in a klystrode output cavity. The source electron beam, injected at low energy and high current, is micropulsed at the cavity eigenfrequency and excites the RF field in a resonant interaction. Calculations have been made for Rhodotron coaxial cavity, leading to interesting efficiency (up to 70%). These good results have been confirmed by experiments.

1. INTRODUCTION

Particule accelerators, except electrostatic accelerators have complicated RF or HF power systems which consist in expensive generators and amplifiers. In the medium power RF domain (some 10's of kW at 100-500 MHz), the most used devices are power grid tubes for which the tunning is not easy and the price often prohibitive. We suggest a device in which a micropulsed electron beam is directly injected into the accelerator cavity and produces RF power at resonance.

2. ELEMENTARY THEORY [1]

The process to generate RF field consists in slowing-down electrons through the cavity. The injected beam is micropulsed at the cavity eigenfrequency and resonant electron power loss contributes to amplify the amplitude of the mode. Generated electromagnetic fields calculations are only possible when electron motion and trajectories are precisely known. The process efficiency can also be evaluated, it only depends on the electron energy loss and on the cavity geometry. In order to maximize the efficiency, the injected beam must have modulated current with a strong Fourier component at the frequency of the excited mode.

2.1. Beam dynamics

We consider the motion of electrons in a resonant cavity. Each electron is described by its position $\mathbf{r}(t)$ and its momentum $\mathbf{p}(t)$. Electrons travel through the cavity, submitted to electromagnetic force. The equation of motion is:

$$\frac{d\mathbf{p}}{dt} = \frac{d(\gamma m_0 \mathbf{v})}{dt} = -e\ (\mathbf{E} + \mathbf{v} \wedge \mathbf{B}) \tag{1}$$

where $\mathbf{E}$ and $\mathbf{B}$ are electric and magnetic fields.

All along the trajectories, not only electron position and momentum are evoluting, but also the electromagnetic field amplitude.

2.2. Electron/RF field interaction efficiency

The interaction efficiency ρ is evaluated by considering the average work of the electrical force applied to one electron averaged on all electrons. This work, calculated for one RF period, is described by:

$$\Delta\overline{W} = \frac{1}{N}\sum_{i=1}^{N}\int_{in}^{out} -e\,\mathbf{E}(r_i,t).\mathbf{v}_i(t)\,dt \tag{2}$$

where N is the total number of electrons injected during one period. We can observe that work is negative maximum when electrical field $\mathbf{E}$ and electron velocity $\mathbf{v}$ have the same direction; in that case the electron is greatly slowed-down. We obtain the following expression for efficiency:

$$\rho = -\frac{\Delta\overline{W}}{W_{in}} \tag{3}$$

where W_{in} represents the injection energy of electrons.

2.3. Energy balance into the cavity

Assuming that the damping of the studied mode (pulsation ω_0 is parametrized by the quantity Q (quality factor of the cavity for this mode) defined as followed:

$$Q = \omega_0 \frac{(\textit{Electromagnetic energy stored in the cavity})}{(\textit{Time average power loss})} \tag{4}$$

the evolution of stored power W_{st} becomes:

$$\frac{dW_{st}}{dt} = -\frac{\bar{I}}{e}\Delta\overline{W} - \frac{\omega_0 W_{st}}{Q} \tag{5}$$

where $\bar{I}$ is the average beam current.

As stored energy is proportional to the square of peak tension U_p of the excited mode, equation (5) can be written:

$$\frac{dU_p}{dt} = -\frac{1}{2}\frac{\bar{I}}{e}\frac{U_p}{W_{st}}\Delta\overline{W} - \frac{1}{2}\frac{\omega_0}{Q}U_p \tag{6}$$

2.4. Transitional regime and equilibrium state

Unfortunately, equation (6) is not sufficient to determine U_p evolution. $\Delta\overline{W}$ calculation requires precise evaluation of the phase φ between RF field and electrons. In simplest cases it is easy to show that φ value corresponds to the maximisation of the electrons slowing-down. But in general case we need another equation.

The solution is to consider the electric field as the sum of two components respectively in phase and in $\pi/2$ phase shifted with the beam:

$$\mathbf{E}(\mathbf{r},t) = (U_{p_1}\cos(\omega_0 t) + U_{p_2}\sin(\omega_0 t))\frac{\boldsymbol{\epsilon}(\mathbf{r})}{\hat{u}_p} \tag{7}$$

where $\boldsymbol{\epsilon}(\mathbf{r})$ represents the spatial normalized mode, $\hat{u}_p$ the peak tension:

$$\hat{u}_p = 2\int_L \boldsymbol{\epsilon}\,dl \tag{8}$$

$\hat{w}_{st}$ is the electromagnetic stored energy of this mode divided by ε_0:

$$\hat{w}_{st} = \frac{1}{2}\iiint_V \boldsymbol{\epsilon}^2 dV \tag{9}$$

Then, stored energy and average work of electrical force appears as the sum of respectively stored energies and works corresponding to the two components U_{pi}.

The decomposition of Maxwell equations on a basis of Laplacien eigenmodes [2] would show that equation (6) is verified by each component U_{pi} of U_p:

$$\frac{dU_{P_i}}{dt} = -\frac{1}{2}\frac{\bar{I}}{\varepsilon_0}\frac{\hat{u}_P}{\hat{w}_{st}}\Delta\hat{W}_i - \frac{1}{2}\frac{\omega_0}{Q}U_{P_i} \tag{10}$$

where

$$\begin{bmatrix}\Delta\hat{W}_1\\ \Delta\hat{W}_2\end{bmatrix} = \frac{1}{N}\sum_{i=1}^{N}\int_{in}^{out}\boldsymbol{e}(\mathbf{r}_i)\begin{bmatrix}\cos(\omega_0 t)\\ \sin(\omega_0 t)\end{bmatrix}.\mathbf{v}_i(t)dt \tag{11}$$

Equation (10) allows us to calcul not only the peak tension evolution but also φ. We can note that there is no more indetermination for $U_{pi} = 0$, the initial derivative being:

$$\frac{dU_{P_i}}{dt}(0) = -\frac{1}{2}\frac{\bar{I}}{\varepsilon_0}\frac{\hat{u}_P}{\hat{w}_{st}}\Delta\hat{W}_i \tag{12}$$

From equation (10), we obtain directly the equilibrium components:

$$U_{P_i}(t\to\infty) = Q\frac{\bar{I}}{\varepsilon_0\omega_0}\frac{\hat{u}_P}{\hat{w}_{st}}\Delta\hat{W}_i \tag{13}$$

The total efficiency is obtained by summing the partial efficiencies relative to the two components of electrical field.

In the case of several eigenmodes in harmonical series, these calculations can be made by taking into account a lot of resonant modes, not only the fondamental one.

2.5. Case of cylindrical coaxial cavity

The interesting mode is TEM_1 for which fields analytical expressions are:

$$E_R = \frac{U_0}{R}\cos\left(\frac{\pi Z}{L}\right)\cos(\omega_0 t + \varphi)$$
$$B_\theta = \frac{U_0/c}{R}\sin\left(\frac{\pi Z}{L}\right)\sin(\omega_0 t + \varphi) \tag{14}$$

Refering to Rhodotron accelerator working principle we define:

$$U_C = 2U_0 Log(R_2/R_1) \tag{15}$$

We can also calculate:

$$Q = \frac{\sqrt{\pi\sigma_C/\varepsilon_0}}{\sqrt{f_0}}\frac{Log(R_2/R_1)}{L/R_1 + L/R_2 + Log(R_2/R_1)} \tag{16}$$

and assuming that:

$$\boldsymbol{e}_R(\mathbf{r}) = \frac{\alpha}{R}\cos\left(\frac{\pi Z}{L}\right) \tag{17}$$

we obtain:

$$\hat{u}_p = 2\alpha Log(R_2/R_1) \quad \text{and} \quad \hat{w}_{st} = \frac{\pi\alpha L\hat{u}_p}{4} \tag{18}$$

Taking $\hat{u}_p = \lambda_{RF} = 2L$, we calculate:

$$\alpha = \frac{L}{Log(R_2/R_1)} \tag{19}$$

When cavity is not exactly cylindrical but with a wide terminated internal cylinder (Fig. 1), we have no more analytical expressions for fields but all the results of Sec. 2 are still valid. Numerical calculations are possible by using a map of electromagnetic field. The advantage of this modified

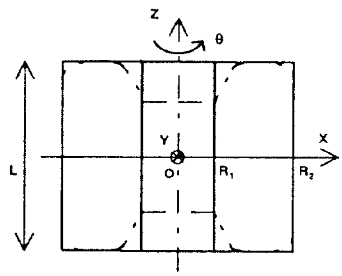

Figure 1 : Cylindrical cavity (continuous line) with a wide terminated internal cylinder (dashed line).

geometry is that higher TEM_n mode frequencies are not multiple of the fondamental TEM_1 mode frequency.

3. Numerical Simulations In Coaxial Cavity

3.1. Generalities

Calculations have been performed for Rhodotron [3] coaxial cavity (Fig. 1).

The aim of this study was to optimize the parameters of the process (injection energy, beam intensity, micropulse duration, place and direction of injection) to maximize efficiency. A numerical code has been developped to solve evolution equations with simplifying assumptions:

- electrons are injected without tranverse velocity ($v_\theta = 0$) with the result that the problem becomes two-dimensional;
- the injected beam radius is zero;
- field variations are calculated with a much slower time-scale than electron motion;
- space-charge effects are not simulated.

It was interesting to search for energy and intensity of the injected beam able to excite typical value of U_c used into the Rhodotron prototype of Saclay (U_p=720 kV needed to reach 500 keV energy gain for each cavity crossing). Good results corresponding to different injection places have been obtained (example on Fig. 2).

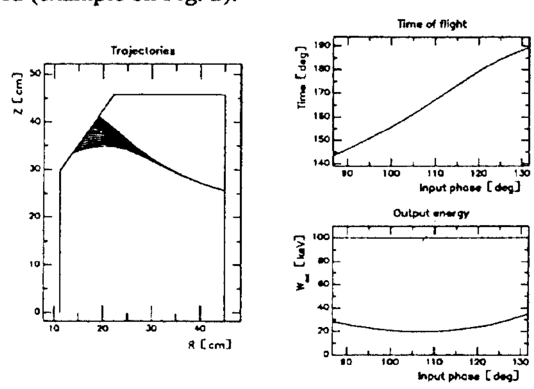

Figure 2 : Injection of a 100 keV-0.6 A beam from external cylinder into a 180 MHz resonant cavity. Excitation efficiency at equilibrium point is 76.1% (Up=737.2 kV). Only a quarter of the cavity is represented.

The efficiency also depends closely on the micropulse phase duration. Without taking account of the space-charge effects, the narrowest pulses give best results (but no improvment with width smaller than some 10's of degrees), with higher peak intensity. Fig. 3 shows efficiency evolution versus pulse duration.

We notice that calculations show the possibily of finding configuration giving efficiency better than 70 %. But the theory doesn't clearly indicate the best compromise between high injection tension and high average current.

Efficiency is obviously limited by space-charge effects leadind to beam blow-up. One solution would be to use several electron guns in parallel all around the cavity. Another one would consist in adapting cavity geometry to the excitation process.

3.2. Simulations for I.B.A. Rhodotron-TT200® cavity

In order to make experimental confirmation of the excitation process, we decided to compare theoritical results to experimentals measurements in the cavity of I.B.A. Rhodotron-TT200® [4]. This accelerator is a high-power electron accelerator producing a 10 MeV-100 kW continuous beam.

The accelerating cavity is a half-wave-length coaxial cavity with wide-terminated internal cylinder. Radius are R_1= 0.21 m and R_2= 0.96 m. The frequency of fondamental TEM_1 mode is about 107.4 MHz and the corresponding measured Q value is about 45000.

As it is not a research machine, it was impossible to inject the beam out of median plan, which is usually used for the beam acceleration. Beam power has been fixed to some 100's of watts, available with the original Rhodotron-TT200® electron gun. Results are presented in Fig. 3.

4. EXPERIMENTAL RESULTS

Experiments corresponding to above simulations have been carried out in Rhodotron-TT200® cavity. Varying parameters were average beam current, duration and frequency of the micropulses.

The beam was produced by a RF gun including a VARIAN-EIMAC Y845 cathode-grid system. The grid was modulated by RF power (some 10's of W) leading to an emitted beam of C-class time structure. For constant beam current, the higher the RF driving power, the narrower micropulses.

Experimental measurements of generated power versus driving power are close to numerical calculations (Fig. 3):

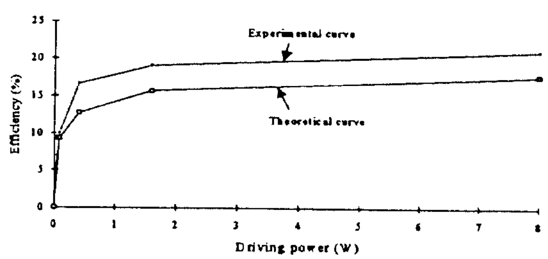

Figure 3 : Efficiency versus driving power for 10 mA-40 keV injected beam.

In order to measure the RF power generated in the cavity, we used an extracting RF loop connected to a 2445 Tektronix oscilloscope. Fig. 4 shows RF signals generated by a 35 kV-10 mA beam injected at different frequencies: at quite exactly the resonance frequency of TEM_1 mode (left), and at a 3-kHz-lower frequency (right)

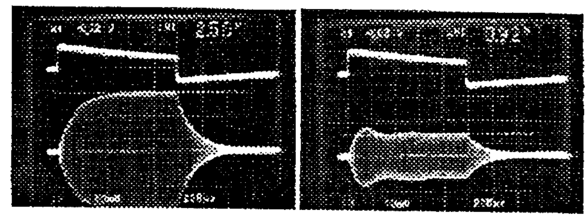

Figure 4 : RF signal generated by a 10 mA-35 keV beam injected at 107.3978 MHz (left) and 107.3948 MHz (right). Upper signal is beam macropulse (1 ms duration) measured by an Intensity Transformer. Lower signal is cavity response. Best results are obtained with driving RF frequency equal to eigenfrequency cavity (left).

The efficiency variation versus RF-driving frequency is shown on Fig. 5. Half-height width corresponds to a Q factor of about 33000.

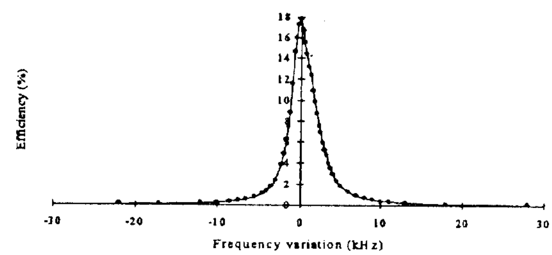

Figure 5 : Efficiency versus RF-driving frequency obtained with 10 mA-40keV beam. Central frequency is 107.4020 MHz.

5. CONCLUSION

We have presented an original method for generating RF field into a resonant cavity without using sophisticated RF power devices. Excitation calculations have been made with simplifying assumptions. They give results in rather good agreement with experimental measurements. Even if hopefull theoretical results (efficiency up to 70%) have been obtained for the case of high-power generation, new experiments must be undertaken. They must confirm that the use of higher beam intensities will be technically possible without degrading theoretical efficiencies.

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