The Space Charge Effect in Slow Extraction by Third Integer Resonance

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1 INTRODUCTION

With the development and construction of high intensive beam accelerators and storage rings more and more attention is being focussed on the problem of the self-field effect of accelerated particles on the stability of their motion. This problem endures second birth, which connected with, on the one hand, requirement to know more exactly the parameters of beam and on other hand, with more powerful computers for an investigation. At first the analytical methods were used in mainly, among which the equations of Kapchinsky and Sacherer take significant place. They gave necessary information about the envelope of high intensive beam with the elliptical distribution. As far as the improvement of computer technology, the new numerical methods have being developed for the solution of the motion and Poisson equations. However, up to right now one run of a modern full dimensions code for ring with some hundreds of elements takes hundreds hours, what makes impossible to optimize the lattice. Many authors synthesize the real tracking with fixed elliptical distribution, when the second moments of density are changed only[1]. Apparently this approach is correcting for the remote resonance system. In the case of being in resonance and even passing through resonance, the elliptical symmetry of distribution can be violated. In the paper [2] we studied the passing through half-integer resonance under different initial distribution with elliptical symmetry, and we have find the different behavior of hallo for each case. Moreover, the change in time of the second moments depends on, what kind of distribution is used.
In this paper we study the excitation of the third integer resonance at high space charge for the slow extraction. We are interested, how to realize the isolated resonance under significant nonlinearity due to space charge.

2 THE CODE DESCRIPTION

It is obviously the third integer resonance changes the distribution significantly, in which it is difficult to recognize the elliptical distribution. We developed and use the four dimensional code for the solution of the motion and the space charge equation together. The motion equation are solved in the Hamilton's form. The Hamiltonian of system is represented in the common form for the curvilinear coordinates:

$$H = (1+hx)[\frac{p_x^2}{2} + \frac{p_y^2}{2}] + (1+hx)\frac{e\Phi_{sc}(x,y)}{m_0c^2\gamma\beta^2} - \frac{e}{p_0c}A_{sc}(x,y) - \frac{e}{p_0c}A_{ex}(x,y), \qquad (1)$$

where A_{sc}, Φ_{sc} - the vector and the scalar potential of the space charge field and A_{ex} -the vector potential of the external field. It is assumed here, that the transverse currents are absent:

$$A_{sc} = \frac{v}{c}\Phi_{sc}(1+hx), \qquad (2)$$

where v is the longitudinal velocity equal for all particles. The vector potential of the external field has components up to the octupole inclusive:

$$-\frac{e}{cp_0}A_{ex}(x,y) = hx + (K+h^2)\frac{x^2}{2} - K\frac{y^2}{2} + \frac{S}{6}(x^3 - 3xy^2) + \frac{O}{24}(x^4 - 6x^2y^2 + y^4) \qquad (3)$$

The Poisson equation is solved on the grid with the metallic boundary:

$$\nabla^2\Phi_{sc}(x,y) = \frac{e}{\varepsilon_0}\rho(x,y) \qquad (4)$$

For the rectangular boundary we use the FFT method for Poisson equation solution. However, there is the option for the arbitrary shape of the boundary, which uses the direct method of the matrix transformation of the equations system written in finite elements. The input coefficient for the code, which defines the space charge parameters and the beam energy, is K:

$$K = 4 * 10^{-7}\frac{I(A)}{\beta^3\gamma^3} \qquad (5)$$

In this representation the Laslet linear shift of frequency equals:

$$\Delta\nu_L = 0.25 * 10^7 K\frac{r_0 R}{2\varepsilon_{rms}} \qquad (6)$$

The output TWISS file of MAD, which describes the lattice parameters, is used as input file of the code. The figure 1 shows the typical screen picture, which gives the visual information in interactive regime about distribution in all planes, losses and the tunes in the horizontal and vertical planes.

3 THE NUMERICAL RESULTS

As example we consider E-ring lattice of the TRIUMF Kaon Factory project, where the sextupoles are used for

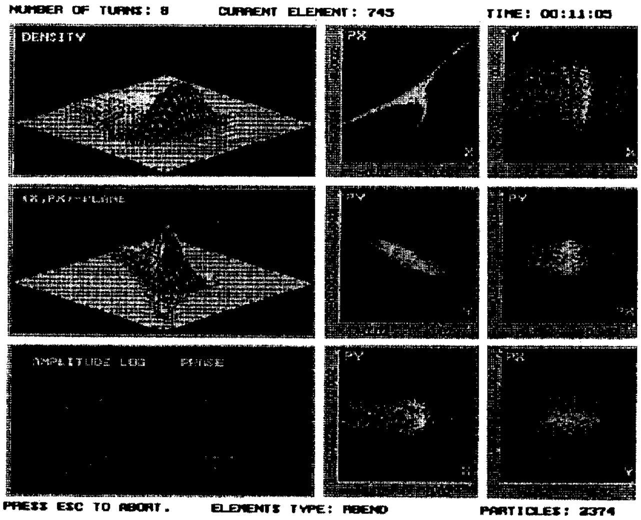

Figure 1: The third integer resonance picture at the interactive regime

the third-integer resonance excitation in the horizontal plane. The extender ring has a racetrack lattice with two straight sections, one of which provides beam the extraction. Both straight sections have zero dispersion function due to special supressors on the arcs. The adjustment of the quadrupoles gives the tune near the integer+1/3 . The influence of the metallic walls on the beam is investigated by changing of the vacuum chamber size. Figures 2 a,b show the phase portraits of the beam in resonance under different significance of the current: a)$\Delta\nu_L$=0.01, b)$\Delta\nu_L$=0.03, c)$\Delta\nu_L$=0.06 and d)$\Delta\nu_L$=0.1. Every time the quadrupoles are set for tune near 10+1/3.From these figures one can see, how the space charge acts on the orientation and the shape of the triangular separatrix. The tails of triangular "star" becomes more wide and with increasing of current we observe at first s-bending and then disappearance of each tail . It is interesting, keeping the total tune on fixed value 10.33, the process of triangular separatrix arising due to the third integer resonance is changed then by the process of the separatrix smearing due to space charge. It has periodical character.The periodicity of the exchanging each by other depends on the current intensity. Due to the smearing the momentum spread of particle in the region of the preseptum magnet became more, than the preseptum magnet make itself, what cases the losses on septum magnet. So C.Ohmori observed the periodical changing of the extracted beam in paper[3].

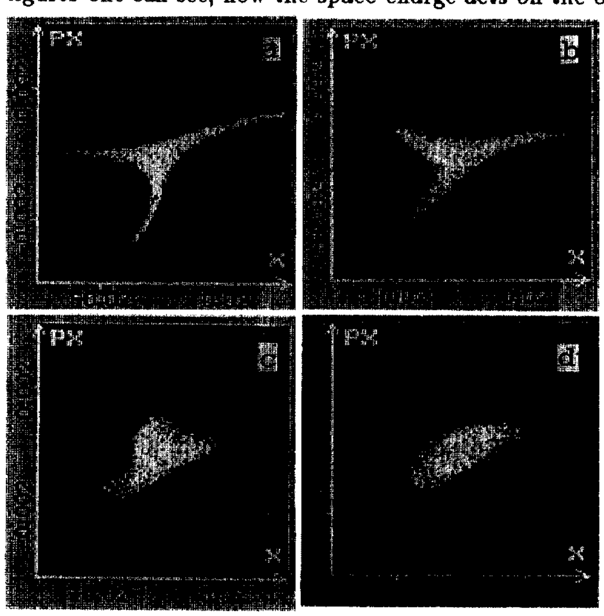

Figure 2: The phase portraits of the beam at a)$\Delta\nu_L$=0.01, b)$\Delta\nu_L$=0.03, c)$\Delta\nu_L$=0.06 and d)$\Delta\nu_L$=0.1

4 HIGH ORDER NONLINEARITY COMPENSATION

So we should compensate the nonlinearity space charge action, in order to increase the beam current threshold of the slow extraction. We suppose, the scalar potential is described by the function:

$$\Phi(x,y,s) = \sum_{m,n}^{N} \alpha_{mn}(s) x^m y^n, \tag{7}$$

Since we study the system with one isolated resonance in x plane only, we can remain in the potential expression all members with n=0 only:

$$\frac{\partial\Phi}{\partial x} = \sum_{m}^{N} \alpha_{m0}(s) m x^{m-1} \tag{8}$$

The smooth approximation $\xi = \sqrt{\beta_x} x$ gives:

$$\frac{d^2\xi}{d\theta^2} + \nu_x^2 \xi = \frac{R^2}{\beta_x^{1/2}}(F_{sc} + F_{ex}), \tag{9}$$

where we assume the length of sextupoles L_s and the octupoles L_o is much less, than the circumference $2\pi R$ of the ring:

$$F_{ex} = \frac{S}{2} f_s(\theta)\beta_x \xi^2 + \frac{O}{6} f_o(\theta)\beta_x^{3/2}\xi^3, \tag{10}$$

$$F_{sc} = \sum_m \alpha_{m0}(\theta) m \beta_x^{\frac{m-1}{2}} \xi^{m-1}, \tag{11}$$

$$f_s(\theta) = \frac{L_s}{2\pi R}[1 + 2\sum_p \cos p\theta] \tag{12}$$

$$f_o(\theta) = \frac{L_o}{2\pi R}[1 + 2\sum_p \cos p\theta] \tag{13}$$

$$\alpha_{m0} = \overline{\alpha_{m0}} + \sum_p \alpha_p \cos p\theta \tag{14}$$

Let's represent the solution of this equation in the standard form of Bogolubov and Metropolsky:

$\xi = a_x \cos\psi$ (15

$$\frac{da_x}{d\theta} = -\frac{1}{2\pi\nu}\frac{R}{\beta_x^{1/2}}\int_0^{2\pi}[F_{ex}(\cos^2\psi, \cos^3\psi, \cos p\theta) + F_{sc}(\cos\psi, ..., \cos^m\psi, ..., \cos p\theta)]\sin\psi d\psi$$

$$\frac{d\psi}{d\theta} = \nu - \frac{1}{2\pi a_x \nu}\frac{R}{\beta_x^{1/2}}\int_0^{2\pi}[F_{ex}(\cos^2\psi, \cos^3\psi, \cos p\theta) + F_{sc}(\cos\psi, ..., \cos^m\psi, ..., \cos p\theta)]\cos\psi d\psi \quad (16)$$

The second integral of averaging don't equal to zero in two cases: 1. for even m, when $< \cos^m\psi >= \frac{C_m^{m/2}}{2^m}$, where $C_m^{m/2} = \frac{m!}{(m/2!)^2}$ is number of the combination of m out of m/2 (for odd m $< \cos^m\psi >= 0$) and
2. for any m, when we are near the resonance , and $\cos(m\psi - p\theta)$ is slowly changing function.
The first integral don't equal zero near resonance only , when $\sin(m\psi - p\theta)$ is slowly changing function.
Then taking into account, that we excite the isolated third integer resonance, we can rewrite the last system of equations for m=3:

$$\frac{da_x}{d\theta} = -\frac{SL_s\beta_s^{3/2}a_x^2}{16\,pi}\sin 3\phi$$

$$\frac{d\psi}{d\theta} = \nu - \frac{SL_s\beta_x^{3/2}a_x}{16\pi}cos3\phi - \frac{OL_o\beta_x^2 a_x^2}{32\pi} + \frac{C_m^{m/2}a_x^{m-2}m\beta_x^{m/2}\overline{\alpha_{m0}}}{2^{m+1}\pi} \quad (17)$$

Thus, knowing the potential expansion in Taylor series,we can always to design the scheme consisting of quadrupoles, octupoles and other even multipoles, which is able to compensate the nonlinearities of space charge.
For example we have estimated, what strength of octupoles are required for a compensation octupole's nonlinearity of the space charge for the distribution:

$$\rho(x, y) = \rho_0 \sum_{n=0}^{\infty}\alpha_{2n}(\frac{x^2}{a^2} + \frac{y^2}{b^2})^{2n} \quad (18)$$

Lowering the intermediate operations, we get:

$$\frac{3}{4}\alpha_4\Delta\nu_L = a^2\sum_i \frac{S_i L_{si}\beta_{xi}}{32\pi}, \quad (19)$$

here index i corresponds to i-th octupole with strength S_i and length L_i, which is placed in the point, where the beta-function is β_i. Since the space charge octupole component is enough high, the correction should be carried out by many octupoles. In particular, for our case we use 40 octupoles. Figure 3 shows the phase portrait of the beam at third integer resonance with and without the octupole correction in the same place after even turns. Comparing these results , we can see more sharp third integer resonance. However, we should correct the strength of octupoles in time turn to turn, since the distribution is changed and the balance of the space charge and the external octupole components is disturbed. Figure 4 shows the dependence of the E_x component of space charge electric field versus transverse coordinate at beginning(a), early(c)

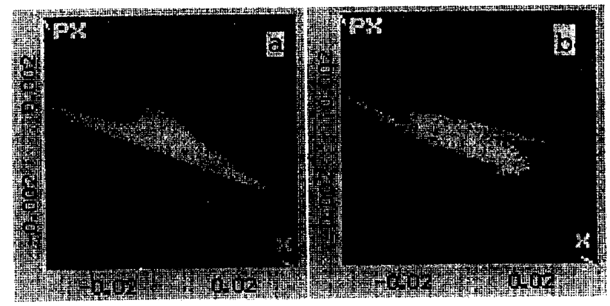

Figure 3: The phase portraits of the beam with (a) and without (b) octupole correction

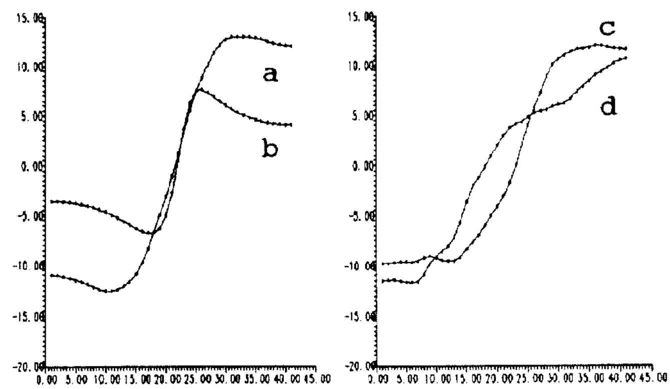

Figure 4: E_x component of space charge force versus transverse coordinate

,steady(d) moments of third integer resonance separatrix forming. One can see, the symmetry of force is disturbed due to the odd orders in distribution. The dipole mode of the space charge force arises. The curve (b) shows begining moment for the free space, when the wall is absent. The significant difference between the results with and without wall says, we should use full scale model for correction of high order nonlinearity.

5 CONCLUSION

We studied the behavior of the beam in the third integer resonance at high influence of the space charge. We suggest to use the octupole correction for the compensation of the space charge nonlinearity. More perfect scheme for correction should include in itself other multimodes. Since only the even orders of the potential give the tune dependence on the radius beam, it is possible to use even multipoles only.

6 REFERENCES

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